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Measuring the Financial Soundness of US Firms 1926-2012∗

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Abstract

Building on the Merton (1974) and Leland (1994) structural models of credit risk, we develop a simple, transparent, and robust method for measuring the financial soundness of individual firms using data on their equity volatility. We use this method to retrace quantitatively the history of firms’ financial soundness during U.S. business cycles over most of the last century. We highlight three main findings: First, the three worst recessions between 1926 and 2012 coincided with insolvency crises, but other recessions did not. Second, fluctuations in asset volatility appear to drive variation in firms’ financial soundness. Finally, the financial soundness of financial firms largely resembles that of non-financial firms.

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1 Introduction

A large literature in macroeconomics argues that financial frictions impair the flow of resources to and across firms and play a key role amplifying and propagating business cycle shocks. Papers in this literature include Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Cooley and Quadrini (2001), Cooley, Marimon, and Quadrini (2004), and many others. One central theme in this literature is that the distribution of financial soundness across firms in the economy at any point in time is an aggregate state variable that has consequences for the response of the macroeconomy to a variety of aggregate shocks. In particular, in these theories, negative macroeconomic shocks are greatly amplified and propagated when they simultaneously deteriorate the distribution of financial soundness across firms.

Motivated by the literature cited above, in this paper we develop a simple, transparent, theoretically-grounded, and broadly-applicable empirical procedure for measuring the distribution of financial soundness across a wide cross-section of firms in the economy and over a long historical time period. We term our measure of an individual firm’s financial soundness at a point in time its Distance to Insolvency or DI. We present a structural interpretation of our measure of a firm’s distance to insolvency in the context of the structural credit risk model of Leland (1994): in that model our measure of distance to insolvency is an approximate measure of a firm’s leverage adjusted for the volatility of innovations to the market valuation of that firm’s underlying assets (also called the firm’s business risk). In this sense, our measure of a firm’s distance to insolvency is a measure of the adequacy of that firm’s equity cushion relative to the riskiness of the firm as a whole.

We then show empirically that our measure of a firm’s distance to insolvency is highly correlated with leading alternative empirical measures of firms’ financial soundness for those time periods in which there is data available for these alternative measures. The alternative measures that we consider include Standard and Poor’s credit ratings, Bank of America - Merrill Lynch data on option-adjusted bond spreads, Markit data on credit default swap rates, Moody’s Analytics data on aggregate bond default rates, and the measure of firms’ distance to default based on the structural credit risk model of Merton (1974) which is commonly used in forecasting individual firms’ bond default and bankruptcy rates (see, for example, Duffie, 2011; Sun, Munves, and Hamilton, 2012). These correlations are strong and relatively stable both in the cross section at a point in time and across time, including even the turbulent financial crisis of 2008.

The primary advantage of distance to insolvency as a measure of firms’ financial soundness relative to leading alternatives is that it requires only data on firms’ equity volatility
and hence can be computed for a very broad set of firms over a very long historical time period. We capitalize on this simplicity of our measure to compute the cross section distribution of distance to insolvency for each month from 1926 through 2012 for all publicly traded U.S. firms listed in the U.S. Stocks Database maintained by the Center for Research in Securities Prices (CRSP) at the University of Chicago. This measurement exercise gives us a new picture of the evolution of the distribution of financial soundness across US firms at a high frequency over a time frame that includes both the Great Depression and a large number of subsequent business cycles.

We find that over the 1926-2012 time period there are a number of episodes in which the entire distribution of distance to insolvency across publicly traded firms in the United States deteriorated sharply. We term these episodes insolvency crises. In particular, we define insolvency crises as months in which the median distance to insolvency in that month’s cross section distribution of DI across firms drops to a level normally associated with firms in extreme financial distress and the 95th percentile of distance to insolvency in the distribution of DI across firms drops to a level normally associated with firms with junk credit ratings or worse.

We then use our measurement procedure to address three empirical questions regarding the relationship between insolvency crises and U.S. business cycles since 1926. First, are U.S. recessions since 1926 systematically associated with insolvency crises? Second, are insolvency crises driven by changes in firms’ leverage or instead by changes in the volatility in the valuation of firms’ underlying assets? Third, are financial firms different from non-financial firms in terms of the behavior of their distance to insolvency both before and during insolvency crises?

With regard to the first question, we find that the largest recessions in our sample, namely 1932-1933, 1937, and 2008, are closely associated with insolvency crises. However, we do not find significant insolvency crises in other recessions outside of these three. This includes even the deep recessions of the late 1970’s and early 1980’s. These findings are not sensitive to the cutoffs used to define insolvency crises — the insolvency crises of 1932-33, 1937, and 2008 are quite distinctive events in the data. These findings are consistent with the hypothesis that financial frictions played a major role in three of the largest recessions in U.S. history. At the same time, these findings cast doubt on the importance of financial frictions for U.S. postwar recessions outside of the most recent one.¹

It has long been recognized in the macroeconomic literature that the distribution of leverage across firms is likely to be a key state variable for determining the effects of

¹See Giesecke, Longstaff, Schaefer, and Strebulaev (2011) for a related quantitative finding based on bond default rates.
financial frictions on the aggregate economy. There is a significant literature that points to the buildup of leverage across firms as a key precursor to the start of a financial crisis (see for example Kindleberger and Aliber, 2005, and Reinhart and Rogoff, 2009). The impact of changes in asset volatility or business risk on firms' financial soundness and financial frictions, on the other hand, has been examined more closely only recently, for example by Bloom (2009), Christiano, Motto, and Rostagno (2010), Gilchrist, Sim, and Zakrajsek (2010), Rampini and Viswanathan (2010), Arellano, Bai, and Kehoe (2011), and others. For the time period 1972-2012, we are able to use our measure of distance to insolvency together with accounting data from COMPSTAT on firms' leverage to examine the role of changes in firms' leverage versus changes in firms' asset volatilities in accounting for the changes in the distribution of firms' distance to insolvency that have occurred over this time period, and in particular, during the insolvency crisis of 2008.

With regard to this second question, contrary to many theories of financial crises, we find that the deterioration of firms' distance to insolvency in the insolvency crisis of 2008 appears to be mainly a result of an increase in asset volatility for all firms. The contribution of an increase in leverage, induced by either “excessive borrowing” or a fall in asset values in this insolvency crisis, was relatively small. In fact, over the entire period for which we have the COMPSTAT accounting data needed to compute firms' leverage (1972-2012), we find that changes over time in the distribution of distance to insolvency across firms are mainly a result of changes in the volatility of firms’ underlying assets rather than of changes in firms' leverage.

We find it striking, moreover, that our measure of the distribution of financial soundness across firms both financial and non-financial rose to historically high levels of soundness in advance of the crisis of 2008 despite the increase in leverage that occurred over this period and that the distribution of financial soundness across firms deteriorates over the periods 1926-1929 and 1995-2000 despite the big increases in stock prices experienced over these time periods. Hence, our findings suggest that measures of leverage traditionally used in macroeconomic analysis that do not adjust for changes in firms’ business risk and a measure of financial soundness such as ours that adjusts leverage for business risk behave very differently over time. On the basis of these findings, we argue that in order to understand insolvency crises, one must account for changes in firms' asset volatility over and above changes in firms' leverage.

The macroeconomic literature cited above highlights the role of financial frictions facing all firms in shaping business cycles. There is also a large literature in macroeconomics making the case that frictions facing financial intermediaries play perhaps an even larger role in shaping the evolution of the macroeconomy. According to this literature, reces-
sions can be caused by a deterioration in the financial soundness of financial intermediaries alone, due to their central role in reallocating resources in the economy. Important papers in this literature include Bernanke (1983), and recent surveys of theory by Gertler and Kiyotaki (2010) and of empirical experience with financial crises by Reinhart and Rogoff (2009).

One of the main virtues of our proposed method for measuring the financial soundness of firms is that it can easily be applied to financial as well as non-financial firms even though the type and reporting of leverage in financial statements varies considerably across the two types of firms. In our empirical work, we apply our method to measure the distribution of financial soundness for publicly traded financial firms from 1926 through 2012. This allows us to address our third question: is the evolution of the distribution of financial soundness significantly different for publicly-traded financial firms than for other firms during and in advance of insolvency crises?

We find that the evolution of the distribution of financial soundness for publicly traded financial firms closely resembles that of non-financial firms. Indeed, over the period 1926-June 1962, the distance to insolvency for the median financial firm is virtually identical to that of the median non-financial firm. Unfortunately, however, there are too few publicly traded financial firms in the data in this time period to draw strong conclusions from this observation. Starting in July 1962, however, there are considerably many more financial firms in the data and one can thus make more meaningful comparisons between the distance to insolvency of financial and non-financial firms. In particular, we find that if one focuses on large firms (as measured by equity market capitalization), then the evolution of the median distance to insolvency for the top 50 financial firms is virtually identical to that for the top 50 non-financial firms over the period July 1962-2012. If instead one focuses on a set of large financial firms identified as “systemically important” ex-post after the 2008 financial crisis, we find that the evolution of the median distance to insolvency for these large financial firms is virtually identical to that of the largest 50 non-financial firms over the period July 1962 - July 2007 and then has been significantly worse over the period August 2007-2012.

Clearly, it is too early to draw strong conclusions from the evidence presented here regarding the question of whether some subset of “systemically important” financial firms plays a special role in insolvency crises — our findings simply show correlations. Despite this, we find these results striking for two reasons.

First, in contrast to the widespread discussion of the role of changes in financial regulation, macroeconomic volatility, and financial innovations over the past several decades in shaping the leverage and risk-taking of large financial firms, we see no evidence in the
distribution of these firms’ distance to insolvency that changes in these factors led to changes in market participants’ perceptions of the financial soundness of large financial firms relative to non-financial firms over the period July 1962-July 2007. It appears that market participants viewed whatever changes in large financial firms’ leverage that may have occurred over this time period as corresponding to offsetting changes in these firms’ business risks that left these firms with effective equity cushions remarkably similar to those of their large non-financial peers.

Second, consistent with many accounts of the current crisis, we do find that starting in August of 2007, institutions recently identified as systemically important ex-post experienced a disproportionate deterioration in their financial soundness and have exhibited a slower recovery of their distance to insolvency relative to their large non-financial peers. Our findings are thus also consistent with arguments in Diamond and Rajan (2011) and Admati, DeMarzo, Hellwig, and Pfleiderer (2012) that increased regulatory efforts have not overcome the problems of debt overhang and moral hazard facing those remaining financial institutions that have been labelled systemically important.

The remainder of this paper is organized as follows. In section 2, we describe the theory underlying our measurement procedure. Our theoretical contribution is to show in the context of Leland’s structural credit risk model that our measure of distance to insolvency, which we define as the inverse of a firm’s instantaneous equity volatility, is an upper bound on the firm’s leverage adjusted by its asset volatility, and that this bound is tight if the firm’s creditors are alert and aggressive in forcing the firm into bankruptcy quickly when the firm becomes insolvent. Because this result is based only on elementary properties of the value of the firm’s equity as a function of the value of the firm’s assets in the model, we conjecture that our result should generalize to a wider class of structural credit risk models.

In section 3 we compare the empirical performance of our measure of firms’ distance to insolvency to alternative measures of firms’ financial soundness. We first map our measure of DI into credit ratings, and provide an interpretation for the levels of DI associated with good and safe, vs. vulnerable ratings. We then provide evidence of a robust and strong linear relationship between the logarithm of a firm’s DI and the logarithm of its credit default swap spreads and likewise with the logarithm of its option adjusted bond spreads. We demonstrate these relationships by constructing each month portfolios of firms by seven credit ratings classes and comparing both in the cross section (across ratings classes) and in the time series the relationship between median log DI and median log CDS swap rates (2001-2012) by ratings class portfolio. We do the same with median log option adjusted bond spreads (1997-2012). Similarly, for these same seven ratings classes, we
compare median DI and median distance to default computed from the Merton model over the 1985-2012 time period and find a strong, stable, monotonic relationship between these two measures. We also document a stable relationship between the distribution of distance to insolvency across firms and aggregate bond default rates as reported by Moody’s Analytics over the full time period 1926-2012.

These comparisons of distance to insolvency to other leading indicators of firms’ financial soundness give us confidence that our measure is both theoretically and empirically meaningful. We then turn to an analysis of the characteristics of the distribution of distance to insolvency across firms as our aggregate state variable of interest. We find that over the entire time period, the distribution of distance to insolvency across firms can be summarized well in a low-dimensional manner. More precisely, we find that, each month over the 1926-2012 time period, the distribution of distance to insolvency across firms is approximately lognormal and hence this distribution of financial soundness across firms can be summarized as an aggregate state variable by two of its moments. Moreover, we find that fluctuations in the median (or mean) of this lognormal distribution account for most of the movements in the entire distribution — the fluctuations in the cross section standard deviation of these lognormal distributions are relatively small. Hence, we find that an insolvency crisis corresponds to a collapse in the median and mean of the lognormal distribution of distance to insolvency across firms with little change in the cross section dispersion of the logarithm of distance to insolvency across firms.

In section 4 we present our empirical results regarding our three questions on the relationship between insolvency crises and business cycles. We conclude with a discussion of the implications of these findings for business cycle research.

2 The Theory Underlying our Measurement

Our empirical work has its theoretical foundations in the structural models of firms’ credit risk pioneered by Merton (1974) and Leland (1994). In those models, a measure of a firm’s leverage adjusted by the volatility of innovations to the market valuation of the firm’s underlying assets is a key state variable summarizing the financial soundness of the firm. This is true both in a statistical sense — this state variable summarizes the probability that the firm will become insolvent in the future — and in an economic sense.

\[2\] We are grateful to Bryan Kelly for pointing this out to us.

\[3\] There is a large literature that uses firms’ leverage adjusted for asset volatility computed from the Merton model to forecast firms’ bond default and bankruptcy rates in a reduced form manner. Duffie (2011) clearly describes one way in which this procedure can be implemented. Moody’s Analytics (a subsidiary of the credit rating agency) has sold the results from a related model under the brand name.
in that this state variable summarizes the distortions to equity holders’ incentives that potentially arise when the firm becomes financially distressed.4

In this section we use a straightforward extension of Leland’s (1994) structural model of credit risk in order to derive two approximation results that dramatically simplify measurement relative to what has been done in the academic literature and in commercial applications. We show that one can approximate a firm’s leverage adjusted for its asset volatility simply with the measure that we call distance to insolvency:

\[ DI \equiv \frac{1}{\sigma_E} , \]

the inverse of the firm’s instantaneous equity volatility. Specifically, we show that in the Leland (1994) model of credit risk, at any point in time, distance to insolvency is an upper bound on the firm’s leverage adjusted for its asset volatility. Second, if the firm’s creditors are aggressive in forcing the equity holders to file for bankruptcy as soon as the firm is insolvent, then this upper bound is tight.5 We argue that because these findings rely on just a few elementary properties of the value of equity, they are likely to hold in a broad class of models.

### 2.1 Leverage Adjusted for Asset Volatility: definition

To define terms, we make use of the following notation. The firm has on the left–hand side of its balance sheet assets which yield at time \( t \geq 0 \) a stochastic cash flow denoted by \( y_t \). Let \( V_{At} \) be the market value of the assets’ future cash flows, measured using state-contingent prices. On the right-hand side of its balance sheet, the firm has liabilities which we model as a deterministic sequence of cash flows \( \{c_t, t \geq 0\} \) which the equity holders of the firm are contractually obligated to pay if they should wish to continue as

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4To generate real costs of financial distress, these models rely on some violation of the Modigliani and Miller Theorem (Modigliani and Miller, 1958). Myers (1977) is an early contribution characterizing the cost of debt financing arising from suboptimal investment. Townsend (1979) studies optimal financing under asymmetric information and shows that debt financing minimizes monitoring costs. Diamond and He (forthcoming) show that investment distortions due to debt overhang vary with debt maturity, and they derive the optimal debt maturity structure. Villamil (2008) presents a survey of some of the important theoretical work which derives violations of Modigliani and Miller from the underlying constraints on contracting and information. Recent work by Almeida and Philippon (2007), Chen (2010), and Bhamra et al. (2010) emphasizes the time varying nature of the costs of financial distress.

5Black and Cox (1976) pioneered the study of structural models of credit risk in which creditors add bond provisions to force equity to exercise their right to limited liability when the firm becomes insolvent. Longstaff and Schwartz (1995) build on the Black and Cox model to incorporate both default and interest rate risk.
owners of the firm. Let $V_{Bl}$ be the market value of the liabilities’ future cash flows, valued as if they were default free. Of course, since the firm may default on its liabilities, $V_{Bl}$ is larger than the market value of the firm’s debt. We say that a firm is solvent if its underlying assets are worth more than the promised value of its liabilities, $V_{At} \geq V_{Bl}$, and insolvent otherwise. Let the asset volatility, $\sigma_{At}$, be the (instantaneous) annualized percentage standard deviation of innovations to $V_{At}$, representing the business risk that the firm faces. Let the leverage be the percentage gap between the value of the firm’s underlying assets and the firm’s liabilities, $\frac{V_{At} - V_{Bl}}{V_{At}}$. A firm’s leverage adjusted for its asset volatility is defined as the ratio of our measure of leverage to our measure of asset volatility, both dated at a point in time $t$:

$$
\left( \frac{V_{At} - V_{Bl}}{V_{At}} \right) \frac{1}{\sigma_{At}},
$$

(1)

This ratio corresponds to the drop in asset value that would render the firm insolvent, measured in units of the firm’s asset standard deviation.

We illustrate these concepts graphically in Figure 1. The solid blue line in the figure denotes the evolution of the value of the firm’s assets, $V_{At}$, over time. The solid blue line ends at the current time $t$. The solid red line denotes the value of the firm’s promised liabilities $V_{Bl}$. The black arrow denotes the distance between $V_{At}$ and $V_{Bl}$ at time $t$. The dashed blue lines denote standard error bands around the evolution of $V_{At+s}$ going forward at different time horizons $s > 0$. The likelihood that the firm becomes insolvent in the near term depends both on the distance between $V_{At}$ and $V_{Bl}$, measured here in percentage terms by the firm’s leverage, and the volatility in percentage terms of innovations to the value of the firm’s assets. We combine these two factors into leverage adjusted for asset volatility which serves as simple one-dimensional index of the firm’s financial soundness.

### 2.2 Leverage Adjusted for Asset Volatility: Measurement

Calculating a firm’s leverage adjusted for its asset volatility is challenging in practice because this calculation requires one to measure the market value and volatility of a firm’s underlying assets, $V_{At}$ and $\sigma_{At}$, and the value of its liabilities, $V_{Bl}$. The former are not directly observable, and the latter is subject to deficiencies and inconsistencies in accounting measures of firms’ liabilities across countries, time, and industries.

One approach to this measurement problem, pioneered by Merton (1974) and Leland (1994), is to use a specific structural model of the cash flows from the firms assets and on the firm’s liabilities together with assumptions about the interest rates and risk prices
used to discount those cash flows to derive formulas for the value of the firm’s equity at \( t \), denoted by \( V_{Et} \), and the standard deviation of the innovations to the logarithm of \( V_{Et} \), denoted by \( \sigma_{Et} \), as functions of the asset value and volatility and the firm’s liabilities, \( V_B, V_{At}, \) and \( \sigma_A \). Given data on the firm’s equity value, equity volatility, and liabilities, one can then invert these formulas to uncover the unobserved asset value \( V_{At} \) and asset volatility \( \sigma_{At} \). Duffie (2011) clearly describes one way in which this procedure can be implemented using the Merton model.

We now describe our two principle results in the specific context of the Leland model.

**The Leland Model.** Let interest rates and the market price of risk be constant. On the left–hand side of the firm’s balance sheet, the cash flows derived from the firm’s underlying assets (lines of business) follow a geometric Brownian motion with constant volatility. In this case, the market value of the firm’s asset, \( V_{At} \), also follows a geometric Brownian motion with constant volatility \( \sigma_A \). In particular, fluctuations in \( V_{At} \) are driven entirely by fluctuations in the firm’s projected cash flows. On the right-hand side of its balance sheet, the firm has liabilities given by a perpetual constant flow of payments \( c > 0 \). Hence, the present value of these payments is constant and equal to \( V_B = c/r \), where \( r > 0 \) denotes the interest rate.

Equity holders have limited liability, in that they can choose to stop making the contractual liability payments, in which case they default and assets are transferred to creditors. Creditors are protected by covenants, allowing them to force equity holders into default if the value \( V_{At} \) of the assets falls below some exogenously given threshold, which we assume is lower than \( V_B \). Using standard arguments, one can show that, when the value of assets falls below some endogenous threshold \( V_A^* \leq V_B \), either equity holders exercise their right to default or creditors exercise their protective covenants. The value of equity can be written as \( V_{Et} = w(V_{At}) \), for some continuous function \( w(V_A) \) with key properties illustrated in Figure 2.

**Lemma 1.** In the Leland (1994) structural model, the value of equity, \( w(V_A) \), is greater than \( \max\{0, V_A - V_B\} \), non-decreasing, convex, and satisfies \( w'(V_A) \leq 1 \) as well as \( w(V_A^*) = 0 \).

The lower bound, \( \max\{0, V_A - V_B\} \), follows from the limited liability assumption: the value of equity has to be greater than zero, and it also has to be greater than \( V_A - V_B \), its value under unlimited liability. Moreover, in line with the original insights from Merton (1974), the value of equity inherits the standard convexity properties of call options.\(^6\)

\(^6\)The Merton model differs from the Leland model only in the assumption that the cash flows on
Note in particular that \( w'(V_A) \leq 1 \), which follows from the fact that the option value of limited liability falls as the value of the firm’s assets rises. Finally, the value of equity must be zero at the default point, \( V_A^\star \).

Armed with these basic properties for the value of equity, we develop our two approximation results which relate distance to insolvency and leverage adjusted for asset volatility. We show each result in turn.

**Proposition 1.** In a Leland (1994) structural model, leverage adjusted for asset volatility is bounded above by the inverse of equity volatility.

\[
\left( \frac{V_A - V_{Bt}}{V_A} \right) \frac{1}{\sigma_A} \leq DI_t = \frac{1}{\sigma_{Et}}.
\]

*Proof.* To prove this result, note first that, by Ito’s formula, the volatility of equity solves:

\[
\sigma_{Et} = \frac{w'(V_A)}{w(V_A)} \sigma_A V_A \Rightarrow \frac{1}{\sigma_{Et}} = \frac{w(V_A)}{w'(V_A)} \frac{1}{\sigma_A V_A}.
\]

By Lemma 1 we have that \( w(V_A) \geq V_A - V_{Bt} \), and \( w'(V_A) \leq 1 \), and the results follow. \( \square \)

Next, consider the question of whether this upper bound on the firm’s leverage adjusted for its asset volatility is tight. To do this, recall that \( V_A^\star \) is the threshold asset value at which equity exercises its option to default: it gives up control of the firm’s assets in exchange for abandoning the firm’s liabilities. We use \( V_A^\star \) to define the concept of *Distance to Default* in the Leland model as

\[
DD_t = \left( \frac{V_A - V_A^\star}{V_A} \right) \frac{1}{\sigma_A}.
\]

(2)

Note that default is distinct from insolvency in our theory and that quite generally a firm’s distance to default exceeds its leverage adjusted for its asset volatility. This is because equity may not walk away immediately from an insolvent firm, but will not choose default if the firm is solvent. With this definition we have our second proposition:

**Proposition 2.** In a Leland (1994) structural model, the inverse of a firm’s equity volatility lies between its Leverage adjusted for its asset volatility and its Distance to Default.

\[
\left( \frac{V_A - V_{Bt}}{V_A} \right) \frac{1}{\sigma_A} \leq DI_t = \frac{1}{\sigma_{Et}} \leq DD_t
\]

liabilities are simply a single cash flow required at a specific date \( T \) in the future. This lemma also applies to the value of equity in the Merton model at dates \( t < T \) with the change that \( V_A^\star = 0 \).
Proof. This proposition follows from the convexity of the value of the firm’s equity as a function of the value of the firm’s assets at each time $t$ and because $w(V_A^*) = 0$. □

We illustrate the proof of these two propositions in Figure 2. At time $t$, the value of the firm’s equity as a function of the value of its assets is a convex function with slope less than or equal to one that lies above the horizontal axis (exceeds zero) and the line $V_A^t - V^t_B$ giving the value of the firm’s equity under unlimited liability. The value of the firm’s equity hits the horizontal axis at the default point $V_A^*$. Define $X_t$ to be the point at which the tangent line to the the value of equity $V_{Et}$ at the current asset value $V_A^t$ hits the $x$-axis. All these lines and points are drawn in this figure.

By the convexity of $w(V_A)$, we have $V_A^* \leq X_t \leq V_B^t$. Simple algebra then delivers that
\[
\frac{1}{\sigma_E t} = \left( \frac{V_A^t - X_t}{V_A^t} \right) \frac{1}{\sigma_A}
\]
which proves the result.

With these two results, we have that the inverse of a firm’s equity volatility, $1/\sigma_E$, i.e., its distance to insolvency, is an accurate measure of its leverage adjusted for its asset volatility if the firm’s distance to default is close to its leverage adjusted for its asset volatility. That is, the bound is tight if creditors quickly force insolvent firms into default. Therefore, as an empirical matter, the economics of creditors’ incentives to force a firm that is insolvent into bankruptcy as soon as possible to avoid further costs of financial distress suggest that firms with alert and aggressive creditors should satisfy this condition.

While we have established our approximations in the context of a simple model, our results rely on just a few elementary properties of the value of equity, which are likely to hold in a broad class of models used in applied work.\(^7\) First, the proof requires that the value of equity be a convex function of the value of assets with slope less than one, a property that is typical of structural credit risk models. Second, the proof requires that the value of equity is the only state variable following a diffusion. Thus our results hold if there are others state variables, for the interest rate, market price of risk, or liability payments, as long as these are “slower moving”, in the sense of being continuous time Markov chains.\(^8\)

In the empirical work that follows, we estimate $1/\sigma_E$ by the inverse of realized volatility,

\(^7\)For example, our first proposition holds in the Merton model. The second proposition holds trivially as well using the definition of distance to default given here. It is standard in applications to use an alternative definition of distance to default in the Merton model reflecting that default can happen only at period $T$ when the single cash flow on liabilities is due.

\(^8\)See for example Chen (2010) and reference therein, for versions of Leland’s model in which the firm’s cash flow process follows, under the risk-neutral measure, a modulated geometric Brownian motion.
which we compute from the CRSP database on daily equity returns for each firm and each month from 1926 to 2012.\textsuperscript{9,10} One may argue that, since the concept of financial soundness is fundamentally forward looking, DI should be measured using implied instead of realized volatility. An important drawback of using implied volatility, however, is that it is available only for selected stocks, and only for recent dates. Moreover, in Appendix C, we compare the distribution of realized vs. implied volatilities, for the available data, and we show that the two track each other closely. We conclude that the benefits of using realized volatility largely outweigh the costs.

3 Distance to Insolvency and Alternative Measures of Financial Soundness

In this section we compare distance to insolvency to leading alternative measures of firms’ financial soundness for those time periods in which we have data for these alternative measures. First, we construct a mapping between the level of DI and Standard and Poor’s credit ratings in the cross section. We use this mapping to interpret the level of DI in terms of these credit ratings.

We next validate our calibration of DI using credit ratings by comparing DI to option adjusted bond spreads and credit default swap spreads. Specifically, we compare the median DI to the median option adjusted bond spreads, and to the median credit default swap rate, every month within portfolios of firms sorted by credit ratings.\textsuperscript{11} This is useful because these market-based measures are more responsive to market conditions than are slow-moving credit ratings.

We find that there is a strong linear relationship between the logarithm of median DI for these portfolios and the logarithm of median option adjusted bond spreads and the logarithm of median credit default swap rates. We find that this relationship is roughly stable both in the cross section and in the time series. This is important because we

\textsuperscript{9}The CRSP daily dataset on equity returns includes NYSE daily data beginning December 1925, Amex (formerly AMEX) daily data beginning July 1962, NASDAQ daily data beginning December 1972, and ARCA daily data beginning March 2006. We estimate \( \sigma_E \) by the square root of the average squared daily returns in the month. We annualize this standard deviation by multiplying by \( \sqrt{252} \) where 252 is the average number of trading days in a year.

\textsuperscript{10}One could also compute realized volatility using a range of alternative methods including a rolling window of returns, the latent-variable approach of stochastic volatility models. We have chosen our measure primarily to ensure that it does not use overlapping daily data and for the convenience of correspondence with the monthly calendar. Moody's uses a much longer window to compute equity volatility in its KMV model.

\textsuperscript{11}By organizing firms into these portfolios, we are able to reduce the impact of sampling error in the estimation of firms’ equity volatility on the empirical relationship between these alternative measures.
are going to argue that one can construct a measure of financial crises based on the unconditional level of an economy-wide measure of DI.

Next we show a strong monotonic relationship between DI and Black and Scholes’ Distance to Default (DD), for our portfolios of firms by credit ratings. Given the large empirical literature that uses distance to default as an indicator of firms’ bond default and bankruptcy risk, we interpret this finding as indicating that DI should also be a strong indicator of firms’ bond default and bankruptcy risk. We directly document the relationship between DI and bankruptcy in the cross section and time series.

### 3.1 DI and credit ratings

To interpret the level of DI, we first study its cross-sectional relationship with credit ratings. Specifically, we compare the inverse of firms’ equity volatility to their credit ratings as reported quarterly in COMPUSTAT. We pool all firm-month observations from 1985 to the present for which we simultaneously have a credit rating from COMPUSTAT and daily stock return data from CRSP. Each month, we place firms into credit ratings bins and then compute the median DI for all firm-month observations by ratings bin.

In Figure 3, we plot the median of the cross-sectional distribution of firms’ DI conditional on Standard & Poors (S&P) credit rating. The figure reveals a clear monotonic relationship between the two: highly rated firms have a higher median DI.\(^{12}\) We emphasize four cutoffs. For highly rated firms (A and above), the median DI is 4. For firms at the margin between investment grade and speculative grade (BBB- vs. BB+), the median DI is 3. For firms that are vulnerable (in the B range), the median DI is 2, while for firms that have filed for bankruptcy and/or have defaulted (C or D) the median DI is 1.

In further support of our calibration, in Figure 4, we plot the frequency in the pooled firm-month data of firms having an investment grade rating (BBB- and above) conditional on values of DI.\(^{13}\) The frequency of firms having an investment grade rating increases sharply with DI for rated firms: it is less than 15% if DI is below 1, and more than 80% if DI is above 4. For DI’s between 1 and 2, this probability is 30%, for DI’s between 2 and 3, it is just under 50% and finally for DI’s between 3 and 4, it is 65%. Thus, a DI below 1 strongly indicates that a firm has a speculative grade rating, and a DI above 4 strongly indicates a firm has an investment grade rating.

Finally, we also consider the frequency in the pooled firm-month data of firms being

\(^{12}\)In fact, our data indicates that monotonicity holds for all percentiles. This means that, in the cross-section, a higher credit rating corresponds to a higher DI, in the sense of first-order stochastic dominance.

\(^{13}\)It is important to note that the unconditional distribution of firms’ credit ratings is biased towards higher ratings since firms select into being rated.
what S&P calls “highly vulnerable” (a rating of CC and below), conditional on values of DI. For firms with DI less than 1, this frequency is about 10% and for firms with DI’s between 1 and 2, it is about 1.4%. To interpret these conditional probabilities, note that the unconditional probability of a rating CC and below is very small, about 0.75%. Taking this into account, a firm with DI below 1 is thirteen times more likely to be highly vulnerable than a randomly chosen firm. A firm with a DI between 1 and 2 is twice more likely to be highly vulnerable.

Given these findings, we propose the following benchmark calibration to interpret the level of DI:

- DI above 4: good and safe.
- DI of 3: borderline between investment and speculative grade.
- DI of 2: vulnerable.
- DI below 1: highly vulnerable.

### 3.2 DI and Bond Credit Spreads

We now consider the relationship between DI and credit spreads in bond yield data. Bank of America-Merrill Lynch (BAML) calculates daily data on option-adjusted bond spreads for a large universe of corporate bonds whose yields underlie BAML’s corporate bond indices. BAML then groups firms into portfolios by rating class, for the seven ratings classes AAA to CCC and below and reports an index of the option adjusted spread on bonds of firms in each portfolio. These daily data on option adjusted bond spreads by ratings class are available from 1997 to 2012.\(^\text{14}\) We compute monthly averages of daily option adjusted spreads on these indices and, in Figure 5, we plot the logarithm of these option adjusted bond spreads against the logarithm of median DI for firms in the same ratings class bin in the same month. We plot separately the pre August 2007 data (with blue triangle) and post August 2007 data (with red circles). Note that there are two sources of variation shown in this figure: variation in DI and bond spreads across credit ratings classes at a point in time and variation over time in DI and bond spreads by ratings class.

Clearly, credit spreads are decreasing in DI, and the relationship is linear in logs. This relationship is relatively tight: the \(R^2\)’s from a regression of log OAS on log DI for data

\(^{14}\)The option adjustment here is intended to correct bond spreads for features of corporate bonds such as callability that do not correspond to default risk and yet might impact observed bond spreads.

\(^{15}\)These data are available in the data repository FRED at the Federal Reserve Bank of St. Louis.
pre and post August 2007 are 0.74 and 0.79, respectively. For another way to see that DI is strongly indicative of credit spreads, note that it is very rare to have a low DI and a low credit spread. In particular, no portfolio has a DI below 1 and an OAS spread below 400bp. Likewise, it is very rare to have a high DI (above 4) and a high credit spread (above 400bp). We conclude from the data DI and OAS, that DI captures a significant amount of the information in credit spreads, and this helps to validate DI as a measure of financial soundness.

Note as well that the linear relationship in logs between DI and bond spreads is quite stable in the data pre August 2007 and post August 2007. We interpret this finding as indicating that during the financial crisis of 2008 both DI and bond spreads as indicators of financial soundness deteriorated over time in the same relative proportion as they do typically at a point in time across the spectrum of firms of different credit qualities.

3.3 DI and Credit Default Swap Rates

In the past decade, a broad market in credit default swaps has emerged. Credit default swaps have a payoff, contingent upon default, which is equal to the value of the defaulted bond relative to its face value. Thus, CDS swap rates offer a natural market-based measure of corporate default risk which we can compare to DI.

We use data from Markit on single-name five-year CDS swap rates from 2001 through 2011. We construct monthly averages of daily swap rates by firm. We then merge this CDS swap rate data with our monthly DI data from CRSP by CUSIP using Markit’s Reference Entity Dataset, then hand and machine-check the results of our merge. Finally, we bin firms by ratings class into seven ratings classes from AAA to CCC and below to reduce noise, and we compute the median CDS spread and median DI monthly by rating class.

Figure 6 plots DI vs. CDS swap rates by ratings class for 2001-2011. We use a log scale because the relationship between DI and CDS spreads is, like the relationship between DI and OAS bond spreads, close to log linear. The plot shows a clear negative relationship between CDS spreads and DI. It also adds further credibility to our calibration, since relatively few observations with a DI less than 2, and very few observations with DI less than one, correspond to a CDS spread below 400bp. Conversely, very few observations have DI greater than 4 and a CDS spread above 200bp. We separate the data pre August 2007 and post August 2007 to support our calibration in levels that are constant over time. The $R^2$ from a regression of log CDS spread on log DI is 0.76 for the data pooled by credit rating pre 2007, and 0.67 post 2007.\(^\text{16}\) Although the slope and intercept coefficients

\(^\text{16}\)The same regression using firm level data for the whole sample yields similar coefficients and an $R^2$ of over 30%.
differ across these two samples, our calibration appears robust since in both time periods a DI below 1 corresponds to a CDS spread of 400bp and a DI above 4 corresponds to a CDS spread below 200bp.

3.4 DI and Bankruptcy

We now consider the relationship between DI and bankruptcy measures. We first compare our measure of DI to Black and Scholes’ Distance to Default (DD). There is a large empirical literature in corporate finance that examines the performance of DD as an indicator of the likelihood that a firm will declare bankruptcy and/or default on a bond. Duffie et al. (2009) and Duffie et al. (2007) document the economic importance of distance to default in determining default intensities.\footnote{Duffie et al. (2007) reports that a 10% reduction in distance to default causes an estimated 11.3% increase in default intensity, and reports that distance to default is the most economically important determinant of the term structure of default probabilities.} Duffie (2011) is an important recent survey of such work. Moody’s Analytics produces and sells estimates of the likelihood that publicly traded firms will default on their bonds using a similar methodology. (See Sun, Munves, and Hamilton, 2012). The work of Bharath and Shumway (2008) suggests that our simple DI measure should capture much of the relevant information in DD about default probabilities.

To calculate DD, we use data on a firm’s equity value and volatility together with accounting data on a firms’s liabilities, and we follow a procedure outlined in Duffie (2011) based on Black and Scholes’s option pricing formula. See Appendix B for details.\footnote{Note that this Distance to Default, which is the one commonly used in the literature, is measured in “log” units. In equation (2) we defined a related measure in levels, to make it directly comparable to DI.} In Figure 7 we show a scatter plot of our computed DD against DI, monthly from December 1985 to December 2012, for the seven rating classes AAA to CCC and below. While the scale obviously differs, since our DI is measured in levels and DD is measured in logs, the figure shows a clear positive relationship between the two.

To confirm that DI has implications for real outcomes, we also briefly document the relationship between DI and bankruptcy.\footnote{See Bharath and Shumway (2008) for a systematic empirical comparison of the of structural and non-structural models of default prediction.} For our purposes, we establish two facts. First, in the cross-section, we show that DI decreases monotonically as firms become closer and closer to bankruptcy. This corroborates our calibration. Then, in the time series, we show that the fraction of firms with low DI is strongly correlated with the aggregate default rate. Thus, even though DI is a market price based measure, and is thus driven both by fundamental risk and potentially time varying risk premia, DI and actual default events...
are related both in the cross section and in the time series. The default decision seems to be related to DI regardless of the driver of DI.

We first examine the evolution of DI as a firm progresses towards bankruptcy. To do so, we merge the data on bankruptcy filings by publicly traded firms collected by Chava and Jarrow (2004) with that in the UCLA-LoPucki bankruptcy database. In Figure 8, we show the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the distribution of the distance to insolvency for those firms that end up filing for bankruptcy in the thirty six months prior to filing for bankruptcy or being delisted. As one can see, these percentiles decline monotonically as bankruptcy approaches. A year prior to bankruptcy, 90% of these firms have a DI that is below the cutoff of 3 for investment grade, and 50% are near the cutoff of 1 for being highly vulnerable. At bankruptcy, all firms have a DI below 2, associated with being vulnerable, and nearly all firms have a DI below 1, associated with being highly vulnerable.

Next, we consider the relationship between the distribution of DI and aggregate default rates. We use Exhibit 30 in Moody’s (2012), which documents annual issuer-weighted corporate default rates for all rated corporations. For comparability, we construct an annual DI series by computing firm level volatilities over an annual window. In Figure 9, we plot the fraction of firms with DI less than one, against Moody’s aggregate default rate series. The figure reveals that the two series are highly correlated, with a correlation of 0.82. Even if we use the fraction of firms with annual DI less than 2, the correlation of this fraction with Moody’s annual default rates is 0.72. Thus, we conclude that the fraction of firms with low DI is highly correlated with realized annual default rates.

4 Financial soundness 1926-2012

We now use our measure of DI to retrace the history of U.S. firms’ financial soundness, from 1926 to the present. Our interest is to characterize the evolution of the cross section distribution of distance to insolvency across firms at a monthly frequency over this time period. We first show that this distribution is approximately log-normal each month from 1926-2012 and hence can be characterized by two moments of the distribution. We then show that most of the movements in this distribution are accounted for by changes in the mean of log DI rather than changes in the cross section standard deviation of log DI. We then define episodes that we term insolvency crises in terms of movements in the distribution of distance to insolvency across firms and examine our three empirical questions regarding the relationship between insolvency crises and business cycles over this long historical time period.
4.1 A log-normal approximation for the cross-sectional of DI

Figure 10 displays the cross-sectional distribution of DI across firms over the 1926-2012 period, by plotting the time series of the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentile cutoffs for firms’ DI. To analyze this distribution in a simple way, we first argue that the log of DI is approximately normally distributed. Figure 10 suggests this, since it shows that the distribution of DI appears to fan out at high levels of DI: the higher percentiles cutoffs are further apart than the lower ones. It is intuitive that taking logs would make the percentile cutoffs more evenly distributed.

More formally, consider the following empirical diagnostic for a log normal distribution, in the spirit of the Kolmogorov Smirnov specification test. If the cross-sectional distribution of DI were truly lognormal, with the estimated mean and standard deviation in each month, then the transformed variable:

\[ N \left( \frac{\log(DI_t) - \text{mean}_t}{\text{dispersion}_t} \right) \]  

should be uniform, where \( N(\cdot) \) is the cumulative distribution of a standard normal distribution. Figure 11 plots the 5th, 10th, 25th, 50th, 75th, 90th, and 95th empirical percentiles cutoffs of this transformed variable over time, for each month 1926-2012. If DI\(_t\) were truly lognormal, then the transformed variable should be uniform and its percentiles should be exactly equal to 0.5, 0.25, 0.5, 0.75, 0.90, and 0.95 in each month. One sees in the figure that this is approximately true: the empirical percentile cutoffs do not deviate much from these values. We thus conclude that DI is approximately log-normal in the cross-section each month. This is convenient, since we can then approximately characterize the entire distribution each month with its mean and standard deviation.

Figure 12 plots the time series of the cross-sectional mean and standard deviation of log DI. One sees that there are some fluctuations in the standard deviation of DI over time, notably as stocks from new exchanges are added to CRSP. However, these fluctuations are much smaller than those of the mean. This suggests that time variations of the cross sectional mean of the log of DI account for most of the time variations of the entire cross sectional distribution of DI. To illustrate this, consider Figure 13, which shows the true 95th percentile of log DI (in red) versus an approximate 95th percentile, calculated assuming that log of DI is normally distributed with a constant standard deviation, equal to the 1926-2012 average. The 95th percentile of a log normal distribution is equal to \( \text{mean}_t + \Phi^{-1}(0.95) \times \text{standard deviation}_t \), so in principle it could be quite sensitive to fluctuations in standard deviation. One sees however that, empirically, most variations of the 95th percentile are accounted for by variations of the mean.
4.2 Insolvency Crises and Recessions

From now on we study time variations in one moment of the DI distribution: the median. We use our calibration to focus on one particular cutoff for median DI, and define a deep insolvency crisis as one in which the median DI falls below 1. In deep crises half of publicly traded firms have a DI associated with highly vulnerable credit rating.

Although we point to particular dates as insolvency crises based on this calibrated cutoff, the entire time series of the median DI can also be used as a continuous measure of the financial soundness of all US firms. This is an advantage over discrete, or “indicator” measures. The median is also a useful summary statistic for the entire distribution. Indeed, recall that log DI is approximately log normal, so the log of median DI is approximately equal to the mean of log DI. Moreover, we established that the cross-sectional standard deviation fluctuates much less than the mean over time. Thus, in a deep insolvency crisis, the large negative shift in median DI is associated with approximately parallel negative shifts, in a log scale, of all other percentiles of DI.

To assess the size of these negative shifts, consider the following calculation. In a deep crisis, log DI is approximately normally distributed, with a cross sectional mean equal to zero, the log of the median, and a standard deviation that is roughly equal to 0.61 (1.8 in level), its historical average. Hence, during a deep crisis about $N\left(\frac{\log 3}{0.61}\right) \approx 96$ percent of publicly traded firm have a DI below 3 associated with speculative grade rating, and about $N\left(\frac{\log 2}{0.61}\right) \approx 87$ percent of publicly traded firms have a DI below 2 associated with a vulnerable credit rating. Hence, deep financial crises are also broad.

Figure 14 plots the median DI over time against a log scale, and shows the deep insolvency crises which occurred in October of 1929 and the Great Depression, the fall of 1937, and the fall of 2008. During these times, 50% of firms became highly vulnerable, with a DI below 1. Thus, in relation to business cycles over this time period, the worst recessions (the Great Depression and the Great Recession) coincide with deep insolvency crises. One can also see in Figure 13 that, in the 1932-33 and 2008 insolvency crises, 95% of firms had a DI below 2, well below the cutoff of 3 for investment grade. Thus, these crises are even broader than suggested by the log-normal approximation with constant cross-sectional standard deviation.

In sum, defining an insolvency crisis as times in which the median DI falls below one, the Great Depression and the Great Recession stand out as singular events. On the other hand, other severe recessions, such as that in 1981-1982, are not associated with insolvency crises. The median DI does indicate one other insolvency crisis in October 1987, corresponding to the dramatic stock market crash of that month, but this episode did not correspond to a recession. This episode also looks milder from the perspective
of how broad the insolvency contraction was; Figure 13 shows that unlike in the Great Depression and Great Recession, the 95th percentile of DI was closer to the cutoff of 3 instead of 2.

This finding that the recessions of 1932-33, 1937, and 2008 are distinctive in being associated with insolvency crises is not particular sensitive to the thresholds used to define insolvency crises. The next candidate episodes that could be termed insolvency crises if a higher threshold is used correspond to the events of WWII and the immediate post-war period leading into the Cold War and the period from 1998-2002 corresponding to the Russian default crisis and the tech stock market boom and bust, neither of which correspond to large sustained recessions.

Next, we investigate the contributions of leverage and asset volatility to movements in the distribution of DI.

4.3 Leverage vs. Asset Volatility

Given the definition of leverage adjusted for asset volatility and the relationship of this concept to DI, an insolvency crisis can occur for two reasons: one due to an increase in leverage (a drop in the equity cushion \( \frac{V_{At} - V_{Bt}}{V_{At}} \)) and the other due to an increase in asset volatility (an increase in business risk, \( \sigma_{At} \)). In this section, we decompose DI into its leverage and asset volatility components to study the contribution of each to the level of DI over time. We provide evidence that the contribution of asset volatility to financial soundness is as much or more important than the contribution of leverage.

Most of the current literature on financial frictions in macroeconomics envisions that the shock that drives a deterioration in the distribution of financial soundness across firms is a decline in asset values \( V_{At} \), and hence an increase in leverage. Moreover, most models of agency costs focus on the effects of changes in leverage alone on managerial and equity holder decisions. Our empirical decomposition below suggests that, in order to understand insolvency crises, this literature must consider shocks that not only increase leverage but also increase firms’ asset volatility.

We begin with a simple benchmark, assuming unlimited liability, and then compare asset volatility under this assumption to asset volatility using Black and Scholes’ model to compute the value of the option to default.\(^{20}\) Under unlimited liability, \( V_{At} = V_{Et} + V_{Bt} \), and thus we can decompose log DI into leverage and asset volatility simply using:

\[
\log \left( \frac{1}{\sigma_{Et}} \right) = \log \left( \frac{V_{At} - V_{Bt}}{V_{At}} \right) + \log \left( \frac{1}{\sigma_{At}} \right).
\]

\(^{20}\)See Appendix B for details.
Specifically, in this decomposition, the mean of log DI, which we have argued is a good proxy for the position of the entire cross section distribution of DI, is equal to the mean of log leverage and the mean of the log of firms’ inverse asset volatility.

As in our calculation of Black and Scholes’ DD above, we use quarterly COMPUSTAT data on total liabilities as an estimate of $V_{Bt}$ and we use daily equity values at the end of the quarter to compute $V_{Et}$. Note that although our estimate of $V_{Bt}$ based on book values might be slow moving, our estimate of leverage moves on a daily basis due to fluctuations in $V_{Et}$. Figure 15 plots the mean log distance to insolvency and asset volatility terms in equation (4), for the 1972-2012 time period. Clearly, most of the changes in the level of DI is due to changes in the level of asset volatility, especially in more recent data.

To see more clearly the relative contribution of leverage and asset volatility, Figure 16 calculates a counterfactual time series by shifting the median log $(1/\sigma_E)$ up by a constant, so that it has the same historical mean as the median log $(1/\sigma_A)$, thus obtaining a “constant-leverage” measure of log $(1/\sigma_E)$. The figure strongly suggests that most of the variation in DI are accounted for by variation in asset volatility. Of particular interest is the role of leverage vs. asset volatility in the insolvency crisis of 2008. Figure 16 shows that this crisis was almost entirely due to an increase in asset volatility. This is in contrast to common narratives in the financial press and academic literature, which emphasize the role of an increase in leverage due to a fall in asset values in driving the deterioration in financial soundness in 2008.

One may wonder whether these decomposition results are biased by our assumption of unlimited liability. To address this concern, Figure 17 plots option adjusted asset volatility. The figure shows that the option adjustment using Black and Scholes’ model is very minor.

As further evidence of the major contributing role of asset volatility over leverage in determining firms’ financial soundness in 2008, we use the decomposition under unlimited liability to compare the percentiles of the cross-sectional distribution of DI in October 2008, to the cross-section distribution of DI in October 2008 that would have occurred if leverage for each firm had remained at its level from October 2007 and only asset volatility had risen to its level in October 2008.

These percentiles are shown in Figure 18. The first column of colored bars shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the cross-sectional distribution of DI in October 2007. The second column of colored bars shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the cross-sectional distribution of DI in October 2008. The third column of colored bars shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the cross-sectional distribution of DI computed firm-by-firm using that
The percentiles of this counterfactual cross-sectional distribution shown in the third column are quite similar to those found for the actual distribution in October 2008 (shown in the second column) and quite different from those found for the cross-sectional distribution in October 2007 (shown in the first column).

This cross-sectional decomposition provides further evidence that the collapse in the distribution of DI in the fall of 2008 is primarily due, in an accounting sense, to an increase in asset volatility rather than an increase in leverage.

The VIX is often used as a measure of financial stability and/or macroeconomic uncertainty. Figure 19 plots our measure of inverse equity volatility for returns on the value weighted portfolio from CRSP along with the median DI in the cross section distribution across individual firms. (We use the DI of the value weighted portfolio rather than VIX since it is available back to 1926. For recent data the comparison using VIX is very similar.) As one can see in the figure, DI for the value weighted portfolio closely tracks median DI in insolvency crises. Thus, one interpretation of our findings is that we are giving a structural interpretation of why high VIX states are bad macroeconomic states. When the VIX is high, firms have smaller effective equity cushions because firms’ underlying business risk is substantially higher, and expected costs of financial distress are higher as a result.

4.4 Financial vs. Non-financial Firms

A large literature in macroeconomics and finance argues that, when financial intermediaries are financially unsound, they amplify and propagate negative shocks to the real economy. In fact, a commonly held view is that the weak financial soundness of financial intermediaries was the root cause of the large recessions of 1932-33, 1937, and 2008. A growing literature also argues that changes in regulation, and/or the introduction of new financial products, changed the risk taking behavior of financial institutions.

To shed light on the relative financial soundness of financial vs. non-financial firms over time, we compare the distribution of DI over time for financial and non-financial firms.\textsuperscript{21} An advantage of our measure is that we do not require accounting or market value information for liabilities, which are hard to measure properly for financial firms. However, like any market based measure, our measure of DI based on equity volatilities is influenced by the presence (implicit or explicit) of government subsidies.\textsuperscript{22} We also

\textsuperscript{21}See Giammarino et al. (1989) for an early contribution using a structural model of default to consider market implied valuations of bank assets, and the value of deposit insurance.

\textsuperscript{22}Kelly et al. (2013) and Lustig and Gandhi (forthcoming) present evidence that government subsidies
acknowledge that the use of market based signals for regulation is subject to the usual caveats regarding adverse feedback loops between agents’ actions and market prices.\textsuperscript{23}

We begin by classifying financial firms as those firms in CRSP with an SIC code in the range of 6000-6999, and comparing the log median DI for financial firms and non-financial firms. We measure the Distance to Insolvency for these financial firms in exactly the same way that we do for all firms. Figure 20 graphs the log median DI for financial and non-financial firms from 1926-2012. The two series coincide for the first half of the sample, and in the second half, financial firms appear more sound than non-financial firms outside of the recent financial crisis. One challenge in interpreting this graph is that the characteristics of the two firm populations are likely to be quite different. Another challenge is that many firms with SIC codes from 6000-6999 are not banks, or at least not typical financial firms. We address these challenges in two ways. First, we compare large financial firms to large non-financial firms. Then, we study a set of firms which we call “government backed large financial firms”, or GBLFI’s.

Figure 21 plots the log median DI for the largest 50 financial and non-financial firms by market capitalization from 1962-2012, the period for which there are enough large firms of each type. The main message from this graph is that the median financial soundness of large financial and large non-financial firms was quite similar over this time period. Taken together, it is hard to argue from the evidence in Figures 20 and 21 that changes in bank regulation or financial innovations, lead large financial firms to add leverage relative to their business risk in a manner different from their large non-financial peers.

Our final comparison is to a set of \textit{ex-post} systemically important institutions which we term \textit{government backed large financial institutions} or GBLFI’s. This set of institutions is comprised of the 18 bank holding companies that currently participate in the Federal Reserve’s annual stress tests and eight large financial institutions that failed during the crisis (AIG, Bear Stearns, Fannie Mae, Freddie Mac, Lehman, Merrill Lynch, Wachovia, and Washington Mutual). The full list of GBLFI’s together with the dates for which data on their equity returns are available is provided in Table 1.

Figure 22 plots the log median DI for the GBLFI’s and the 50 largest non-financial firms by market capitalization from 1962 to 2012. Again, there does not seem to be evidence in market prices of increased risk taking by the GBLFI’s relative to non-financial firms over the period July 1962-July 2007. However, it does appear that the distance to insolvency for the GBLFI’s deteriorated relative to their non-financial peers starting in

\textsuperscript{23}See Bond et al. (2010), which provides an equilibrium analysis of the use of market signals in regulation.
August of 2007 and fell to an extremely low level in the depth of the crisis from October 2008-March 2009. Moreover, these firms have been slower to recover their DI since that time.

One goal of financial regulation is to identify relatively weak financial institutions in the cross-section either before a crisis begins or during the crisis. We are skeptical that regulators can achieve this goal because we find that most of the movements of DI even for the GBLFI’s are systemic in nature — DI for all of these institutions moves closely together. Figure 23 plots the 90th, 50th, and 10th percentiles of the distribution of DI for the GBLFI’s. The figure presents clear evidence that the cross-sectional variation in DI for these GBLFIs in any given month is quite small relative to the movement in the distribution of DI over time: during this time period the risk that any one GBLFI is unsound relative to the others is small relative to the risk that the whole group of GBLFIs becomes unsound together. This pattern is particularly apparent in the fall of 2011: these figures indicate that the whole group of GBLFIs was nearly as unsound at that time as they were in early 2008 or mid 2009.

5 Conclusion

This paper is intended as a contribution to measurement: we propose a simple and transparent method for measuring the financial soundness of firms that can be broadly applied to all publicly traded firms in the economy.

We identify three recessions in which a macroeconomic downturn coincides or follows shortly after a substantial insolvency crisis: 1932-33, 1937, and 2008. We find that the other recessions in this time period are not associated with significant deteriorations insolvency crises. Of course, since our findings uncover only a correlation (or lack thereof) between insolvency crises and recession, they do not establish causation. We do, however, see our findings as consistent with the hypothesis that financial frictions may have played a significant role in the recessions of 1932-33, 1937, and 2008, and that financial frictions (as envisioned by current theories) did not play a significant role in other recessions during this time period. We hope that our research will provoke more detailed studies of the differences between these three recessions and other recessions to see if a stronger empirical and theoretical basis for causal links between financial frictions and the evolution of the macroeconomy can be developed.

A decomposition of our distance to insolvency measure into its leverage and asset volatility components attributes most all of the 2008 insolvency crisis to an increase in asset volatility, or business risk. Distortions to managerial and equity holder decisions
occur when the likelihood of insolvency is high for either reason. Thus, considering the only effects of leverage on agency costs may leave out quantitatively important variation due to time varying asset volatility. We see it as a question of first-order importance for future research to understand the sources of these large changes in asset volatility.

We also find little or no evidence that the evolution of financial soundness across financial firms differs meaningfully from that for all firms, even during the three crisis episodes. In the recessions of 1932-33, 1937, and 2008, the timing and magnitude of the insolvency crisis was the same as that for all firms, financial or non-financial, large or small. We find only weak evidence that the distribution of financial soundness for a set of “systemically important financial institutions” deteriorated in a distinctive manner in advance of the most recent financial crisis.

Finally, we find it distressing that government-backed large financial institutions continued to appear weak in terms of their financial soundness since the summer of 2007, in spite of the heightened regulatory scrutiny they have received. Why it is that these firms continue to look financially weak relative to their peers is an open question that calls for further research.
A Leland (1994) structural model

Under the true "physical" measure, the value of the firm’s assets, $V_A$, follows a Geometric Brownian motion with drift $\mu_A$ and volatility $\sigma_A$. The firm pays a dividend $\delta V_A$ per period. Under the risk-neutral measure, the value of the firm’s assets follows

$$dV_{At} = (r - \delta)V_{At} dt + \sigma_A V_{At} dB^Q_t.$$ 

The intuition for the risk neutral drift of $r - \delta$ is simply that, under the risk neutral measure, the expected return from buying the assets at $V_{At}$, selling at $V_{At+dt}$ and receiving the dividend flow $\delta V_{At} dt$, should be equal to $r dt$. Assume that the equity holders have to pay $c$ (per unit of time) to the debt holders until either (i) equity holders choose to default or, (ii) creditor exercise their right to force equity holders to default, when the value of asset reaches a protective covenant threshold $V_{A^P}$. Let $\tau_P$ be the first time asset value falls below the protective covenant threshold, $V_{A^P}$. Equityholders’ problem is to choose a stopping time $\tau$ in order to solve

$$w(V_A) = \sup_\tau \mathbb{E}^Q \left[ \int_0^{\tau \land \tau_P} (\delta V_{At} - c) e^{-rt} dt \right].$$

Consider equity holders starting with two different initial levels of assets, $V_{A0} < V_{A0}'$. Clearly, the equity holders starting with $V_{A0}'$ can always mimic the policy of equity holders and creditors starting at $V_{A0}$ and would earn a higher payoff, implying that $w(V_A)$ is non-decreasing. This also shows that an optimal policy is of the threshold form: there is a $V_A^*$ such that when $V_A \leq V_A^*$, equity holders default, or are forced into default by creditors, and continue operating the firm otherwise. Thus, the Bellman equation for the value of equity is:

$$V_A \leq V_A^* : w(V_A) = 0$$

$$V_A \geq V_A^* : rw(V_A) = -c + \delta V_A + w'(V_A)(r - \delta)V_A + w''(V_A)\frac{\sigma_A^2}{2} V_A^2.$$ 

A particular solution to the second-order ODE is $V_A - V_B$, where $V_B = c/r$. The general solution of the corresponding homogenous ODE is of the form $K_1 V_A^\theta + K_2 V_A^{-\theta}$, where $K_1$
and $K_2$ are constant, while $\phi$ and $\theta$ are the positive roots of:

$$
\frac{\phi^2 \sigma_A^2}{2} + \phi \left( r - \delta - \frac{\sigma_A^2}{2} \right) - r = 0
$$

$$
\frac{\theta^2 \sigma_A^2}{2} - \theta \left( r - \delta - \frac{\sigma_A^2}{2} \right) - r = 0.
$$

When $V_A \to \infty$, the value of equity has to asymptote to $V_A - V_B$, implying that $K_1 = 0$. The constant $K_2$ is found by value matching $w(V_A^*) = 0$, which delivers:

$$
K_2 = f(V_A^*) \text{ where } f(x) = -(x - V_B) x^\theta.
$$

The optimal threshold maximizes $f(x)$ subject to $x \geq V_A^P$. Differentiating $f(x)$ with respect to $x$ reveals that it is hump shaped and reaches a unique maximum at $\frac{\theta}{1+\theta} V_B$. Therefore, the optimal threshold is:

$$
V_A^* = \max \left\{ V_A^P, \frac{\theta}{1 + \theta} V_B \right\} \text{ and } w(V_A) = V_A - V_B - (V_A^* - V_B) \left( \frac{V_A}{V_A^*} \right)^{-\theta}.
$$

Convexity follows because $V_A^* \leq V_B$ by our assumption that $V_A^P \leq V_B$. Simple calculation show that $w'(V_A^*) \geq 0$ and that $w'(\infty) = 1$, implying that $w(V_A)$ is non-decreasing and has a slope less than one.

### B Option adjusted asset values and volatilities

We calculate option-adjusted asset values and volatilities based on Black and Scholes’ model, using an iterative algorithm that closely follows Duffie (2011) and Vassalou and Xing (2004). For each publicly traded firm in our sample, we assume that the value of asset, $V_A$, is a geometric brownian motion with volatility $\sigma_A$. We view equity as a call option with an underlying equal to the value of asset, $V_A$, and a maturity equal to one year. The strike price is taken to be $V_B$, the face value of total liabilities. The Black and Scholes’ formula gives:

$$
V_E = N(d_1) V_A - N(d_2) V_B e^{-r}, \quad (5)
$$
where \( N(\cdot) \) is the cumulative distribution function of a standard normal random variable, and

\[
d_1 \equiv \frac{\log(V_A) - \log(V_B) + r + \frac{\sigma_A^2}{2}}{\sigma_A}, \quad \text{and} \quad d_2 = d_1 - \sigma_A.
\]

### The iterative algorithm.

We initialize our iterative algorithm for calculating \( V_A \) and \( \sigma_A \) by setting \( V_A^{(0)} = V_E + V_B \), where \( V_E \) is the market capitalization of the firm, calculated using the CRSP data on price and number of shares outstanding, and \( V_B \) is the value of total liabilities, as given in COMPUSTAT.\(^{24}\) At step \( n \) of our algorithm, we have a candidate time series \( V_A^{(n)} \) for the value of asset on each day of our sample. Given \( V_A^{(n)} \), we obtain a daily time series for asset realized volatility, \( \sigma_A^{(n)} \), by computing the annualized square root of the average squared daily returns on asset during the month. Given \( V_A^{(n)} \) and \( \sigma_A^{(n)} \), we use equations (6) to calculate a time series for \( d_1^{(n)} \) and \( d_2^{(n)} \). In applying the formula, we take the interest rate to be the 1-year Treasury constant-maturity (daily frequency) from the Federal Reserve’s H.15 report. We then use (5) to obtain a new candidate time series for the value of asset:

\[
V_A^{(n+1)} = (1 - \omega)V_A^{(n)} + \omega \left( N(d_2^{(n)}) V_B e^{-r} \right),
\]

where \( \omega \) is a relaxation parameter which we set equal to 0.2 to improve convergence. We terminate our algorithm when the norm of \((V_A^{(n+1)} - V_A^{(n)})/V_A^{(n)} \) is less than \( 10^{-5} \). Convergence occurs for over 95% of the stocks in the sample.

### Option adjusted leverage adjusted for asset volatility.

The option adjusted leverage adjusted for asset volatility is simply \( \frac{V_A - V_B}{\sigma_A V_A} \), where \( V_A \) and \( \sigma_A \) are calculated using the iterative algorithm described above.

### Black and Scholes’ DD.

Following the literature, the Black and Scholes’ and Merton distance to default is defined as:

\[
DD \equiv \frac{\log(V_A) - \log(V_B) + \mu_A - \frac{\sigma_A^2}{2}}{\sigma_A}
\]

where \( V_A \) and \( \sigma_A \) are calculated using the iterative algorithm described above, and \( \mu_A \) is the mean return on asset over the time period.

\(^{24}\)If there is no quarterly data, we use the annual data instead as in Duffie (2011). In all cases, we linearly interpolate between points to obtain daily data, as in Gilchrist and Zakrajsek (2012).
C  Realized vs. implied volatility

As a robustness check on our empirical implementation of our DI measure, we compare median DI computed using realized and option implied volatilities from option metrics. We focus on the median log DI since, as argued in Section 4.1, its fluctuations account for most of the fluctuations in the overall DI distribution. Figure 24 plots the time series of the median log DI measured using implied and realized volatility from OptionMetrics for the available data from 1996 to 2013. We use their daily data for both series to ensure that the same firms are included in both samples. We use the implied volatility from OptionMetrics’ standardized, at the money, options with 30 days to maturity, and pool both calls and puts. The figure shows that realized volatility closely tracks fluctuations of implied volatility.
Figure 1: The value of equity as a function of the value of assets.

\[ \left( \frac{V_{At} - V_{Bt}}{V_{At}} \right) \frac{1}{\sigma_{At}} \]

Figure 2: The value of equity as a function of the value of assets.
Figure 3: The empirical relationship between credit rating and Distance to Insolvency.

Figure 4: The empirical relationship between Distance to Insolvency and credit rating.
Figure 5: A scatter plot of monthly DI vs. monthly averages of option adjusted spreads for the Bank of America-Merrill Lynch Corporate Bond Indices by ratings class for January 1997- December 2012, in log scale. Each point represents a single month and data for one of seven ratings from AAA to CCC and below. Pre August 2007 data points are blue triangles, and post August 2007 data points are red circles.

Figure 6: A scatter plot of monthly DI vs. monthly averages of 5 year single name CDS swap rates for 2001-2011, in log scale. Data is pooled by credit rating. Each point represents a single month and one of seven ratings classes from AAA to CCC and below. Pre August 2007 data points are blue triangles, and post August 2007 data points are red circles.
Figure 7: A scatter plot of monthly average DI vs. monthly average Black and Scholes (1973) Distance to Default by month and ratings class from December 1985 to December 2012. Each point represents a single month and data for one of seven ratings from AAA to CCC and below.

Figure 8: The distribution of Distance to Insolvency for firms that declare bankruptcy in the 60 months prior to bankruptcy or delisting.
Figure 9: Annual Distance to Insolvency vs. annual issuer weighted corporate default rates from Moody’s Investor Service Annual Default Study 2012.

Figure 10: The distribution of Distance to Insolvency, 1926-2012.
Figure 11: The y-axis plots the percentiles of a true log-normal distribution for $\frac{1}{\sigma_{E,t}}$ with the estimated cross-sectional mean and standard deviation for each month 1926-2012. The colored lines display the empirical percentile cutoffs on each date.

Figure 12: The mean and standard deviation of log DI 1926-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.
**Figure 13:** The 95th percentile of log DI 1926-2012 with time varying (red) versus constant (pink) standard deviation. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

**Figure 14:** Deep and Broad Insolvency Crises: The log of the median of DI, 1926-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale. The 50th percentile DI hits 1, associated with a highly vulnerable rating, in the Great Depression, 1937, 1987, and the Financial Crisis of 2008.
Figure 15: Leverage and asset volatility under the assumption of unlimited liability, 1971-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

Figure 16: DI versus constant leverage DI, under the assumption of unlimited liability, 1971-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.
Figure 17: Asset volatility under the assumption of unlimited liability, and using Black and Scholes to compute the value of equity’s default option, 1971-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

Figure 18: The percentiles of Distance to Insolvency for all firms in October 2007 and October 2008 together with the counterfactual alternative percentiles of Distance to Insolvency that would have arisen from October 2007 leverage and October 2008 asset volatility.
Figure 19: The DI for the CRSP value weighted portfolio and the DI of the median firm 1926-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

Figure 20: A comparison of the log median DI for financial and non-financial firms, 1926-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.
Figure 21: A comparison of the log median DI for the largest 50 financial and non-financial firms in terms of market capitalization, 1962-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

Figure 22: A comparison of the log median DI for the Government Backed Large Financial Institutions and the largest 50 non-financial firms in terms of market capitalization, 1962-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.
Figure 23: The 90th percentile, median, and 10th percentile of DI for the GBLFI’s from 1997-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

Figure 24: The mean of the log of inverse realized vs. option implied volatilities, 1993-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.
**Table 1: Government Backed Large Financial Institutions**

<table>
<thead>
<tr>
<th>Name</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Express</td>
<td>Amex 01/02/1969 to today</td>
</tr>
<tr>
<td>American Insurance Group (AIG)</td>
<td>AIG 12/14/1972 to today</td>
</tr>
<tr>
<td>Bank of America</td>
<td>BOA 01/02/1969 to today</td>
</tr>
<tr>
<td>Bank of New York</td>
<td>BONY 12/04/1969 to today</td>
</tr>
<tr>
<td>Branch Banking and Trust</td>
<td>BB&amp;T 12/14/1972 to today</td>
</tr>
<tr>
<td>Bear Stearns</td>
<td>BST 10/29/1985 to 05/30/2008</td>
</tr>
<tr>
<td>Capital One</td>
<td>COF 11/16/1994 to today</td>
</tr>
<tr>
<td>City</td>
<td>C 12/31/1925 to today</td>
</tr>
<tr>
<td>Fifth Third Bancorp</td>
<td>FITB 04/23/1975 to today</td>
</tr>
<tr>
<td>Fannie Mae</td>
<td>FNMA 08/31/1970 to 07/07/2010</td>
</tr>
<tr>
<td>Freddy Mac</td>
<td>FRE 08/10/1989 to 07/07/2010</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>GS 05/04/1999 to today</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>JPM 03/05/1969 to today</td>
</tr>
<tr>
<td>Key Banks</td>
<td>KEY 02/23/1972 to today</td>
</tr>
<tr>
<td>Lehman Brothers</td>
<td>LEH 05/31/1994 to 09/17/2008</td>
</tr>
<tr>
<td>Merrill Lynch</td>
<td>MERRILL 07/27/1971 to 12/31/2008</td>
</tr>
<tr>
<td>MetLife</td>
<td>MET 04/05/2000 to today</td>
</tr>
<tr>
<td>Morgan Stanley</td>
<td>MS 03/21/1986 to today</td>
</tr>
<tr>
<td>PNC Financial Services</td>
<td>PNC 12/14/1972 to today</td>
</tr>
<tr>
<td>Regions Financial Corp</td>
<td>REG FIN 12/14/1972 to today</td>
</tr>
<tr>
<td>Suntrust Banks</td>
<td>SUNTRUST 07/01/1985 to today</td>
</tr>
<tr>
<td>State Street Boston Corporation</td>
<td>STATESTREET 12/14/1972 to today</td>
</tr>
<tr>
<td>US Bancorps</td>
<td>USB 12/14/1972 to today</td>
</tr>
<tr>
<td>Wachovia Corporation</td>
<td>WACH 12/14/1972 to 12/31/2008</td>
</tr>
<tr>
<td>Washington Mutual</td>
<td>WaMu 03/11/1983 to 09/26/2008</td>
</tr>
</tbody>
</table>
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