Discussion of “Economic Catastrophe Bonds”
by Coval, Jurek and Stafford

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GSAM-QIS

NBER - July 2007
• Summary

• CDS/CDX Market

• CDO Market

• Credit Spread Puzzle

• The model

• The story

• Final Thoughts
Summary

- Propose simple story for large growth in structured product markets (specifically pooling and tranching):
  - Posit that ratings are sufficient statistic for expected loss.
  - Tranching process pools risky securities (e.g., BBB) to create lower risk (e.g., AAA) and higher risk (e.g., Z) securities by creating different levels of subordination (tranches).
  - By nature of that process senior tranches have more systematic risk and therefore should have higher expected return for given expected loss (∼ rating).
  - However investors care only about expected loss (∼ rating).

⇒ Effectively, the banking sector exploits the “stupidity” of investors by manufacturing portfolios with same expected loss but different systematic risk and selling them at identical prices.

- Provide evidence for their story using CDX.IG synthetic tranche prices:
  - Use a simple pricing model for tranches based on the one-factor Gaussian copula market standard.
  - Instead of assuming that the common factor has a Gaussian density (as in the market model), the authors extract its density from long-term S&P500 option prices.
  - Their results suggest that observed market spreads on all mezzanine and senior tranches are substantially lower than model-implied 'fair' spreads.
Credit markets characterized by rapid financial innovation

- Innovation in contracts,
  - from traditional *funded* securities: corporate bonds
  - to new *unfunded* derivatives: credit default swaps (CDS)

- And increased liquidity,

- Allow investors to express views on:
  - Single-names CDS
  - Baskets of names (CDX.IG, CDX.HV, iTraxx)
  - Correlation (Synthetic liquid CDO, Bespoke CDO, CDO$^2$...)
  - Emerging Market Countries (EMCDS)
  - Basket of Countries (EMCDX)
  - Asset Backed Securities such as credit card receivables or Home equity loans (ABS-CD$S$)
  - Baskets of Asset Backed Securities (ABX)
  - Correlation (TABX)
  - Senior secured Loans (LCDS)
  - Basket of Loans (LCDX)
CDS Contract Structure

- A CDS is an insurance contract against a credit event of Counterparty:
  - Prior to credit event:
    
    ![Diagram](A CDS is an insurance contract against a credit event of Counterparty: Prior to credit event)

  - Upon arrival of credit event:
    
    ![Diagram](A CDS is an insurance contract against a credit event of Counterparty: Upon arrival of credit event)

- Definition of credit event:
  - Bankruptcy
  - Failure to pay
  - Obligation acceleration or default
  - Repudiation/moratorium
  - Restructuring (Full R, Mod R, ModMod R, No R)
## Arbitrage Relation

- Buy XYZ bond + Buy XYZ protection $\sim$ Earn risk-free rate

- Buy risk-free bond + Sell XYZ protection $\sim$ Earn XYZ bond yield

$$\text{CDS spread} \approx Y_{XYZ} - R_f$$

\[ \Rightarrow \text{CDS allows pure unfunded play on credit risk.} \]

- Empirical evidence on \( Basis = \text{CDS spread} - (Y_{XYZ} - R_f) \).

<table>
<thead>
<tr>
<th></th>
<th>Basis wrt Tsy (bp)</th>
<th>Basis wrt Swap (bp)</th>
<th>implied ( R_f / Tsy )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.E. (of mean)</td>
<td>Mean</td>
</tr>
<tr>
<td>Aaa/Aa</td>
<td>-51.30</td>
<td>1.97</td>
<td>9.55</td>
</tr>
<tr>
<td>A</td>
<td>-64.33</td>
<td>1.82</td>
<td>5.83</td>
</tr>
<tr>
<td>Baa</td>
<td>-84.93</td>
<td>3.63</td>
<td>2.21</td>
</tr>
<tr>
<td>All Categories</td>
<td>-62.87</td>
<td>1.38</td>
<td>6.51</td>
</tr>
</tbody>
</table>

\[ \text{source: Hull, Pedrescu, White (2006)} \]
CDS Market Statistics

Exhibit 1.1: The notional amount of credit derivatives globally is larger than the global amount of debt outstanding.

Sources: British Bankers' Association Credit Derivatives Report 2006, Bank for International Settlements and ISDA.
Note: Cash bonds through June 2006.

Exhibit 7.1: Participants in the credit derivatives market. Some favor one direction over the other.

Protection sellers (long risk) Protection buyers (short risk)

- 35% Banks, Trading Activities 39%
- 9% Bank, Loan Portfolio 20%
- 32% Hedge Funds 28%
- 4% Pension Funds 2%
- 1% Corporates 2%
- 8% Mono-line Insurers 2%
- 4% Re-Insurers 2%
- 5% Other Insurers 2%
- 3% Mutual Funds 2%
- 1% Other 1%

The CDX index

- The CDX index is an insurance contract against credit events of a portfolio of counterparties (e.g., 125 names in CDX.IG):
  - Prior to credit event:
    - protection buyer ——— outstanding notional $\times$ spread ——— protection seller
  
  - Upon arrival of credit event of XYZ:
    - protection buyer ——— XYZ deliverable bond ——— protection seller
    - protection buyer ——— XYZ notional ——— protection seller

- Following credit event outstanding notional is reduced by notional of XYZ in portfolio (i.e., $\frac{1}{125}$ in CDX.IG).

- Contract expires at maturity or when notional exhausted.

- N.B.: CDX contract $\neq$ equally weighted portfolio of single name CDS contracts
  CDX spread $\neq$ average of single name CDS spreads
CDX Market Statistics

Growth Rate (notional)

Industry Composition of CDX.IG

CDX.IG Moody's Ratings

End Users

source: BBA & White (2006)
Synthetic CDO Tranches

- Selling protection on CDO tranche with attachment points \([L, U]\) (i.e., notional = \(U - L\)) written on underlying basket of 125 single names (CDX):
  - Prior to a credit event:
    - Protection buyer ➔ outstanding notional × spread ➔ protection seller
  - Upon arrival of credit event (\(LGD = \text{notional} - \text{deliverable bond price}\)), if cumulative loss exceeds lower attachment point (i.e., \(L_t = \sum_{i=1}^{125} LGD_i 1_{\{\tau_i \leq t\}} > L\)) then
    - Protection buyer \(\leftarrow \min(LGD, \text{outstanding notional})\) ➔ protection seller
  - Following credit event outstanding tranche notional is reduced by LGD (up to exhaustion of outstanding notional).
  - Also, super senior tranche notional is reduced by recovery (to satisfy 'adding up constraint').
  - Contract expires at maturity or when tranche notional is exhausted.
- Tranche payoff is call spread on cumulative loss: \(\max(L_t - L, 0) - \max(L_t - U, 0)\).

⇒ Tranche valuation depends on entire distribution of cumulative portfolio losses and crucially on default event correlation model.
Market Size

- Liquid tranche market is growing steadily

- Bespoke portfolio tranche market is much larger (ten times?) than synthetic tranche market:
  - Investors sell or buy protection on a portfolio of specific names for speculative or hedging motives.
  - Dealers take the other side and turn to the synthetic tranche market to hedge their resulting net exposure (keep some basis risk).
  - Hedge funds and other dealers participate in synthetic tranche market to redistribute risks.
Market Model: Implied Gaussian Copula Correlation

- Market standard for quoting CDO tranche prices is the *implied correlation* of the Gaussian Copula framework.

- Intuition builds on structural model of default (CDO model due to Vasicek 1987 who combines Merton (1974) with CAPM idea):
  - Each name in basket characterized by an ‘asset value’ driven by two factors:
    - a common market factor and an idiosyncratic factor
      \( V_i = \sqrt{\rho_i} M + \sqrt{1 - \rho_i} \epsilon_i \) with \( M, \epsilon_i \) independent centered Gaussian).
  - Pairwise ‘asset correlation’ is the product of the individual asset betas (\( \sqrt{\rho_i \rho_j} \)).
  - Default occurs when asset value falls below a constant barrier (\( \text{DefProb} = P(V_i \leq B_i) \)).

- Market convention for quoting tranche values in terms of *implied correlation* assumes:
  - The individual beta is identical across all names in the basket.
  - The default boundary is identical and calibrated to CDX level.
  - All firms have identical LGD of 60%.

⇒ With these heroic assumptions, a single number, the *implied correlation* (\( \rho \)), allows to match a given tranche’s model price with the market price (for a given CDX level).
The implied correlation smile

Market Quotes on Aug. 4, 2004 (CDX index spread 63.25 bp)

<table>
<thead>
<tr>
<th>Tranche</th>
<th>0-3%</th>
<th>3-7%</th>
<th>7-10%</th>
<th>10-15%</th>
<th>15-30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDX.IG</td>
<td>41.38%</td>
<td>3.49%</td>
<td>1.355%</td>
<td>0.46%</td>
<td>0.14%</td>
</tr>
</tbody>
</table>

The market displays an *implied correlation smile*:

<table>
<thead>
<tr>
<th>Tranche</th>
<th>0-3%</th>
<th>3-7%</th>
<th>7-10%</th>
<th>10-15%</th>
<th>15-30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDX.IG</td>
<td>21.7%</td>
<td>4.1%</td>
<td>17.8%</td>
<td>18.5%</td>
<td>29.8%</td>
</tr>
</tbody>
</table>

The smile shows that the Gaussian copula model is mis-specified (≈ option skew).

Market quotes on June 1st 2005 IG4-5Y (CDX index spread of 42 bp):

<table>
<thead>
<tr>
<th>Tranche</th>
<th>0-3%</th>
<th>3-7%</th>
<th>7-10%</th>
<th>10-15%</th>
<th>15-30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDX.IG</td>
<td>30.5%</td>
<td>0.66%</td>
<td>.095%</td>
<td>.075%</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

The corresponding *implied correlation smile*:

<table>
<thead>
<tr>
<th>Tranche</th>
<th>0-3%</th>
<th>3-7%</th>
<th>7-10%</th>
<th>10-15%</th>
<th>15-30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDX.IG</td>
<td>9.08%</td>
<td>5.8%</td>
<td>10.02%</td>
<td>16.77%</td>
<td>27.62%</td>
</tr>
</tbody>
</table>

Market quotes on July 7 2007 IG8-5Y (CDX index spread of 48 bp):

<table>
<thead>
<tr>
<th>Tranche</th>
<th>0-3%</th>
<th>3-7%</th>
<th>7-10%</th>
<th>10-15%</th>
<th>15-30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDX.IG</td>
<td>33.5%</td>
<td>1.80%</td>
<td>.40%</td>
<td>.225%</td>
<td>0.065%</td>
</tr>
</tbody>
</table>
Correlation ‘trading’

- Selling protection on the equity Tranche (delta-hedged) \(\sim\) long correlation:
  - Selling protection on equity is equivalent to being long a put on aggregate losses with strike equal to 3%. The value is increasing in the volatility of total losses which increases with default correlation.
  - The equity tranche is exposed to idiosyncratic Jump-to-default risk since it gets hit at the first default.

- Selling protection on the senior tranches \(\sim\) short correlation:
  - Selling protection on super senior tranche is short a call option on aggregate portfolio losses struck at 30%. Its value is decreasing in loss volatility and hence decreasing in correlation.
  - The Super senior tranche is exposed to systematic (cataclysmic?) risk: What is the probability that > 30% of investment grade default within a year?

- At least two reasons for the rapid development of CDS/CDX/CDO markets:
  - Credit spread puzzle
  - Rating ‘arbitrage’
The Credit spread puzzle

- Investment-grade (IG) firms rarely default:
  
  Average Issuer-Weighted Cumulative Default Rates 1970-2004

<table>
<thead>
<tr>
<th>Years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>0.12</td>
<td>0.21</td>
<td>0.30</td>
<td>0.41</td>
<td>0.52</td>
<td>0.63</td>
</tr>
<tr>
<td>Baa</td>
<td>0.19</td>
<td>0.54</td>
<td>0.98</td>
<td>1.55</td>
<td>2.08</td>
<td>2.59</td>
<td>3.12</td>
<td>3.65</td>
<td>4.25</td>
<td>4.89</td>
</tr>
</tbody>
</table>

- Further, recovery rates are substantial:
  
  Average Recovery Rates by Seniority Class, 1982-2004

<table>
<thead>
<tr>
<th>Year</th>
<th>Sr. Sec.</th>
<th>Sr. Unsec.</th>
<th>Sr. Subord.</th>
<th>Jr. Subord.</th>
<th>Subord.</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.574</td>
<td>0.449</td>
<td>0.391</td>
<td>0.320</td>
<td>0.289</td>
<td>0.422</td>
</tr>
</tbody>
</table>

- Structural models, when calibrated to match average loss rate, tend to underpredict yield spreads (relative to Treasury)

⇒ Structural models underestimate the risk-premium component of credit spreads, and/or

⇒ Spreads compensate for other factors (i.e, liquidity, taxes) in addition to credit risk
A Simple Calibration Exercise

Consider simple Merton (1974) model

\[
\frac{dV}{V} + \delta \, dt = (r + \theta \sigma) \, dt + \sigma \, dz
\]

where \( \theta \) is the asset value Sharpe ratio.

Default occurs at \( T \) if \( V(T) \) falls below \( B \). In that case recover \( 1 - L \).

Risky debt payoff is:

\[
\min(F, V_T) = F - \max(F - V_T, 0)
\]

\Rightarrow \text{ risky debt is equal to risk-free debt minus a put option.}

Spread \((y - r)\) on a date-\( T \) zero coupon bond is:

\[
(y - r) = -\left( \frac{1}{T} \right) \log \left\{ 1 - L \, N \left[ N^{-1} \left( \pi^P \right) + \theta \sqrt{T} \right] \right\}.
\]

\Rightarrow \text{ Even though the model is specified by 7 parameters \( \{r, \mu, \sigma, \delta, V(0), B, L\} \), credit spreads only depend on historical default probability, recovery and asset sharpe ratio \( \{\pi^P, L, \theta\} \).}
### A Simple Calibration Exercise

<table>
<thead>
<tr>
<th>Sharpe</th>
<th>T = 4Y</th>
<th></th>
<th>T = 10Y</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baa</td>
<td>Aaa</td>
<td>Baa-Aaa</td>
<td>Baa</td>
</tr>
<tr>
<td>0.15</td>
<td>44.0</td>
<td>1.6</td>
<td>42.4</td>
<td>67.7</td>
</tr>
<tr>
<td>0.20</td>
<td>54.9</td>
<td>2.2</td>
<td>52.7</td>
<td>88.1</td>
</tr>
<tr>
<td>0.25</td>
<td>68.1</td>
<td>3.0</td>
<td>65.1</td>
<td>112.8</td>
</tr>
<tr>
<td>0.30</td>
<td>83.7</td>
<td>4.1</td>
<td>79.6</td>
<td>141.7</td>
</tr>
<tr>
<td>0.35</td>
<td>102.0</td>
<td>5.5</td>
<td>96.5</td>
<td>175.1</td>
</tr>
<tr>
<td>0.40</td>
<td>123.4</td>
<td>7.4</td>
<td>116.0</td>
<td>212.9</td>
</tr>
</tbody>
</table>

**Table:** (Baa - Aaa) spreads as a function of Sharpe ratio. 4Y Baa default rate = 1.55%. 4Y Aaa default rate = 0.04%. 10Y Baa default rate = 4.89%. 10Y Aaa default rate = 0.63%. Recovery rate = 0.449.

- Typical Baa firm asset value Sharpe ratio estimated around 0.22.
- The credit spread puzzle says that historically, strategy going long corporate bonds seems very appealing (i.e., typical models cannot explain the level of observed spreads) because:
  - (i) historical expected loss rates have been low, and
  - (ii) Idiosyncratic (diversifiable!) risk on typical IG bonds is quite high (roughly half of the total risk).
A Simple Calibration Exercise

Yield Spreads, Default Loss Rates, and Calculated Credit Spreads (10-Year Bonds)

source: Huang and Huang (2003)
The modeling framework

- The contribution (vs. Gaussian copula model) is to extract the density of the market factor $M$ from SP500 Equity options.
  - Fit a parametric implied volatility function to observed implied vols on ATM and 30% OTM SP500 put options.
  - Use the Breeden Litzenberger (1978) formula to extract the density of market return ($\sim$ state price density) from option prices.

- Potential issues:
  - Prices of senior tranches very sensitive to tail events, i.e., very sensitive to the extrapolation of the implied vol curve where there is no data.
  - Results for senior tranche seem pretty sensitive to functional form of implied vol function (10bps difference on 40bps price).
    ⇒ Important to insure that the implied vol parametric form is arbitrage-free. Not easy (e.g., Lee (2003), Fengler (2005)).
  - Better to work in price space (use an option pricing model?).
    ⇒ Inconsistency between underlying SP500 return and IG portfolio (different names, leverage).

- Shares well-known short-comings of the Gaussian Copula model:
  - Assume covariance structure driven by one factor model
  - Assume that all firms in the basket are homogeneous (same distance to default, same recovery rate, same pair-wise correlation).
  - The model is purely static: gives loss distribution only at maturity.
  - Infinite $N$ (large portfolio) approx. is not needed given homogeneity assumption.
    ⇒ Cannot ‘explain’ major repricing events such as May 05 (Auto sector), July 07 (Subprime).
Quantitative implications

- The calibration procedure is a bit imprecise:
  - All securities handled as zero-coupon prices (no timing of cash-flows).
  - CDX is priced as a zero coupon bond (equation 21). But it is an unfunded product \sim basket of single-name CDS.
  - The calibration procedure has one constraint (matching CDX price) and three parameters (average leverage, average beta, average idiosyncratic risk). Therefore ‘typically require that the model implied equity beta equal one’ (p.15)?

  ⇒ Interesting to report idiosyncratic risk and distance to default numbers.

- Find that all senior and mezzanine tranches (3-7, 7-10, 10-15, 15-30) are well under-priced by the market. Differences are very large:
  - Fair value of selling protection on 3% to 30% should be approximately 23bps vs. 9.55bps in the market.
  - Most dramatic for senior 15-30 tranche: fair value of 47bps vs. 9bps in the market.
  - To be compared to an average spread on CDX of 47.8bps!

- The authors should report all tranches, i.e., 0-3 and 30-100. The implication (if model is correct) is that the 0-3 tranche must be seriously over-priced by the market! Selling protection on the equity tranche should be a very profitable venture...

- Would be interesting to look at the CDX-tranche basis predicted by the model (potential impact of LGD risk).
Are senior tranches priced inefficiently by naive investors?

▷ Investors care only about expected losses (∼ ratings) and not about covariance (ironic since they trade in correlation markets!).

⇒ Spreads across AAA assets should be equalized. Are they?

![AAA spreads by asset](image)

⇒ All spreads should converge to **Physical** measure expected loss.
  ▷ We observe large risk-premium across the board ($\lambda^Q / \lambda^P > 6.$)
  ▷ Large time-variation in that risk-premium.

⇒ Time-variation in spreads should be similar to that of rating changes (smoother?).

▷ Evidence seems inconsistent with marginal price setters caring only about expected loss (∼ ratings).
Who is the marginal investor in synthetic tranche markets?

- Arbitrage relation ties price of all tranches to price of underlying. If some tranches are expensive other tranches are cheap.

⇒ Opportunities for smart investors (hedge funds, Harvard professors...?) to trade.

- Who is marginal investor in synthetic tranche market?

Evidence from most markets is that “Crash risk premium” is very high (OTM put options, catastrophe bonds, reinsurance markets).

⇒ This would be ‘unique’ opportunity to purchase cheap crash insurance.
What drives differences between structured AAA spreads?

- Wall street’s favorite story: Rating-stability premium (‘reaching for yield’ by rating constrained investors who want to take more risk - ratings simply do not reflect risk and/or expected payoff b/c to coarse and persistent).

- My favorite story: Moral hazard and marking to market.
  - Absent MtM risk, moral hazard would drive spreads on senior tranches to zero.
  - Positive aspect of limits to arbitrage?

- Additional Consideration: Borrowing Constraints/funding costs.
  - Remote risks need to offer higher premium to attract capital which is in limited supply (given limited term-financing availability and variability of short term collateralized financing conditions; agents care about dollar NPV not IRR).
Conclusion

- Provocative paper with interesting idea to use option prices to extract common factor to replace Gaussian factor of market Copula model.
- Promising results with respect to fitting CDX ‘out of sample’.
- More work needed to convince that model prices are realistic.
- Story seems hard to reconcile with simple facts (at least when taken at face value).