"Corporate Cash Holdings and Credit Line Usage"

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Friday, February 22nd, 2013, 10.30-12.00
Room 126, 1st floor of the Extranef building at the University of Lausanne
Corporate Cash Holdings and Credit Line Usage

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November 2012

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Keywords: Dynamic capital structure, cash holdings, precautionary savings, corporate liquidity.
JEL Classifications: G31, G32, G35, E21, E22

*We thank Gian Luca Clementi, Adlai Fisher, Rick Green, Burton Hollifield, Lars-Alexander Kuehn, Toni Whited, as well as seminar participants at the American Finance Association meetings, Carnegie-Mellon University, the Federal Reserve Bank of Dallas, the Northern Finance Association meetings, the T2M annual conference, the University of British Columbia Summer Finance Conference, the University of Colorado at Boulder, the University of Wisconsin at Milwaukee, Victoria University Wellington, and Wilfrid-Laurier University for helpful comments.
1 Introduction

North American firms increasingly use liquidity instruments to manage the risk they face. It is well known that cash holdings as a proportion of total assets of North American COMPUSTAT firms have increased dramatically since the 1970’s. Figure 1 documents this large and steady increase for the average North American firm in our sample.\(^1\) For the period covering 1995 to 2006, cash represents about 17 percent of total assets on average.

Alongside the unprecedented level of cash liquidities, Sufi (2009) documents the widespread use of credit lines. Among a random sample of 300 COMPUSTAT firms, 85 percent of firms have access to a credit line between 1996 and 2003 and their line usage amounts to about six percent of total assets on average. For firms with a Standard and Poor’s credit rating, 94.5 percent of them have a credit line where usage represents 4.66 percent of total assets.

The literature highlights that, because of financial frictions, firms must rely on liquidity instruments to manage risk. This suggests that the larger importance of liquidities is attributable to three possible causes. Firms may rely more on liquidity instruments because financial frictions have increased, because the cost of using liquidities has decreased, or because firms face more risk. So far, the literature concludes that the latter is more important: Bates, Kahle, and Stulz (2009) conclude that the large cash increase is attributable to a precautionary motive, in the general sense that higher cash flow risk generated the higher cash holdings.

In this paper, we use a structural approach to investigate which economic factors changed over time to produce the large increase in corporate cash holdings amidst prevalent credit lines and other liquidity management tools. In our structural model, firms face financial frictions and different types of risk. We recognize that it is more costly for firms to alter their investment, dividend, debt and equity policies. For example, firms can transfer funds between interest-earning assets and cash at the beginning of each week to meet their liquidity requirements during that week. One possible mechanism explaining the cash increase is a concurrent increase in firms’ earnings volatility which

\(^1\)Cash refers to COMPUSTAT Mnemonic CHE and is composed of cash (CH) and short-term investments (IVST). It includes, among others, the following items: cash in escrow; government and other marketable securities; letters of credits; time, demand, and certificates of deposit; restricted cash.
would escalate the need for liquidities. Such a liquidity mechanism is featured in the seminal work of Miller and Orr (1966), where firms must manage cash inventories to face liquidity needs generated by income fluctuations. The liquidity mechanism is also similar to that discussed in Telyukova and Wright (2008), where liquidity needs yield a motive for consumers to accumulate liquidities.

We also recognize that firms face various taxes, as well as adjustment and issuing costs. Firms not only smooth payouts to avoid extreme taxes and costs, but also behave “prudently” and accumulate liquidities to self-insure against future adverse shocks. In this paper, we use the term “precautionary” to denote strictly the firm’s preference for prudence. In this sense, the firm may accumulate precautionary liquidities over and above those required by immediate funding needs discussed above. With a precautionary motive, an increase in firms’ idiosyncratic risk underscores the need to self-insure and therefore increases liquidities. The precautionary mechanism is similar to that discussed in Leland (1968) and Carroll and Kimball (2006), where a convex marginal utility generates prudence and yield a motive for consumers to accumulate liquidities.

Our analysis employs a standard dynamic model of a firm’s investment and financial decisions augmented by two classic motivations to hold liquidities: a Miller and Orr (1966) liquidity motive and a Leland (1968) precautionary motive. We show that the model offers a reasonable description of firms’ behavior, and can be used to study firms’ liquidities. We focus our analysis on two extreme periods: the first third of our sample period from 1971 to 1982 and the last third from 1995 to 2006 before the financial crisis.

Our quantitative analysis replicates the increase in cash holdings and the widespread use of credit lines. Interestingly, our quantitative analysis suggests that changes in financing and payout frictions do not play a large role in explaining the increase in liquidities. While it might have been tempting to attribute the large and steady accumulation of internal funds to a large and steady increase in the magnitude of financial frictions, we show that the dramatic increase in liquidities since the 1970s is less related to changing financial frictions and more focused on changing economic fundamentals.

Our analysis highlights that both lower costs to using liquidities and increases in risk have
contributed to the large increase in cash holdings. A key financial innovation was the rapid adoption of sweep accounts in the early to mid 1990s, as documented in Anderson (2003). Cash became much more attractive because sweep accounts effectively raised its real rate of return. This cannot be the entire story, however, because it does not address the widespread use of credit lines.

The key increase in risk originates from a large increase in net income risk that is unrelated to recurring capital, payout, debt and equity decisions. We find that the large increase in net income risk produces a large increase in cash holdings and widespread credit line use. Dichev and Tang (2008) first noted that earnings volatility nearly doubled between 1967 and 2003. While Donelson et al. (2011) attribute this increased volatility to the increased occurrence of special items, Srivastava (2011) specifically points to three fundamental changes in operating strategies of US firms, namely increases in research and development expenses, a shift toward service industries, and reductions in manufacturing costs through outsourcing. Expenses to maintain a research lab, a customer service platform, or an outsourcing operation overseas act more like fixed costs than unit-costs and therefore cannot be controlled through production quantity decisions. This type of time-varying operating leverage adds much volatility to the firm’s bottom line. While it is true that firms’ revenues also became more volatile, the increased volatility of revenues does not explain much of the change in liquidities.

Our model is related to a growing literature on corporate cash holdings. There are complementary papers examining other motives for holding cash. For example, our neoclassical framework does not feature the agency motive to hold cash, as Bates, Kahle, and Stulz (2009) fail to support the agency problem explanation for the overall increase in cash holdings of US firms. The relationship between higher cash holdings (or lower cash value) and higher agency costs is well documented in other contexts – see Dittmar and Mahrt-Smith (2007), Dittmar et al. (2003), Faulkender and Wang (2006), Harford (1999), Harford et al. (2008), Lins et al. (2010), Nikolov and Whited (2010), and Pinkowitz et al. (2006), and Yun (2009).

Our model does not feature the tax motive to hold cash. Foley et al. (2007) show that US multinationals accumulate cash in their subsidiaries to avoid the tax costs associated with repatri-
ated foreign profits. On that front, Bates, Kahle, and Stulz (2009) note that the large increase in cash holdings is also observed for domestic firms.

The model of Gamba and Triantis (2008) is closely related to ours in that a firm can finance its investment through debt issues, equity issues, and internal funds, where only internal funds do not trigger transaction costs. Their focus is on valuing financial flexibility, while we explore which economic factors are responsible for the large increase in cash holdings and the widespread use of credit lines.

A number of papers in the corporate cash literature document the relationship between financial constraints and cash hoarding, where the cash saving behavior of constrained firms is shown to be different from that of other firms. This literature includes Acharya, Almeida, and Campello (2007), Almeida, Campello, and Weisbach (2004), Bolton, Chen, and Wang (2011), Han and Qiu (2007), Morellec and Nikolov (2009) and Riddick and Whited (2008). For example, Bolton, Chen, and Wang (2009) propose a dynamic model of investment decisions for financially constrained firms with constant returns to scale in production. Among other contributions, they find that Tobin's marginal q may be inversely related to investment, and that financially constrained firms may have lower equity betas because of precautionary cash holdings.

Another strand of the corporate liquidity literature centers on credit lines, including Boot et al. (1987), DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), Holmstrom and Tirole (1998), Martin and Santomero (1997), Sannikov (2007), and Tchistyi (2006). For example, DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), Sannikov (2007) and Tchistyi (2006) show that a credit line is an optimal security for a firm with agency problems. In these models, the firm defaults when it suffers losses greater than the unused portion of the credit line.

In our model, we represent the main empirical features of credit lines, such as covenants and credit limits, as described in Agarwal et al. (2004), Berger and Udell (1995), Ham and Melnik (1987), Disatnik et al. (2011), Shockley and Thakor (1997), and Sufi (2009). We examine the usage of credit lines in a rich framework that also considers cash savings in addition to capital investments, payout policies, and equity and debt financing.
The paper is organized as follows. Section 2 presents the model, and provides some analytical results characterizing the behavior of liquidities. The model, however, does not possess an analytical solution. Section 3 discusses the calibration of the model. The formal structural estimation and resulting parameters are discussed in the appendix. Section 4 presents our simulation results, where we first ensure that the model broadly reproduces observed facts. We then use the model to study the increase in cash holdings and the usage of credit lines. Finally, we study the sensitivity of our results to a number of extensions. Section 5 offers some concluding remarks.

2 The Model

We study how a firm manages its cash holdings and credit line in an otherwise standard dynamic model of financial and investment decisions. The firm does not consider cash as a perfect substitute for debt: cash is not negative debt. Instead, cash savings can serve two purposes. They may provide self-insurance against future adverse shocks, and they may provide liquidity to meet current adverse shocks. The line of credit may also provide liquidity to meet current adverse shocks, but only if the firm has not yet violated its financial covenant. In this sense, the extent to which cash holdings and lines of credit can be substitute is limited.

To operationalize the precautionary and liquidity motives, we assume that the firm faces shocks to its revenues and to its funds more generally. Knowing the current realization of the revenue shock, the firm chooses how much to invest, how much cash to save, how much debt to issue, how much dividend to pay out (or how much equity to raise). During the year, however, the firm may face another shock that turns out to be either lower or higher than expected back at the beginning of the year. When this funding shock is lower than expected, the firm cannot scale back its investment commitments, take back its distributed dividend, or go back to external markets with more favorable issuing conditions. Unexpectedly low funding shocks are met with cash, or a line of credit if the latter is available. As is explained in details below, the firm holds sufficient cash to pay shortcomings that may occur during the year. The firm may also hold additional cash as a precaution against future adverse shocks to revenues.
2.1 The Firm

The firm, acting in the interest of shareholders, maximizes the discounted expected stream of payouts $D_t$ taking into account taxes and issuing costs. When payouts are positive, shareholders pay taxes on the distributions according to a tax schedule $T(D)$. The schedule recognizes that firms can minimize taxes for smaller payouts by distributing them in the form of share repurchases. Firms, however, have no choice but to trigger the dividend tax for larger payouts. Following Hennessy and Whited (2007), the tax treatment of payouts is captured by a schedule that is increasing and convex:

$$T(D_t) = \tau_D D_t + \frac{\tau_D}{\phi} \exp(-\phi D_t) - \frac{\tau_D}{\phi},$$

where $\phi > 0$ controls the convexity of $T(D)$ and $0 < \tau_D < 1$ is the tax rate. When payouts are negative, shareholders send cash infusions into the firm as in the case of an equity issue. The convex schedule $T(D)$ also captures the spirit of Altinkilic and Hansen (2000), where equity issuing costs are documented to be increasing and convex. Figure 2 displays the schedule $T(D)$ for different values of $\tau_D$ and $\phi$.

Net payouts are

$$U(D_t) = D_t - T(D_t).$$

This function is increasing $U'(D) = 1 - \tau_D + \tau_D \exp(-\phi D) > 0$, concave $U''(D) = -\phi \tau_D \exp(-\phi D) < 0$, and its third derivative is positive $U'''(D) = \phi^2 \tau_D \exp(-\phi D) > 0$. As a result, the net payout function ensures that the firm is risk averse and has a precautionary motive. In this context, the parameter $\phi$ is the coefficient of absolute prudence: $\phi = -U'''(D)/U''(D)$.

The firm faces two sources of risk. The first source of risk comes from stochastic revenues. Revenues $Y_t$ are generated by a decreasing returns to scale function of the capital stock $K_t$:

$$Y_t = \exp(z_t) \Gamma K_t^\alpha,$$

where $z_t$ is the current realization of the revenue shock, the parameter $0 < \alpha < 1$ denotes the capital intensity, and $\Gamma > 0$ is the productivity scale parameter. The revenue shock follows the
autoregressive process
\[ z_t = \rho z_{t-1} + \sigma_z \epsilon_{zt}, \quad (4) \]
where \( \epsilon_{zt} \) is the innovation to the revenue shock, \( 0 < \rho < 1 \) denotes the persistence of the revenue shock, and \( \sigma_z \) is the volatility of the revenue innovations. The innovations \( \epsilon_{zt} \) are independent and identically distributed random variables drawn from a standard normal distribution: \( \epsilon_{zt} \sim N(0, 1) \).

As will be discussed below, the revenue innovation \( \epsilon_{zt} \) influences the marginal productivity of the firm’s capital. In contrast, the second source of risk is not multiplicative but enters additively into the sources and uses of funds equation. As such, it represents any level shock to funds unrelated to the investment, payout, debt and equity financing decisions of the firm already modeled. Our model is sufficiently complex that we cannot explore the different sources of the additive risk in this paper, but we point to Srivastava (2011) who links the net income volatility to expenses incurred in maintaining a research lab, a customer service platform, or an outsourcing operation in another country. Accordingly, the additive funding shock is given by:
\[ F_t = \bar{F} + f_t, \quad (5) \]
where \( \bar{F} \geq 0 \) is the predictable level and \( f_t = \sigma_F \epsilon_{ft} \) is the innovation. The funding innovations are assumed to be independent of the revenue innovations and drawn from a uniform distribution: \( \epsilon_{ft} \sim U[-1, 1] \).

The firm chooses how much to invest \( I_t \), how much cash to hold \( M_{t+1} \), how much debt to raise \( B_{t+1} \), how much credit line to use \( L_{t+1} \), how much to pay out (or how much equity to issue) \( D_t \). The sources and uses of funds defines the firm’s liquidities for the beginning of the following year:
\[ M_{t+1} = Y_t + F_t - I_t + B_{t+1} + L_{t+1} - D_t - (1 + r)B_t - (1 + \xi)L_t + (1 + \iota)M_t - T^C_t - \Omega^K_t - \Omega^B_t, \quad (6) \]
where \( B_t, L_t \) and \( M_t \) are the beginning-of-the-year level of debt, credit line and cash holdings. \( T^C_t \) represents corporate taxes, \( \Omega^K_t \) and \( \Omega^B_t \) denote the adjustment costs to capital and debt. The interest rates \( r, \xi, \) and \( \iota \) are associated with debt, credit line, and cash holdings.

Capital accumulates as follows:
\[ K_{t+1} = I_t + (1 - \delta)K_t, \quad (7) \]
where $0 < \delta < 1$ denotes the depreciation rate. The firm faces quadratic capital adjustment costs:

$$
\Omega^K_t = \frac{\omega_K}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t, \quad (8)
$$

where $\omega_K \geq 0$ is the capital adjustment cost parameter.

Debt issuance is given by $\Delta B_{t+1} = B_{t+1} - B_t$. The firm faces quadratic costs to varying the debt level away from its long-run level $\bar{B} > 0$:

$$
\Omega^B_t = \frac{\omega_B}{2} (B_{t+1} - \bar{B})^2, \quad (9)
$$

where $\omega_B \geq 0$ is the debt cost parameter.

In contrast to investment, debt issuance, and equity issuance, the firm does not incur any cost when changing its cash holdings or line of credit usage. The interest rate earned on cash $\iota$, however, is assumed lower than interest rate on debt $r$, while the interest charged on the line of credit $\xi$ is assumed higher: $\iota < r < \xi$.

Finally, corporate taxes are imposed on revenues after depreciation, interest payments, and interest income:

$$
T^C_t = \tau_C \left( Y_t + F_t - \delta K_t - \tau r B_t - \xi L_t + \iota M_t \right), \quad (10)
$$

where $0 < \tau_C < 1$ is the corporate tax rate.

Credit lines cannot always substitute for cash holdings. Credit line loans may not exceed a preset upper limit $\bar{L}$ so that the firm’s financing is constrained: $0 \leq L_{t+1} \leq \bar{L}$. Credit lines also come with covenants requiring the firm to keep in good standing with respect to creditors. We specify a covenant where the firm may use its credit line only when revenue shocks are not too low, i.e., when $z_t \geq \bar{z}$.\footnote{For reasons to be explained below, the covenant cannot depend on the realization of the funding shock.}

The after-tax discount factor is $\beta = 1/(1 + (1 - \tau) r)$, where $\tau$ is the personal tax rate on interest income. Because individuals pay taxes on their interest income at a lower rate than the rate at which corporations deduct their interest payment, debt financing is tax-advantaged. To counter this benefit of debt financing, the convex cost in Equation (9) bounds the debt level. In this sense, deviation costs play a role similar to a collateral constraint.
2.2 The Intertemporal Problem

At the beginning of the year, the firm makes decisions knowing the current realization of the revenue shock, but not the current realization of the funding shock. During the year, the firm reacts to the funding shock realization when it is different from its expected value. The shock triggers a need for liquidity. Of course, foreseeing all this, the firm may already have been cautious and invested less, paid out less in dividends, or raised more funds externally so that its liquidities (cash plus credit line) could cover the realization. As a result, the stock of cash at the end of the year, $M_{t+1}$, is equal to the firm's choice of cash savings $S_t$ at the beginning of the period plus the used line of credit $L_{t+1}$ plus the funding shock, i.e.:

$$M_{t+1} = S_t + L_{t+1} + (1 - \tau_C)\sigma f \epsilon_{ft},$$  \hspace{1cm} (11)$$

where the cash saving chosen at the beginning of the period is defined as

$$S_t = (1 - \tau_C) \left( Y_t + F - \delta K_t - r B_t - \xi L_t + \iota M_t \right) - \Delta K_{t+1} + \Delta B_{t+1} + M_t - L_t - \Omega^K - \Omega^B - D_t$$  \hspace{1cm} (12)$$

and $\Delta K_{t+1} = K_{t+1} - K_t$.

When the firm has sufficiently high cash flows, it can use its line of credit. In these circumstances, the firm must nevertheless set aside enough cash to cover the gap between the worst possible funding shock and the upper limit on the credit line. Because the firm makes its cash saving decision $S_t$ without knowledge of the realization of the funding shock $\epsilon_{ft}$, the firm must save enough to cover all possible realizations. The juxtaposition of the end-of-the-year cash holdings Equation (11) and the non-negativity constraint $M_{t+1} \geq 0$ requires that $S_t + \bar{L} - (1 - \tau_C)\sigma f \geq 0$.

When the firm's revenue shock is too low ($z_t < \bar{z}$), the firm cannot use its line of credit. In these circumstances, the funding shock must be met with cash only: $S_t - (1 - \tau_C)\sigma f \geq 0$. Altogether, the liquidity constraint at the beginning of the period becomes:

$$S_t + \bar{L} \mathbf{1}_{(z_t \geq \bar{z})} - (1 - \tau_C)\sigma f \geq 0,$$  \hspace{1cm} (13)$$

where $\mathbf{1}_{(z_t \geq \bar{z})}$ is an indicator function that takes a value of one when $z_t \geq \bar{z}$.
We note that the credit line is contingent on the revenue shock only. If it were contingent on the funding shock that gets realized later in the period, the firm would never use its credit line. The covenant would block the firm’s access to the line of credit when the funding shock turns out to be too low. The covenant would effectively constrain the firm to accumulate enough cash at the beginning of the year to cover all possible funding shock realization including the worst shock. As a result, the firm would rely on cash only. To make the line of credit relevant in the model, we do not make it contingent on the funding shock realization.

The firm’s intertemporal problem can therefore be described by the following dynamic programming problem. Solving backwards, in mid-period when the revenue shock is high enough so that the line of credit is available, the firm’s problem can be stated as:

$$W(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \max_{\{M_{t+1}, L_{t+1}\}} \beta E_{t+1} [V(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_t)]$$  \hspace{1cm} (14)

subject to equation (11), the non-negativity constraints $M_{t+1} \geq 0$ and $L_{t+1} \geq 0$ and the upper limit on the loan $L_{t+1} \leq \bar{L}$. Note that the conditional expectation $E_{t+1}$ is taken on an information set $\Phi_{t+1}$ that includes all beginning-of-the-period choices of capital $K_{t+1}$, debt $B_{t+1}$, cash saving $S_t$ plus the revenue shock $z_t$ and the realization of the funding shock $f_t$.

When the revenue shock is too low for accessing the line of credit, the firm’s problem simplifies to holding cash $M_{t+1} = S_t + (1 - \tau_C) \sigma_f \epsilon f_t$ and no credit $L_{t+1} = 0$.

At the beginning of the period, the firm’s problem is to choose payouts $D_t$, capital stock $K_{t+1}$, next period’s debt level $B_{t+1}$ and cash savings $S_t$ to solve

$$V(K_t, B_t, L_t, M_t; z_t, f_{t-1}) = \max_{\{D_t, K_{t+1}, B_{t+1}, S_t\}} U(D_t) + E_t [W(K_{t+1}, B_{t+1}, S_t; z_t, f_t)]$$  \hspace{1cm} (15)

subject to Equations (1) to (5), (7) to (9), (12) and (13). Here, the conditional expectation $E_t$ is taken on an information set $\Phi_t$ that includes the beginning of the period values for the capital stock $K_t$, debt level $B_t$, line of credit $L_t$, cash holdings $M_t$, and revenue shock $z_t$, but not the mid-year realization of the funding shock $f_t$. The appendix presents the optimality conditions for this problem. In what follows, we summarize the salient results.

The necessary optimality conditions include the complementary-slackness conditions associated
with the liquidity constraint (13):

\[ \lambda_t \geq 0, \quad S_t + \bar{L}1_{(z_t \geq \bar{z})} - (1 - \tau_C)\sigma_f \geq 0, \quad \text{and} \quad \lambda_t \left[ S_t + \bar{L}1_{(z_t \geq \bar{z})} - (1 - \tau_C)\sigma_f \right] = 0. \]  

When the multiplier is positive \( \lambda_t > 0 \), the firm saves just enough cash to satisfy the liquidity constraint with equality. That is, all cash holdings are driven by the liquidity motive. When \( \lambda_t = 0 \), the firm may precautionarily save more than required to meet immediate liquidity needs.

We summarize these findings in the following proposition:

**Proposition 1.** When \( \lambda_t > 0 \), \( S_t = (1 - \tau_C)\sigma_f - \bar{L}1_{(z_t \geq \bar{z})} \) so that the firm saves cash only as a safeguard against the funding shock realization that cannot be covered by the credit line. When \( \lambda_t = 0 \), \( S_t \geq (1 - \tau_C)\sigma_f - \bar{L}1_{(z_t \geq \bar{z})} \) so that the firm may precautionarily hold more cash than required.

The optimality conditions also include an Euler equation with respect to cash holdings. The beginning-of-the-period cash decision \( S_t \) is characterized by

\[ U'(D_t) - \lambda_t = \beta E_t \left[ U'(D_{t+1}) (1 + (1 - \tau_C)\xi_t) \right] + E_t \left[ \gamma_t^M \right], \]

where \( \gamma_t^M \) is the multiplier on the non-negativity constraint \( M_{t+1} \) present in the mid-period problem.

We can restate the Euler equation above in terms of return:

\[ R^M E_t [m_{t+1}] + \left\{ E_t \left[ \gamma_t^M \right] + \lambda_t \right\} / U'(D_t) = 1 \]

where

\[ R^M = 1 + (1 - \tau_C)\xi_t \]

and \( m_{t+1} = \beta U'(D_{t+1})/U'(D_t) > 0. \)

The credit line usage is characterized by a similar equation:

\[ U'(D_t) - \lambda_t = \beta E_t \left[ U'(D_{t+1}) (1 + (1 - \tau_C)\xi_t) \right] - E_t \left[ \gamma_t^L - \gamma_t^U \right], \]

where \( \gamma_t^L \) is the multiplier on the non-negative constraint \( L_{t+1} \geq 0 \) and \( \gamma_t^U \) is the multiplier on the upper limit \( L_{t+1} \leq \bar{L} \).

Similarly, the Euler equation can be restated as:

\[ R^L E_t [m_{t+1}] - \left\{ E_t \left[ \gamma_t^L - \gamma_t^U \right] + \lambda_t \right\} / U'(D_t) = 1, \]

12
where

\[ R^L = 1 + (1 - \tau_C)\xi \]  \hspace{1cm} (22)

In both Euler Equations (17) and (20), there is no cost to adjusting cash holdings or credit line usage. Cash holdings are taxed at the corporate rate \( \tau_C \), the same rate at which the credit line interest is tax-deductible. Because the interest rate earned on cash accumulations \( \iota \) is lower than the interest rate charged on the credit line \( \xi \), the multipliers are pivotal in determining the equilibrium solutions. Proposition 2 highlights the substitution between the two sources of liquidity.

**Proposition 2.** The firm will never hold cash \( M_{t+1} > 0 \) and use its credit line \( L_{t+1} > 0 \). Either \( M_{t+1} \geq 0 \) and \( L_{t+1} = 0 \), or \( M_{t+1} = 0 \) and \( L_{t+1} > 0 \).

*Proof.* See Appendix 6.2.

Moreover, Proposition 3 states that the firm will hold cash at the end of the period when it has enough cash savings or when the funding shock is high enough. In these two cases, the firm will not use its credit line.

**Proposition 3.** The firm will hold cash \( M_{t+1} > 0 \) when the funding shock is high, \( f_t > \bar{L}1(z_t \geq \bar{z})/(1 - \tau_C) - \sigma_f \), or when beginning cash savings are high, \( S_t > (1 - \tau_C)\sigma_f \).

*Proof.* See Appendix 6.2.

If the firm is sufficiently prudent, it will save enough at the beginning of the period so that it accumulates cash holdings by the end of the period, and it will not use the credit line. Proposition 4 embodies the precautionary savings mechanism.

**Proposition 4.** When \( U'(D_t) \) is sufficiently convex, then cash saving is high, \( S_t > (1 - \tau_C)\sigma_f \), and the firm holds cash, \( M_{t+1} > 0 \), but no credit line, \( L_{t+1} = 0 \), for all values of the funding shock \( f_t \).

*Proof.* See Appendix 6.2.

Cash holding and credit line usage decisions also interact with debt and capital investment decisions through the stream of payouts. The Euler equations describing the debt financing and
capital investment decisions are:

\[ U'(D_t) [1 - \omega_B (B_{t+1} - \bar{B})] = \beta E_t \left[ U'(D_{t+1}) (1 + (1 - \tau_C)r) \right] , \]

and

\[ U'(D_t) \left[ 1 + \omega_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \right] = \beta E_t \left[ U'(D_{t+1}) \left\{ 1 + (1 - \tau_C)(\alpha \exp(z_{t+1})\Gamma K_{t+1}^{\alpha-1}) - \delta + \frac{\omega_K}{2} \left( \frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right\} \right] . \]

Stated in terms of

\[ R_B^t = \frac{[1 + (1 - \tau_C)r]}{[1 - \omega_B (B_{t+1} - \bar{B})]} \]

and

\[ R_K^{t+1} = \frac{1 + (1 - \tau_C)\left[ \alpha \exp(z_{t+1})\Gamma K_{t+1}^{\alpha-1} - \delta \right] + \frac{\omega_K}{2} \left[ \left( \frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right]}{1 + \omega_K \left( \frac{K_{t+1}}{K_t} - 1 \right)} , \]

the Euler equations (23) and (24) become:

\[ R_B^t E_t [m_{t+1}] = 1 \]

and

\[ E_t [R_{t+1}^K] E_t [m_{t+1}] + \text{Cov}_t [R_{t+1}^K, m_{t+1}] = 1 . \]

The firm’s choices of debt and capital investment influence its liquidities and vice-versa. For example, when comparing the debt issuing decision to the cash saving decision, we can see that the cost of issuing debt \( R_B^t \) is always greater than the return on accumulating cash \( R_M^t \) when the debt is at its target \( B_{t+1} = \bar{B} \). However, it is possible that the cash return is not dominated by debt \( R_B^t = R_M^t \) when the debt policy is sufficiently conservative. In this case, Proposition 5 shows that the firm accumulates cash and does not use its credit line.

**Proposition 5.** When \( R_B^t = R_M^t \) [conservative debt policy], beginning-of-the-period cash saving is high, \( S_t > (1 - \tau_C)\sigma_F \), the firm holds cash at the end of the period, \( M_{t+1} > 0 \), and it does not use its credit line, \( L_{t+1} \), for all values of the funding shock \( f_t \).

**Proof.** See Appendix 6.2. 

\[ \square \]
Similarly with capital, Proposition 6 shows that when the return to capital and the covariance risk are sufficiently low, the firm accumulates cash and does not use its credit line. A positive covariance risk implies that the return to capital net of taxes and adjustment costs $R_{t+1}^K$ is high when payouts $D_t$ are low. With a large positive covariance, the firm may be able to self-insure against future adverse shocks without accumulating cash. Conversely as stated in Proposition 6, when capital does not provide this insurance, the firm saves with cash.

**Proposition 6.** When $\mathbb{E}_t [R_{t+1}^K] - R_M + \frac{\text{Cov}[m_{t+1}, R_{t+1}^K]}{\mathbb{E}_t [m_{t+1}]} = 0$ [no self-insurance], beginning liquidity is high, $S_t > (1 - \tau_C)\sigma_F$, and the firm holds cash, $M_{t+1} > 0$ for all values of the funding shock $f_t$.

Proof. See Appendix 6.2. □

### 3 Data

The data comes from the North American COMPUSTAT file and covers the period from 1971 to 2006 excluding the crisis period. To explain the large change in cash holdings, the data is split in two extreme time periods: the first third of the sample period from 1971 to 1982 and the last third from 1995 to 2006. The COMPUSTAT sample includes firm-year observations with positive values for total assets (COMPUSTAT Mnemonic AT), property, plant, and equipment (PPENT), and sales (SALE). Our measure of cash holdings is COMPUSTAT Mnemonic CHE, and it is composed of cash (CH) and short-term investments (IVST). The sample includes firms from all industries, except utilities and financials, with at least five years of consecutive data. The data is winsorized to limit the influence of outliers at the 1 percent and 99 percent tails. The final sample contains 53,067 firm-year observations for the 1971 to 1982 period and 67,720 firm-year observations for the 1995 to 2006 period.

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3 The cash (CH) item includes: bank and finance company receivables; bank drafts; bankers’ acceptances; cash on hand; certificates of deposit included in cash by the company; checks; demand certificates of deposit; demand deposits; letters of credit; and money orders. The short-term investments (IVST) item includes accrued interest included with short-term investments by the company; cash in escrow; cash segregated under federal and other regulations; certificates of deposit included in short-term investments by the company; certificates of deposit reported as separate item in current assets; commercial paper; gas transmission companies’ special deposits; good faith and clearing house deposits for brokerage firms; government and other marketable securities (including stocks and bonds) listed as short term; margin deposits on commodity futures contracts; marketable securities; money market fund; real estate investment trusts’ shares of beneficial interest; repurchase agreements (when shown as a current asset); restricted cash (when shown as a current asset); time deposits and time certificates of deposit, savings accounts when shows as a current asset; treasury bills listed as short term.
We seek to explain the large increase in cash holdings using model-simulated data. The model does not possess an analytical solution and is solved numerically. Once the model is solved, the policy functions for capital $K_{t+1}$, debt $B_{t+1}$, credit line $L_{t+1}$, cash $M_{t+1}$, and the equity value $V_t$ of Equation (15) are simulated from random outcomes of the innovations $\epsilon_{zt}$ and $\epsilon_{ft}$. These simulated series serve to build other series including dividends $D_t$, operating income $OI_t = Y_t - F_t$, net income $NI_t = (1 - \tau_C) (Y_t - F_t - \delta K_t - r B_t - \xi L_t + \iota M_t)$, and the total firm value $A_t = V_t + (1 + (1 - \tau) r) B_t - B_{t+1}$. For each of the two time periods, we construct five panels that have roughly the same number of firm-year observations as observed in the COMPUSTAT panel.

The numerical method requires values for all parameters. To this end, a number of parameters are estimated directly from the data, and others are estimated in a moment matching exercise. The appendix provides an extensive description of the numerical and estimation methods, as well as a complete discussion of the parameter estimates. The resulting parameter values appear in Tables 1, 2, and 3.

Table 1 presents parameters whose values are estimated directly from the data. The capital intensity $\alpha$, the productivity scale $\Gamma$, the persistence of the revenue shock $\rho_z$, and the volatility of its innovations $\sigma_z$ are estimated directly from Equation (3) and the autoregressive process (4). The table also presents parameters whose values are observed directly from the data, including the corporate tax rate $\tau_C$, the interest income tax rate $\tau_r$, the dividend tax rate $\tau_D$, the real interest rate $r$, and the real interest rate on cash holdings $\iota$, and the real interest rate on the credit line use $\xi$. In calibrating the interest rate on cash $\iota$, we recognize that by 1995 sweep money market accounts became available to firms (see Anderson, 2003). For a fee, firms could have their cash savings in excess or in shortage of a minimum balance automatically transferred to and from money market accounts. During the 1971-1982 period, cash not held in short-term investments would simply lose value at the inflation rate. The way we calibrate $\iota$ therefore changes from the first to the last period to account for the sweep account innovation. Appendix 6.5 provides more detail.

For the line of credit parameters including the interest rate $\xi$, the upper limit $\bar{L}$ and the cutoff level $\bar{z}$, there are no comprehensive data available and therefore we calibrate to reasonable values.
For the interest rate $\xi$, we apply a premium above the real interest rate $r$, and use the premium reported in Ham and Melnik (1987) for the 1971-1982 period and in Sufi (2009) for the 1995-2006 period. We allow the preset limit on the credit line to include all funding shocks: $\bar{L} = (1 - \tau_C)\sigma_F$.

We specify only that the firm must be experiencing average or better revenue shocks $\bar{z} \geq 0$ to use its line of credit.

The remaining parameters are set to values chosen to ensure that simulated series from the model replicate important features of the data. Table 2 shows the parameter values and the target moments for the period of 1971 to 1982, while Table 3 does so for the period of 1995 to 2006. The parameters are the depreciation rate $\delta$, the capital adjustment cost $\omega_K$, the constant debt level $\bar{B}$, the debt deviation cost $\omega_B$, the average funding shock level $\bar{F}$, the funding shock volatility $\sigma_F$, and the coefficient of absolute prudence $\phi$. The target moments are based on the average and the standard deviation of the investment-to-total assets ratio, the average and standard deviation of the leverage ratio, the average operating income-to-total assets ratio, and the standard deviation of the net income-to-total assets ratio.

Of special interest, the coefficient of absolute prudence $\phi$, which dictates the strength of the firm’s precautionary motive, is estimated in the first time period from 1971 to 1982 so that the average of cash holdings-to-total assets matches the data. This choice ensures that we have the right starting point to investigate the increase in cash holdings. We are interested in predicting average cash holdings during the 1995-2006 period without changing the coefficient of prudence from its 1971-1982 parameter value.

The average of COMPUSTAT firms’ cash holdings-to-total assets $M_{t+1}/A_t$ was 8.9 percent during the 1971-1982 years. Matching this moment requires a convexity parameter estimate of $\phi = 0.004$. Note that the estimated value is much smaller than the values ranging from 0.73 to 0.83 estimated in the different environment of Hennessy and Whited (2007). In that sense, our explanation for cash holdings does not rely on a large coefficient of absolute prudence.
4 Results

4.1 Do Simulated Financial Policies Behave as in the Data?

Before studying cash holdings in detail, it is important to verify that the model provides a reasonable description of firms’ observed behavior. Admittedly, the moment matching exercise ensures that specific simulated moments match a number of targeted moments in the data. In what follows, the analysis moves on to controversial moments related to dividend smoothness and debt countercyclicality. These results appear at the bottom of Tables 2 and 3.

It has long been recognized that firms smooth dividends (see Lintner, 1956). In our COMPUSTAT data, payout policies are smooth in the first period. The average standard deviation of payouts is only 3.2 percent during the 1971 to 1982 period, and rises to 10.5 percent during the 1995 to 2006 period. The model replicates the smooth payout policies in the first period and the rise in volatility in the last period. The standard deviation is only 3.6 percent for the first period calibration, and rises to 14.1 percent for the last period calibration. We note, however, that the model overestimates the volatility of payouts in the last period.

In the model, firms smooth payouts to avoid large taxes on payouts and large equity issuing costs. Specifically, the net payout function $U(D)$ recognizes the convexity of taxes and issuing costs and therefore describes firms as risk averse. Firms smooth payouts well in the first period. Unavoidably, the observed increase in net income volatility in the last period affects the volatility of payouts.

It has also been recognized that debt issues are countercyclical (see Choe, Masulis, and Nanda, 1993, Covas and Den Haan, 2010, and Korajczyk and Levy, 2003). In our COMPUSTAT data, the correlation between debt issues and revenues is $-0.242$ for the 1971 to 1982 period and $-0.188$ for the 1995 to 2006 period. In the model, the correlations are $-0.333$ and $-0.123$. Both in the data and in the model, debt issues are countercyclical, and the negative correlation attenuates in the recent time period.

The countercyclicality of debt issues in the model is surprising because standard dynamic capital structure models with a tax benefit of debt generate procyclical debt. In these models, firms take
on more debt in persistent good times to benefit from the tax advantage because their stronger abilities to repay the debt. In our model, the “risk averse” firm chooses to smooth the effect of adverse revenue shocks on payouts by issuing more debt.

4.2 Do Simulated Liquidity Policies Behave as in the Data?

The analysis of cash holdings begins by verifying that the model provides a reasonable overall description of cash holdings. In COMPUSTAT data, cash (including short-term investments) represents 8.9 percent of total assets in the 1971-1982 period. That number dramatically rises to 17.1 percent in the 1995-2006 period. In the model, the average ratio of cash holdings-to-total assets is specifically targeted by our calibration using the convexity parameter $\phi$ for the first period. Holding $\phi$ constant, the model predicts cash holdings of 17.5 percent of total assets in the last period. Interestingly, as dramatic as the observed cash increase has been in the data, the model predicts even more cash than observed in the data. This result suggests that firms have become less prudent over time.

Another quantity of interest is the used line of credit. Unfortunately, we have no data for the 1971-1982 period and limited data from Sufi (2009) who manually examined the credit line usage of 300 firms during the 1996-2003 period. Conditional on using the line of credit, the model predicts an average loan during the 1971-1982 period of 2.2 percent of total assets. In the later period, the percentage increases significantly to 9.4, overshooting Sufi’s (2009) average of 4.7 percent among firms with a credit rating. Our 9.4 percent, however, is well within the disperse range across industries documented by Sufi (2009), including 15 percent among capital-intensive firms (agriculture, minerals, and construction).

4.3 What Does Liquidity Look Like?

To gain some intuition about the model, Figures 3 to 6 show different aspects of the liquidity decisions. The figures are constructed by first drawing one series of revenue shock innovations ($\epsilon_{zt}$) and one series of funding shock innovations ($\epsilon_{ft}$). The same series of innovations is used to describe the firm behavior in both periods. Figures 3 to 6 illustrate how a firm may allocate
liquidity optimally over time in response to a particular string of shocks in an economic environment describing the 1971-1982 period and alternatively describing the 1995-2006 period.

Figure 3 shows that the model-simulated cash holdings are on average higher and more volatile in the last period parametrization compared to the first period parametrization. Figure 4 shows that the line of credit is used more often and to a greater extent in the last period. Together Figures 3 and 4 illustrate the substitution between the two sources of firm liquidity, as described in Equations (17) and (20). When the firm uses its line of credit when \( L_{t+1} > 0 \) in Figure 4, it does not hold any cash \( M_{t+1} = 0 \), and conversely when the firm accumulates cash \( M_{t+1} > 0 \), it does not draw down its credit line \( L_{t+1} = 0 \).

Figures 5 and 6 discuss the role of the liquidity constraint (13) in generating the observed patterns for cash holdings and credit line usage. Figure 5 plots the multiplier \( \lambda_t \) of the constraint under both parameterizations. Recall that cash saving decisions \( S_t \) are entirely driven by the liquidity motive when \( \lambda_t > 0 \). The figure shows that strictly positive multiplier values (\( \lambda_t > 0 \)) occur in both parameterizations of the model. For the particular string of shocks simulated in the figures, the multiplier is strictly positive for six of the observations in the last period but only five in the first period. In the latter period, the firm more frequently saves only the minimum amount to meet liquidities needs. Considering all simulated firms rather than only the one shown in the figures, the liquidity constraint is binding 16.8 percent of the time in the 1971-1982 calibration and 32.9 percent of the time in the 1995-2006 calibration. This indicates that, all else equal, the firm feels less compelled to save more than required in the latter period. This is a surprising result given the large increase in cash holdings. One would have expected that firms’ precautionary desire to save more than required would also have increased.

Figure 6 shows how movements of the multiplier \( \lambda_t \) translate into beginning-of-the-period cash savings \( S_t \). The figure graphs the cash saving decision scaled by mean total assets \( S_t / \bar{A} \). Standardizing each observation by the overall mean of total assets \( \bar{A} \) rather than standardizing each observation by its corresponding total assets \( A_t \) focuses attention on the variations in cash savings \( S_t \) while maintaining the appropriate scale. With the calibration of \( \bar{L} \) and \( \bar{z} \) discussed in the
previous section, the liquidity constraint (13) reduces to $S_t \geq (1 - \tau_C) \sigma_F$ when the credit line is unavailable (i.e., when $z_t < 0$), or to $S_t \geq 0$ when the firm can count on the credit line ($z_t \geq 0$).

When revenue shocks are low, the firm must save at least $(1 - \tau_C) \sigma_F$ at the beginning of the period. In Figure 6, we can detect this threshold, as there are a number of realizations at 6.5 percent of total assets in the 1971-1982 calibration and at 17.9 percent in 1995-2006. The firm often saves precautionarily more than this amount. When $z_t \geq 0$, the firm can save less because it can rely instead on the credit line. In this case, the firm could choose to save nothing $S_t = 0$ but it never does. Cash savings are always strictly positive for all simulated firm-year observations in the sample. When the revenue shock is high enough, the firm, being prudent and averse to paying large issuing costs, uses the opportunity to save for a rainy day.

Together, Figures 3 to 6 suggest that the firm saves more in recent times, but the savings are more often set to the minimum that satisfies the liquidity constraint. The increase in cash savings and line of credit usage are related to the increase in overall liquidity needs, not to an increase in precautionary savings over and above the liquidity needs.

### 4.4 What Drives Liquidity?

Given that the model replicates the large increase in cash holdings and produces significant credit line usage, the analysis now turns to identifying which economic forces are responsible for the increase in firm liquidities. Table 4 presents the results of a sensitivity analysis, which proceeds on the basis of the first period parametrization. In turn each parameter is reset from its first period value to its last period value, leaving all other parameters to their first period values. The results of the sensitivity analysis focuses on three groups of parameters: liquidity, debt and capital parameters.

#### 4.4.1 Liquidity Parameters

Because the liquidity constraint $S_t \geq (1 - \tau_C) \sigma_F - \bar{L} \mathbf{1}_{(z_t \geq \bar{z})}$ is binding more often in the latter period, it seems natural to investigate first the liquidity policy parameters embedded in the constraint. The firm’s cash saving decision depends directly on the volatility of the funding shock $\sigma_F$ and the
corporate tax rate $\tau_C$. Over the two periods, the shock volatility (standardized by mean total assets) grows from 0.123 to 0.274, while the corporate tax rate decreases from 0.473 to 0.350. The associated increase in the cash-saving constraint forces the firm to save more cash to meet its current liquidity needs. Accordingly, Table 4 shows that, by changing only the funding shock volatility $\sigma_F/\bar{A}$ to its last period value, the calibration otherwise based on the first period increases its predicted cash holdings from 8.9 percent of total assets to 15.9 percent, close to explaining the total increase in cash. Changing only the funding shock volatility also increases the credit line usage from 2.2 percent to 7.9 percent, because the upper limit $\bar{L} = (1 - \tau_C)\sigma_F$ increased. Table 4 shows that the reduction in the corporate tax rate increases cash holdings and credit line usage, but to a far lesser extent. Also, the large increase in average funding shock $\bar{F}/\bar{A}$ barely affects cash and credit.

Euler Equation (17) suggests that the coefficient of absolute prudence $\phi$ and the impatience regarding cash holdings $\beta(1 + (1 - \tau_C)\iota)$ are important factors in determining the cash saving decision. The coefficient of absolute prudence $\phi$ controls the convexity of the marginal net pay-out function $U'(D)$. Because the coefficient remains constant over the two periods, it obviously cannot explain the increase in cash holdings and credit line usage. As for the impatience factor $\beta(1 + (1 - \tau_C)\iota)$, it grows from 0.968 in the first period to 0.976 during the last period. Because firms become more patient, it should be easier for the cash Euler Equation (17) to hold with $\lambda_t = 0$. That is, the firm should be more willing to accumulate precautionary savings, rather than only the minimum threshold as is the case more often in the last period.

Table 4 shows that the large increase in volatility $\sigma_F/\bar{A}$ is not the only driver of the dramatic increase in cash holdings. The most important driver is the interest rate on cash holdings $\iota$. The return to holding cash increases dramatically from a real rate of $-5.319$ percent during the inflation years of 1971-1982 to 0.352 percent in the 1995-2006 period. When the return to holding cash increases so much, the firm reacts accordingly. All else equal, the increase in $\iota$ alone would spur an otherwise similar firm in the 1971-1982 period to hold 22.26 percent of its assets in cash, far overshooting the observed average cash holdings of 17.09 percent from 1995 to 2006. With such
large cash liquidities, the firm substitutes away entirely from the credit line. The puzzle henceforth to investigate is: which economic force has limited the increase in cash and generated the significant credit line usage?

The dividend tax rate $\tau_D$ is part of the answer. Figure 2 graphs the schedules of taxes and equity issuing costs in the first ($\tau_D = 0.395$) and last ($\tau_D = 0.233$) period. For a given value of $\phi$, the reduction of $\tau_D$ experienced across the two calibrations makes the schedule $T(D)$ less convex. This is consistent with the US experience. On the taxation side, Piketty and Saez (2007) discuss how changes to the US tax system have made the federal tax system somewhat less progressive. On the equity issuing side, Chen and Ritter (2000) show that initial public offerings on average have become less costly over time. The firm therefore has less incentive to save to avoid the frictions. Accordingly, Table 4 reports lower cash holdings with liquidities substituted toward a greater usage of the credit line.

In Equation (20), the firm is patient with respect to the credit line $\beta(1 + (1 - \tau_C)\xi) > 1$, which means the firm would like to invest and earn the interest $\xi$. Given that the credit line is a debt rather than an investment, the patient firm would like to use no credit line were it not for the liquidity constraint. In fact, the patience factor $\beta(1 + (1 - \tau_C)\xi)$ increases from 1.003 to 1.008 over the two periods, making the credit line even less attractive to use. When the interest rate on the credit line $\xi$ increases from 1.422 percent to 3.109 percent, the credit line usage naturally decreases and the firm substitute their liquidities to more cash holdings.

Disentangling the effect of the discount factor $\beta = 1/(1 + (1 - \tau_r)r)$, Table 4 shows that the increase in the interest rate $r$ and the decrease in the interest income tax rate $\tau_r$, which depress the discount factor, should reduce cash holdings and the credit line usage, but interestingly they do not. A more detailed analysis reveals that the increase in the interest rate $r$ does reduce cash holdings and credit line usage, but it also reduces total assets, such that the cash holdings-to-total asset ratio increases. Total assets decrease because they are valued at a higher discount rate $r$. As for the reduction of the interest income tax rate $\tau_r$, it does not significantly affect cash holdings or credit line usage.
4.4.2 Debt Parameters

The firm’s liquidity decisions may be affected by its debt decisions through the interdependence of the Euler Equations (17), (20) and (23). In Equation (23), the term \( \omega_B (B_{t+1} - \bar{B}) \) is related to debt costs. Table 4 shows that cash holdings are not significantly affected by changes in the debt cost parameter \( \omega_B \) and target leverage \( \bar{B}/\bar{A} \). Because debt is relatively stable over time, frictions on debt financing does not seem to propagate significantly to the firm’s liquidity decisions.

4.4.3 Capital Parameters

In Equation (28), we see that capital decisions are affected by two terms: the expected return, \( E_t [R_{t+1}^K] \), and the covariance risk, \( \text{Cov}_t [R_{t+1}^K, m_{t+1}] \). An analysis of the conditional moments reveals that the second term is negligible: the covariance varies between -0.0001 and -0.0004. In the first period, the conditional expected return fluctuates wildly. For some realizations, the expected return on capital is so low that it is close to the return on cash. In this case, \( \lambda_t = 0 \) and the firm may save in a precautionary manner. In the last period, the conditional expected return to capital fluctuates much less, and it often dominates cash by a large margin so that the firm often saves only the minimum required by the constraint.

The effects of parameter changes on the conditional average net return to capital \( E_t [R_{t+1}^K] \) are difficult to study analytically because they cause an endogenous investment reaction. To gain some insight, we examine the deterministic steady state value of the net return to capital. Equation (28) reduces to \( \beta R^K* = 1 \) so that \( R^K* = 1/\beta = 1 + (1 - \tau_r)r \). The deterministic net return to capital has increased from 1.004 in the first period calibration to 1.012 in the last period calibration. However, the extent to which capital dominates cash in steady state return \( R^K* - R^M \) has fallen from 0.032 in the first period to 0.025 in the last period. This suggests that the occurrence of cash savings above the minimum threshold should have increased, which is not the case.

The important parameter effects must therefore reside with the stochastic behavior of \( E_t [R_{t+1}^K] \) rather than its steady state. Its volatility is directly linked to the conditional volatility of the revenue shock innovation \( \sigma_z \). The increase in the standard deviation of the innovation to the revenue shock,
from 0.210 in the first period calibration to 0.309 in the last period calibration, stimulates cash holdings above the minimum threshold and substitute part of the credit line away. In contrast, the reduction in the adjustment costs to capital $\omega_K$ makes it easier to use capital to smooth dividends rather than cash holdings or credit line, but the effect is small. In addition, Table 4 shows that changes in the depreciation rate $\delta$ and the persistence $\rho_z$ do not significantly affect liquidities.

Parameter effects related to the productivity scale parameter $\Gamma$ and the capital intensity parameter $\alpha$ are interesting. The productivity scale parameter $\Gamma$ and the capital intensity $\alpha$ directly affect the scale of revenues, raising the possibility that $\Gamma$ and $\alpha$ affect liquidities via the overall volatility of revenues. The volatility of revenues is increasing in the productivity scale of the firm but decreasing in the capital intensity $\alpha$. When the firm acts prudently, more volatile revenues prompt the firm to accumulate more cash holdings. The higher productivity scale $\Gamma$ yields higher and more volatile revenues as well as higher cash holdings (20.6 percent) which substitute out the credit line, while the smaller capital intensity $\alpha$ yields lower and less volatile revenues as well as lower cash holdings (5.1 percent) but a much larger use of the credit line (5.8 percent). Overall, however, the effect of the increasing productivity $\Gamma$ on liquidities is muted by the counter-effect of the decreasing capital intensity $\alpha$.

4.4.4 Discussion

Our sensitivity analysis suggests that the large increase in liquidities is unrelated to changes in firms’ external financing and payout parameters such as adjustment/issuing costs, the debt target or tax rates. The analysis suggests that the increase in liquidities results from the sweep money market innovation and from the increased net income risk that is unrelated to capital, payout, debt and equity. It is interesting that the increase in liquidities does not appear to be much affected by the change in revenue risk.

The sensitivity analysis thus offers testable predictions regarding the sources of risk propagation. For example, industries that have witnessed the largest increase in net income volatility should also exhibit the largest cash increase. Second, changes in revenue volatility should be mostly unrelated to changes in cash holdings. The aggregate data suggest that these two testable predictions may well
hold. The increase in cash holdings and the increase in net income volatility have been substantial over time, while the increase in the volatility of revenues has been less dramatic. Cash holdings-to-total assets have nearly doubled from 8.9 percent in the 1971-1982 period to 17.1 percent in the 1995-2006 period; the average standard deviation of net income-to-total assets has more than doubled from 0.067 to 0.180; but the average standard deviation of revenues-to-total assets has increased from 0.247 to only 0.297.

Admittedly, a full test of these predictions is outside the scope of this paper. As an illustration of the evidence, consider a simple regression of the increase in average cash holdings within an industry between the first period 1971-1982 and the last 1996-2005 on a constant, the industry change in the standard deviation of net income between the two periods, and the industry change in the standard deviation of revenues between the two periods, using the cross-section of industries defined in Fama and French (1997). The regression is:

$$\Delta Mean_i(M'/A) = a + b\Delta SD_i(NI/A) + c\Delta SD_i(Y/A) + \text{error}_i$$

(29)

The first prediction above requires that the estimated coefficient on the change in net income volatility b be positive and significant, and it is: the cross-sectional estimate of the coefficient (standard deviation) is $b = 0.787$ (0.223). The second prediction requires that the estimated coefficient on the change in revenue volatility c be zero. The estimate is not significantly different from zero at the five percent confidence level: the cross-sectional estimate (standard deviation) is $c = -0.010$ (0.169). The regression $R^2$ is 0.435.

4.5 Extensions

Table 5 reports the sensitivity of firm liquidities to two extensions. The extensions we consider are: introducing asymmetric costs to changing capital and debt, and using the coefficient of absolute prudence $\phi$ to match the volatility of payouts.

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4In this regression, the sweep market innovation is not industry specific, and would thus difference to a constant.
4.5.1 Introducing Asymmetric Costs

The model assumes quadratic and thus symmetric costs to changing capital and debt levels. However, it is not uncommon to consider asymmetric costs. For example, Gamba and Triantis (2008) assume that a reduction in the capital stock is more costly than a symmetric increase. They also assume that issuing new debt is more costly than retiring it.

These considerations are important insofar as the costs to changing capital and debt directly affect the firm’s precautionary cash saving behavior. If the firm cannot easily sell assets or issue more debt, it may depend more heavily on cash savings to self-insure against future adverse shocks. To study whether these asymmetric costs are quantitatively important, the capital adjustment cost function is changed to

\[ \Omega^K_t = \frac{\omega_K}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \left[ 1 - \mathbf{1}(I_t/K_t < \delta) \right] + \frac{\omega_a^K}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \mathbf{1}(I_t/K_t < \delta), \]  

(30)

where \( \mathbf{1}(I_t/K_t < \delta) \) is an indicator function that takes a value of one when the investment-to-capital ratio is less than the depreciation rate and zero otherwise. For simplicity, we assume that \( \omega_a^K = 2 \omega_K \). Similarly, the debt issuance cost function is changed to

\[ \Omega^B_t = \frac{\omega_B}{2} \left( B_{t+1} - \bar{B} \right)^2 \left[ 1 - \mathbf{1}(B_{t+1} > \bar{B}) \right] + \frac{\omega_a^B}{2} \left( B_{t+1} - \bar{B} \right)^2 \mathbf{1}(B_{t+1} > \bar{B}), \]  

(31)

where \( \mathbf{1}(B_{t+1} > \bar{B}) \) is an indicator function that takes a value of one when debt is above its long-run level and zero otherwise. For simplicity, we assume that \( \omega_a^B = 2 \omega_B \).

Table 5 reports the sensitivity of the average cash holdings and credit line usage to introducing asymmetric adjustment costs, keeping all other parameters at their benchmark values. The result of this experiment suggests that asymmetric costs do not significantly affect liquidities. Cash holdings increase, but the increase is small. Average cash holdings grow from 8.9 percent of total assets to 9.3 percent in the first period, and from 17.5 percent to 17.8 percent in the last period. The use of the credit line does not change much: it becomes 2.0 percent in the first period and 9.3 percent in the last period. While it is true that the firm depends more on cash and less on credit when it cannot easily sell assets or issue more debt, the magnitude of these effects is small.
4.5.2 Matching Payout Volatility

The coefficient of absolute prudence was set to match average cash holdings in the 1971-1992 period \((\phi = 0.004)\). The coefficient value was kept constant in the 1995-2006 period so that the model could generate an out-of-sample prediction on the firm’s liquidities. As a result, the simulated cash holdings of 17.5 percent of total assets for the last period does not exactly match the observed cash holdings of 17.1 percent.

In this robustness check, we consider setting the coefficient of absolute prudence \(\phi\) to match another moment: the volatility of payouts. The coefficient \(\phi\) affects not only prudence but also the firm’s degree of risk aversion and therefore its desire to smooth payouts. As expected, matching payout volatility requires higher values of \(\phi\). To match an average standard deviation of payouts of 0.032 in the first period requires \(\phi = 0.005\) (compared to the benchmark value of 0.004 estimated to match cash holdings). To match an average standard deviation of payouts of 0.105 in the last period requires \(\phi = 0.007\). Table 5 shows that the larger values of prudence \(\phi\) generate larger cash holdings by stimulating the precautionary motive. Average cash holdings grow to 0.100 in the first period (from 0.089 using the benchmark calibration), and to 0.225 in the last period (from 0.175 using the benchmark calibration). The firm’s use of the credit line decreases in response to the liquidity substitution toward cash savings. Matching the payout volatility generates cash holdings that are far above observed levels. It therefore remains a quantitative challenge to explain both liquidity and payout policies in the same framework.

5 Conclusion

The cash hoarding behavior of US firms since the 1970s provides an interesting setting in which to evaluate the different motives for corporate liquidities. Our results document that the increase in cash holdings is mostly attributable to two economic forces. First, the interest rate on cash holdings has increased tremendously, with our calibration recognizing not only the lower inflation in the latter period but also the innovation of sweep money market accounts. Second, there is more earnings volatility in the firms’ bottom lines that is not related to physical capital investment,
payout decisions, debt and equity financing. In accord with Dichev and Tang (2008), Donelson et al. (2011) and Srivastava (2011), we have estimated this increased volatility through a funding shock to net income. We note that the increase in revenue volatility by itself does not explain much of the increase in cash holdings.

The results also shed light on two widely acknowledged motives for holding cash. The precautionary motive (Leland, 1968) has become less important over time. Given the large increase in firms’ liquidities over time, it is surprising to discover that firms have become less willing to save in a precautionary manner over and above their required liquidity needs. Firms facing various taxes, as well as adjustment and issuing costs, acted more prudently in the 1970s and accumulated cash more often to self-insure against future adverse shocks. The liquidity motive (Miller and Orr, 1966), however, has become more important over time. The larger volatility in net income in the recent period increased liquidity requirements and forced firms to hold more liquidities, both in terms of cash and credit line.

6 Appendix

6.1 The Intertemporal Problem

At the beginning of the year, the firm chooses $D_t$, $K_{t+1}$, $B_{t+1}$, $S_t$ knowing an information set $\Phi_t$ that includes all state variables ($K_t$, $B_t$, $L_t$, and $M_t$) and the current realization of $z_t$, but not $f_t$. During the year, the firm chooses the allocation of $S_t$ between $M_{t+1}$ and $L_{t+1}$ knowing an information set $\Phi_{t+1}$ that includes all the information included in $\Phi_t$, plus all the new relevant states ($K_{t+1}$, $B_{t+1}$, and $S_t$) as well as the realization of $f_t$.

Given the firm’s choice of cash savings $S_t$, the firm’s problem during the period consists of choosing the allocation between cash holdings and credit line. Of course, if $z_t < \bar{z}$, the firm does not have access to the credit line. In this circumstance, the solution is $L_{t+1} = 0$ and $M_{t+1} = S_t + (1 - \tau_C) f_t$. If $z_t \geq \bar{z}$, the firm solves

$$W(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \max_{\{M_{t+1}, L_{t+1}\}} \beta E_t [V(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_t)]$$

(32)
subject to
\[ M_{t+1} - L_{t+1} = S_t + (1 - \tau_C) f_t, \] (33)
\[ M_{t+1} \geq 0, \] (34)
\[ L_{t+1} \geq 0, \] (35)
\[ L_{t+1} \leq \bar{L}, \] (36)

where the \( E_{t+} \) denotes that the expectation is taking conditional on the information set \( \Phi_{t+} \).

The solution must satisfy the following first-order conditions:
\[ \zeta_{t+} - \gamma^M_{t+} = \beta E_{t+} [V_M(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+}, f_t)] \] (37)
\[ \zeta_{t+} + \gamma^L_{t+} - \gamma^U_{t+} = -\beta E_{t+} [V_L(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+}, f_t)] \] (38)
\[ \gamma^M_{t+} \geq 0, \quad M_{t+1} \geq 0, \quad \gamma^M_{t+} M_{t+1} = 0 \] (39)
\[ \gamma^L_{t+} \geq 0, \quad L_{t+1} \geq 0, \quad \gamma^L_{t+} L_{t+1} = 0 \] (40)
\[ \gamma^U_{t+} \geq 0, \quad \bar{L} - L_{t+1} \geq 0, \quad \gamma^U_{t+} (\bar{L} - L_{t+1}) = 0, \] (41)

where \( \zeta_{t+} \) is the multiplier associated with constraint (33), \( \gamma^M_{t+} \) is associated with (34), \( \gamma^L_{t+} \) with (35), and \( \gamma^U_{t+} \) with (36).

At the optimum, we also have that
\[ W_K(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \beta E_{t+} [V_K(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+}, f_t)] \] (42)
\[ W_B(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \beta E_{t+} [V_B(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+}, f_t)] \] (43)
\[ W_S(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \zeta_{t+}. \] (44)

At the beginning of the year, the firm’s problem is
\[ V(K_t, B_t, M_t; z_t, f_{t-1}) = \max_{\{D_t, K_{t+1}, B_{t+1}, S_t\}} U(D_t) + E_t [W(K_{t+1}, B_{t+1}, S_t; z_t, f_t)] \] (45)

subject to
\[ S_t = (1 - \tau_C) \left( Y_t + \bar{F} - \delta K_t - r B_t - \xi L_t + \tau M_t \right) - \Delta K_{t+1} + \Delta B_{t+1} - L_t + M_t - \Omega^K_t - \Omega^B_t - D_t \] (46)
\[ S_t + \bar{L} \mathbf{1}(z_t \geq \bar{z}) - (1 - \tau_C) \sigma_F \geq 0, \]  

where the \( E_t \) denotes that the expectation is taking conditional on the information set \( \Phi_t \).

The first-order conditions of this problem are

\[ \eta_t = U'(D_t) \]  

\[ \eta_t - \lambda_t = E_t [W_S (K_{t+1}, B_{t+1}, S_t; z_t, f_t)] \]  

\[ \eta_t \left[ 1 + \omega_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \right] = E_t [W_K (K_{t+1}, B_{t+1}, S_t; z_t, f_t)] \]  

\[ \eta_t \left[ 1 - \omega_B (B_{t+1} - \bar{B}) \right] = E_t [W_B (K_{t+1}, B_{t+1}, S_t; z_t, f_t)] \]  

\[ \lambda_t \geq 0, \quad S_t + \bar{L} \mathbf{1}(z_t \geq \bar{z}) - (1 - \tau_C) \sigma_F \geq 0, \quad \lambda_t \left[ S_t + \bar{L} \mathbf{1}(z_t \geq \bar{z}) - (1 - \tau_C) \sigma_F \right] = 0, \]  

where \( \eta_t \) is the multiplier associated with constraint (46) and \( \lambda_t \) is associated with (47).

We also have that

\[ V_K (K_t, B_t, L_t, M_t; z_t, f_{t-1}) = \eta_t \left\{ 1 + (1 - \tau_C) \left( \alpha \exp(z_t) \Gamma K_t^{(\alpha - 1)} - \delta \right) + \frac{\omega_K}{2} \left( \frac{K_{t+1}}{K_t} \right)^2 - 1 \right\} \]  

\[ V_B (K_t, B_t, L_t, M_t; z_t, f_{t-1}) = -\eta_t \left( 1 + (1 - \tau_C) r \right) \]  

\[ V_L (K_t, B_t, L_t, M_t; z_t, f_{t-1}) = -\eta_t \left( 1 + (1 - \tau_C) \xi \right) \]  

\[ V_M (K_t, B_t, L_t, M_t; z_t, f_{t-1}) = \eta_t \left( 1 + (1 - \tau_C) \iota \right) \]  

6.2 Proofs

**Proof of Proposition 1**

We first rewrite the first-order conditions of the second subperiod problem as follows.

If \( z_t < \bar{z} \), the solution is simply that \( L_{t+1} = 0 \) and \( M_{t+1} = S_t + (1 - \tau_C) f_t \geq 0 \).

If \( z_t \geq \bar{z} \), the solution must satisfy the following first-order conditions:

\[ \zeta_t + \gamma_t^M = \beta R^M E_t^+ [\eta_{t+1}] \]  

\[ \zeta_t + \gamma_t^L - \gamma_t^U = \beta R^L E_t^+ [\eta_{t+1}] \]  

\[ \gamma_t^M \geq 0, \quad M_{t+1} \geq 0, \quad \gamma_t^M M_{t+1} = 0 \]
The first two conditions impose that

\[
\gamma_{t+}^L + \gamma_{t+}^U = 0, \quad L_{t+1} \geq 0, \quad \gamma_{t+}^L L_{t+1} = 0
\]  

(60)

\[
\gamma_{t+}^U \geq 0, \quad \bar{L} - L_{t+1} \geq 0, \quad \gamma_{t+}^U (\bar{L} - L_{t+1}) = 0.
\]  

(61)

The first two conditions impose that

\[
\gamma_{t+}^L - \gamma_{t+}^U + \gamma_{t+}^M = \beta (R^L - R^M) E_{t+} [\eta_{t+1}] > 0
\]  

(62)

because \( R^L > R^M \) and \( \eta_t = U'(D_t) > 0 \).

We note that the constraints on the credit line are such that

- If \( \gamma_{t+}^L > 0 \), then \( L_{t+1} = 0 < \bar{L} \) and \( \gamma_{t+}^U = 0 \)
- If \( \gamma_{t+}^U > 0 \), then \( L_{t+1} = \bar{L} > 0 \) and \( \gamma_{t+}^L = 0 \)
- If \( \gamma_{t+}^L = \gamma_{t+}^U = 0 \), then \( 0 \leq L_{t+1} \leq \bar{L} \)

Then,

- If \( \gamma_{t+}^M > 0 \) and \( \gamma_{t+}^L > 0 \) and \( \gamma_{t+}^U = 0 \), then \( M_{t+1} = L_{t+1} = 0 \)
- If \( \gamma_{t+}^M > 0 \), \( \gamma_{t+}^L = 0 \), and \( \gamma_{t+}^U = 0 \), then \( M_{t+1} = 0 \) and \( 0 \leq L_{t+1} \leq \bar{L} \)
- If \( \gamma_{t+}^M > 0 \), \( \gamma_{t+}^L = 0 \), and \( \gamma_{t+}^U > 0 \), then \( M_{t+1} = 0 \) and \( L_{t+1} = \bar{L} \)
- If \( \gamma_{t+}^M = 0 \) then \( \gamma_{t+}^L > 0 \), \( \gamma_{t+}^U = 0 \), \( M_{t+1} \geq 0 \), and \( L_{t+1} = 0 \)

These regions suggest that we can only have \( M_{t+1} > 0 \) and \( L_{t+1} = 0 \), \( M_{t+1} = L_{t+1} = 0 \), or \( M_{t+1} = 0 \) and \( 0 < L_{t+1} \leq \bar{L} \).

**Proof of Proposition 2**

The relevant set of equations here is

\[
M_{t+1} = L_{t+1} + S_t + (1 - \tau_C) f_t
\]  

(63)

\[
S_t \geq -\bar{L} 1(z_t \geq \tilde{z}) + (1 - \tau_C) \sigma_F.
\]  

(64)

To obtain \( M_{t+1} > 0 \) and \( L_{t+1} = 0 \):
• If \( z_t < \bar{z} \), then \( M_{t+1} = S_t + (1 - \tau_C)f_t \geq (1 - \tau_C)(\sigma_F + f_t) \geq 0 \)

1. \( M_{t+1} > 0 \) \((L_{t+1} = 0)\) when \( f_t > -\sigma_F \) for all values of \( S_t \geq (1 - \tau_C)\sigma_F \)

2. \( M_{t+1} > 0 \) \((L_{t+1} = 0)\) when \( S_t > (1 - \tau_C)\sigma_F \) for all values of \( f_t \)

• If \( z_t \geq \bar{z} \), then \( M_{t+1} = S_t + L_{t+1} + (1 - \tau_C)f_t \geq L_{t+1} - \bar{L} + (1 - \tau_C)(\sigma_F + f_t) \)

1. \( M_{t+1} > 0 \) \((L_{t+1} = 0)\) when \( f_t > -\sigma_F \) for all values of \( S_t \geq -\bar{L} + (1 - \tau_C)\sigma_F \)

2. \( M_{t+1} > 0 \) \((L_{t+1} = 0)\) when \( S_t > (1 - \tau_C)\sigma_F \) for all values of \( f_t \).

**Proof of Proposition 3**

The relevant Euler equation is

\[
U'(D_t) - \lambda_t = \beta R^M E_t \left[ U'(D_{t+1}) \right] + E_t \left[ \gamma_{t+1}^M \right].
\]

(65)

Recall that \( \beta R^M < 1 \). If \( U'(\cdot) \) is sufficiently convex to ensure that \( E_t \left[ U'(D_{t+1}) \right] = [1/(\beta R^M)]U'(D_t) > U'(D_t) \), then \( \lambda_t + E_t \left[ \gamma_{t+1}^M \right] = 0 \). Because the multipliers are non-negative, this requires that both \( \lambda_t = 0 \) and that \( E_t \left[ \gamma_{t+1}^M \right] = 0 \). The last requirement effectively means that \( \gamma_{t+1}^M = 0 \) for all values of \( f_t \) when \( z_t \geq \bar{z} \) (of course, \( \gamma_{t+1}^M = 0 \) when \( z_t < \bar{z} \)). As a result, \( S_t > (1 - \tau_C)\sigma_F \) and \( M_{t+1} > 0 \) for all values of \( f_t \).

Note that the condition \( E_t \left[ U'(D_{t+1}) \right] > U'(D_t) \) can occur because our assumptions on the schedule of taxes and equity issuing costs \( T(D_t) \) ensures that \( U'(D_t) \) is convex (Jensen’s Inequality).

**Proof of Proposition 4**

A comparison of cash and debt Euler equations yields

\[
\frac{E_t \left[ \gamma_{t+1}^M \right] + \lambda_t}{U'(D_t) E_t [m_{t+1}]} = R^B_t - R^M.
\]

(66)

When \( R^B_t = R^M \), the firm is indifferent between holding cash and holding debt. As a result, \( E_t \left[ \gamma_{t+1}^M \right] + \lambda_t = 0 \) which requires both \( \lambda_t = 0 \) and \( \gamma_{t+1}^M = 0 \) for all values of \( f_t \). Then, the firm chooses a high level of liquidity \( S_t > (1 - \tau_C)\sigma_F \) to ensure that \( M_{t+1} > 0 \) for all values of \( f_t \).
When $R_t^B > R^M$, then $E_t \left[ \gamma_t^M \right] + \lambda_t > 0$. This occurs because $\lambda_t \geq 0$ and $\gamma_t^M > 0$ for some values of $f_t$.

**Proof of Proposition 5**

A comparison of cash and capital Euler equations reveals that

$$\frac{E_t \left[ \gamma_t^M \right] + \lambda_t}{U'(D_t) E_t \left[ m_{t+1} \right]} = E_t \left[ R_{t+1}^K \right] - R^M + \frac{Cov_t \left[ m_{t+1}, R_{t+1}^K \right]}{E_t \left[ m_{t+1} \right]}.$$ (67)

When $E_t \left[ R_{t+1}^K \right] - R^M + \frac{Cov_t \left[ m_{t+1}, R_{t+1}^K \right]}{E_t \left[ m_{t+1} \right]} = 0$, the firm is indifferent between holding cash and holding capital. As a result, $E_t \left[ \gamma_t^M \right] + \lambda_t = 0$ which requires both $\lambda_t = 0$ and $\gamma_t^M = 0$ for all values of $f_t$. Then, the firm chooses a high level of liquidity $S_t > (1 - \tau_C)\sigma_F$ to ensure that $M_{t+1} > 0$ for all values of $f_t$.

### 6.3 Numerical Method

The model is solved numerically using a finite element method as described in Coleman’s (1990) algorithm. Accordingly, the policy functions $K_{t+1}$, $M_{t+1}$, $B_{t+1}$, and co-states $\lambda_t$, $V_t$ are approximated by piecewise linear interpolants of the state variables $K_t$, $M_t$, $B_t$, as well as $z_t$. The numerical integration involved in computing expectations is approximated with a Gauss-Hermite quadrature rule with two quadrature nodes.

This state space grid consists of 625 uniformly spaced points for the beginning-of-the-year state variables. The lowest and highest grid points for the endogenous state variables $K_t$, $M_t$, and $B_t$ are specified outside the endogenous choices of the firm. The lowest and highest grid points for the income shock $z_t$ are specified three standard deviations away, at $\exp(-3\sigma_{\epsilon_1})$ and $\exp(+3\sigma_{\epsilon_1})$.

The approximation coefficients of the piecewise linear interpolants are chosen by collocation, i.e., to satisfy the relevant system of equations at all grid points. The approximated policy interpolants are substituted in the equations, and the coefficients are chosen so that the residuals are set to zero at all grid points. The time-stepping algorithm is used to find these root coefficients. Given initial coefficient values for all grid points, the time-stepping algorithm finds the optimal coefficients that minimize the residuals at one grid point, taking coefficients at other grid points as given. In turn, optimal coefficients for all grid points are determined. The iteration over coefficients stops when
the maximum deviation of optimal coefficients from their previous values is lower than a specified tolerance level, e.g., 0.0001.

6.4 Estimation by Simulation

Our estimation procedure follows a moment matching procedure similar to Ingram and Lee (1991). We compute moments in the data and in the simulation as

\[ \tilde{H}(x) = \frac{1}{F} \sum_{f=1}^{F} \left[ \frac{1}{T} \sum_{t=1}^{T} h(x_{f,t}) \right] \quad \text{and} \quad \tilde{H}_s(\theta) = \frac{1}{F} \sum_{f=1}^{F} \left[ \frac{1}{T} \sum_{t=1}^{T} h(x_{s,f,t}(\theta)) \right], \]

where \( \tilde{H}(x) \) is an \( \tilde{m} \)-vector of statistics computed on the actual data matrix \( x \) and \( \tilde{H}_s(\theta) \) is an \( \tilde{n} \)-vector of statistics computed on the simulated data for panel \( s \). The simulated statistics depend on the \( k \)-vector of parameters \( \theta \). We use these statistics to construct the \( m < \tilde{m} \) moments \( H(x) \) and \( H_s(\theta) \) on which our estimation is based. The estimator \( \hat{\theta} \) of \( \theta \) is the solution to

\[ \min_{\theta} \left[ H(x) - \frac{1}{S} \sum_{s=1}^{S} H_s(\theta) \right]^{\top} W \left[ H(x) - \frac{1}{S} \sum_{s=1}^{S} H_s(\theta) \right], \]

where \( W \) is a positive definite weighting matrix.

For the first period, we compute \( \tilde{m} = 9 \) statistics to form the \( m = 7 \) targeted moments and identify the \( k = 7 \) parameters of the first period (see Table 2). For the last period, we compute \( \tilde{m} = 8 \) statistics to form the \( m = 6 \) targeted moments to identify the \( k = 6 \) parameters of the last period (see Table 3). We construct simulated samples that have the same number of firm-year observations as the data. The actual data sample has 5,469 firms and 53,067 firm-year observations for the first period and 7,220 firms and 67,720 firm-year observations for the second sample. To replicate the data, the simulated samples have 53,070 firm-year observations (\( F = 5,307 \) firms and \( T = 10 \) years) for the first period and 67,720 firm-year observations (\( F = 6,772 \) firms and \( T = 10 \) years) for the last period. In practice, we simulate 50 years, but keep only the last 10 years. In both periods, we construct \( S = 5 \) simulated panels. We use weighting matrix \( W = [(1 + 1/S)\hat{\Omega}]^{-1} \).

Our estimates of the covariance matrix is \( \hat{\Sigma} = [B^\top \hat{W} B]^{-1} \), where \( \hat{W} = [(1 + 1/S)\hat{\Omega}]^{-1} \) and \( B \) contains the gradient of the \( m \) moments with respect to the \( k \) parameters. We construct \( \hat{\Omega} \) as \( D'\hat{\Omega}_{\tilde{m}} D \) where \( D \) contains the gradient of the \( m \) moments with respect to the \( \tilde{m} \) statistics and \( \hat{\Omega}_{\tilde{m}} \) is an heteroscedasticity-consistent covariance matrix of the \( \tilde{m} \) statistics in the simulation.
6.5 Parameters Estimated from the Data

Table 1 presents the first set of parameter estimates for both periods. The capital intensity $\alpha$, the productivity scale $\Gamma$, the persistence of the revenue shock $\rho_z$, and the volatility of its innovations $\sigma_z$ are estimated from the revenue Equation (3) and the autoregressive process (4). For each of the two time periods, the four parameters are estimated for each firm, and then averaged over all firms. Revenues $Y_t$ are measured as sales, and the beginning-of-the-period capital stock $K_t$ is measured as lagged property, plant, and equipment.\(^5\)

For the 1971-1982 period, the averages are $\alpha = 0.579$, $\Gamma = 2.273$, $\rho_z = 0.245$, and $\sigma_z = 0.210$. For the 1995-2006 period, the averages are $\alpha = 0.423$, $\Gamma = 3.180$, $\rho_z = 0.206$ and $\sigma_z = 0.309$. The share $\alpha$ of physical capital that explains revenues has decreased over time but the productivity scale $\Gamma$ has increased. Note that the values for the capital intensity $\alpha$ are in line with the values used in Moyen (2004), Hennessy and Whited (2005, 2007), and Gamba and Triantis (2008). Over time, shocks to revenues became less persistent ($\rho_z$) and their innovations more volatile ($\sigma_z$). The parameters of the stochastic process indicate a significant increase in the unconditional variance of the revenue shock $\sigma_z^2/(1 - \rho_z^2)$ from 4.7 percent during the 1971-1982 period to 10 percent during the 1995-2006 period.

The corporate tax rate $\tau_C$ is set to the top marginal rate. The top marginal tax rate was 48 percent from 1971 to 1978 and 46 percent from 1979 to 1982. The top corporate marginal tax rate has been constant at 35 percent since 1993. As a result, the corporate tax rate is set to its twelve year average of $\tau_C = 0.473$ for the first period and to $\tau_C = 0.35$ for the last period. The personal tax rates are set to the average marginal tax rates reported in NBER’s TAXSIM. Over the 1971-1982 period, the marginal interest income tax rate averaged $\tau_r = 0.276$ while the marginal dividend tax rate averaged $\tau_D = 0.395$. Over the 1995-2006 period, the marginal interest income tax rate averaged $\tau_r = 0.244$ while the marginal dividend tax rate averaged $\tau_D = 0.233$.

\(^5\)As an alternative, the capital stock could be reconstructed from the accumulation Equation (7) using capital expenditures (CAPX) assuming an initial value for the capital stock and a value for the depreciation rate. We do not pursue this alternative approach for two reasons. First, it requires a value for the depreciation rate, which would prevent our estimation of the depreciation rate. Second, our results obtained by measuring the capital stock as lagged property, plant, and equipment yield parameter estimates similar to those obtained elsewhere in the literature.
The real interest rate $r$ is set to the average of the monthly annualized t-bill rate deflated by the consumer price index. High inflation characterized much of the period from 1971 to 1982. As a result, the real interest rate was quite low, at 0.585 percent. As for the later period of 1995 to 2006, the real interest rate was higher, at 1.609 percent. For the interest rate earned on cash holdings $\iota$, we disentangle the two components of cash (CHE): short-term investments (IVST) and cash (CH). In 1971-1982, firms held 30.7 percent of their cash in short-term investments earning a rate of return $r$ and 69.3 percent in cash earning a zero nominal interest rate. Given an average inflation rate of 7.9 percent, the interest rate on cash holdings is set to $\iota = 0.307r + 0.693(0 - 0.079) = -5.3$ percent. By 1995, sweep money market accounts became available to firms. Therefore we calibrate the interest rate on cash holdings as primarily dependent on the risk free rate minus 100 basis points of expense ratio and FDIC rate, so that $\iota = 0.92(r - 0.01) + 0.08(0 - 0.026) = 0.352$ percent. We assume that the minimum balance is eight percent, inspired from the Basel requirement, where the minimum balance depreciates at the inflation rate during the 1995-2006 period of 2.6 percent. Finally, for the interest rate on the credit line, we apply a premium above the real interest rate. For the 1971-1982 period, we use Ham and Melnik (1987) and set $\xi = r + 0.008$. For the 1995-2006 period, we use Sufi (2009) and set $\xi = r + 0.015$.

### 6.6 Parameters Estimated by Matching Moments

The last set of parameters cannot be estimated in isolation directly from the data. The estimation procedure (described above) is based on the simulated method of moments. In spirit, the estimation strategy targets a particular moment for each parameter. In practice, a change to one parameter affects all simulated moments. The parameters to be estimated are the depreciation rate $\delta$, the capital adjustment cost $\omega_K$, the long-run debt level $\bar{B}$, the debt deviation cost $\omega_B$, the average funding shock level $\bar{F}$, the funding shock volatility $\sigma_F$, and the coefficient of absolute prudence $\phi$. The estimation seeks to replicate the important features of the data, namely moments of investment, debt, and cash policies.

Two moments of the capital policy are targeted to estimate the depreciation rate $\delta$ and the adjustment cost parameter $\omega_K$. The estimate of the depreciation rate $\delta$ ensures that the average of
investment-to-total assets simulated from the model matches the average investment found in the data. In the COMPUSTAT data, the ratio is computed as capital expenditures (CAPX) divided by total assets (AT). In the model simulated data, the ratio is computed as investment \( I_t \) divided by total assets \( A_t \). The estimate of the adjustment cost parameter \( \omega_K \) ensures that the simulated standard deviation of investment \( I_t / A_t \) normalized by the standard deviation of revenues \( Y_t / A_t \) matches that of the data. We normalize by the standard deviation of revenues so that the capital adjustment cost \( \omega_K \) can target the volatility of investment in reference to the volatility of the revenue shock \( \sigma_z \).

For the long-run debt level \( \bar{B} \) and the cost parameter \( \omega_B \), we target two moments of the debt policy. Our estimate of \( \bar{B} \) ensures that the simulated average leverage \( B_t / A_t \) matches that of COMPUSTAT firms. Leverage is measured by the sum of long-term debt (DLTT) and debt in current liabilities (DLC) divided by total assets. Similarly to \( \omega_K \), our estimate of \( \omega_B \) ensures that the simulated standard deviation of debt relative to the standard deviation of revenues matches that of the actual data. Because costs are more relevant to long-term debt than to short-term debt, we focus on the standard deviation of long-term debt-to-capital stock. This standard deviation is then normalized by the standard deviation of revenues-to-capital stock.

The estimate of the average funding shock level \( \bar{F} \) ensures that the average of operating income-to-total assets ratio \( OI_t / A_t \) matches the data, where operating income \( OI_t \) is measured before depreciation (OIBDP). The estimate of \( \sigma_F \) ensures that the standard deviation of net income-to-total assets \( NI_t / A_t \) matches the data, where net income is measured as \( NI_t \). We target net income because we want to allow for expenses similar to extraordinary items: expenses that may not be part of the regular operations of the firm but that can affect the firm’s financial health.

Finally, the convexity parameter \( \phi \) is the coefficient of absolute prudence. We estimate \( \phi \) to ensure that the average of cash holdings-to-total assets \( M_{t+1} / A_t \) matches the data in the first time period of 1971-1982, where cash holdings are measured by cash and short-term investments (CHE).

Tables 2 and 3 present the results of the moment matching exercise. Table 2 shows the parameter values and the target moments for the period covering 1971 to 1982, while Table 3 does so for the
period covering 1995 to 2006.

In the data, the average investment-to-total assets is 8 percent in the first time period and 6 percent in the last time period. To hit these moments, the estimated depreciation rate $\delta$ is set to 0.176 in the first time period and to 0.174 in the last time period. The results indicate that the reduction in capital investment from 8 percent to 6 percent results mostly from the lower capital share $\alpha$.

In COMPSTAT data, investment has an average standard deviation of 16.8 percent of the average standard deviation of revenues during the 1971-1982 years and a relative average standard deviation of 12.1 percent during the 1995-2006 years.\(^6\) To replicate these moments, the estimates of the capital adjustment cost $\omega_K$ are 1.273 in the first time period and 0.673 in the last time period. These estimates are of magnitudes similar to those obtained by Cooper and Haltiwanger (2006). All else equal, the lower capital adjustment cost in recent years stimulates the volatility of investments $I_t/A_t$. This higher volatility, however, is overwhelmed by the increased volatility of revenues $Y_t/A_t$. This denominator effect explains why the lower capital adjustment cost parameter replicates the lower ratio of the standard deviation of investment to the standard deviation of revenues in recent years.

The average leverage of COMPSTAT firms has been fairly constant over time: 0.305 during the 1971-1982 period and 0.280 during the 1995-2006 period. To replicate these moments, the estimates of the long-run debt level (standardized by mean total assets) $\bar{B}/\bar{A}$ are 0.305 in the first time period and to 0.280 in the last time period.

The long-term debt-to-capital stock of COMPSTAT firms has an average standard deviation of 16.8 percent of the average standard deviation of revenues-to-capital stock during the 1971-1982 years and a relative average standard deviation of 21.5 percent during the 1995-2006 years. To replicate these moments, the debt cost estimates $\omega_B$ are 0.028 in the first time period and 0.012 in the last time period.

\(^6\)The relative standard deviation computed in the data differs considerably from the relative standard deviation commonly shown using macroeconomic data. This simply results from different measurements. For example, we compute the standard deviation of the ratio of investment-to-total assets, while macroeconomists compute the standard deviation of the logarithm of investment detrended using the Hodrick-Prescott filter.
In COMPUSTAT data, operating income has declined from an average of 13 percent of total assets during the 1971-1982 period to an average of 0.7 percent of total assets during the 1995-2006 period. A lower average funding shock is required to explain the reduction in the average operating income over time. The estimates of the average expense level (standardized by mean total assets) $\bar{E}/\bar{A}$ are $-0.017$ for the first period and $-0.158$ for the last period.

In the data, the standard deviation of net income-to-total assets has greatly increased over time from an average of 0.067 during the 1971-1982 years to an average of 0.180 during the 1995-2006 years. The estimates of the volatility parameter (standardized by mean total assets) $\sigma_{E/\bar{A}}$ are 0.124 for the first period and 0.276 for the last period.

7 References


Sannikov, Y., 2007. Agency problems, screening and increasing credit lines. mimeo.


Table 1
Parameter Estimates of the Calibration

The parameter estimates are based on North American data from COMPUSTAT for the sample periods 1971 to 1982 and 1995 to 2006. The COMPUSTAT samples include firm-year observations with positive values for total assets (COMPUSTAT Mnemonic AT), property, plant, and equipment (PPENT), and sales (SALE). The sample includes firms from all industries, except for utilities and financials, with at least five years of consecutive data. The data are winsorized to limit the influence of outliers at the 1% and 99% tails. For each of the two time periods, we estimate the four parameters of the revenue function per firm, and then average the estimates over all firms. The corporate tax rates are calibrated to the top marginal rate, while the personal tax rates are calibrated to the average marginal tax rates reported in NBER’s TAXSIM. The real interest rates are calibrated to the average of the monthly annualized t-bill rate deflated by the consumer price index. For the 1971-1982 period, the interest rate earned on cash holdings is calibrated as the proportion of cash held in short-term investments (CHE) which earns the the real interest rate, plus the proportion held in cash (CH) which earns a zero nominal interest rate deflated by the consumer price index. For the 1995-2006 period, the interest rate earned on cash holdings is calibrated as the real interest rate minus 100 basis points of expense ratio and FDIC rate, except for a minimum balance that earns a zero nominal interest rate deflated by the consumer price index. The interest rates paid on credit line debt are calibrated as the real interest rate plus a credit line premium.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.579</td>
<td>0.423</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>2.273</td>
<td>3.180</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.245</td>
<td>0.206</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.210</td>
<td>0.309</td>
</tr>
<tr>
<td>Tax Rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_C$</td>
<td>0.473</td>
<td>0.350</td>
</tr>
<tr>
<td>$\tau_P$</td>
<td>0.276</td>
<td>0.244</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>0.395</td>
<td>0.233</td>
</tr>
<tr>
<td>Interest Rates (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>0.585</td>
<td>1.609</td>
</tr>
<tr>
<td>$\iota$</td>
<td>-5.319</td>
<td>0.352</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.593</td>
<td>1.624</td>
</tr>
</tbody>
</table>

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Table 2
Matching Moments for the 1971-1982 Period

The observed moments are computed using a sample of North American data from COMPUSTAT for the sample period 1971 to 1982. The simulated moments are computed using 5 simulated panels of 5,307 firms over 10 years. I denotes investment, A total assets, Y revenues, B debt level, K capital stock, OI operating income, NI net income, M cash holdings, D dividends, ΔB′ debt issues, and primed variables refer to time t + 1 values rather than time t values. The model is solved using a finite-element method. The parameters are estimated using a just identified system of moment matching. The numbers in parenthesis are the standard deviations of the estimated parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Targeted Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ</td>
<td>0.176</td>
<td>Mean(I/A)</td>
<td>0.080</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω_K</td>
<td>1.273</td>
<td>SD(I/A)/SD(Y/A)</td>
<td>0.168</td>
<td>0.168</td>
</tr>
<tr>
<td></td>
<td>(1.936)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B/Á</td>
<td>0.305</td>
<td>Mean(B′/A)</td>
<td>0.305</td>
<td>0.305</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω_B</td>
<td>0.028</td>
<td>SD(B′/K′)/SD(Y/K′)</td>
<td>0.168</td>
<td>0.168</td>
</tr>
<tr>
<td></td>
<td>(0.264)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F/Á</td>
<td>−0.017</td>
<td>Mean(OI/A)</td>
<td>0.130</td>
<td>0.130</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ_F/Á</td>
<td>0.124</td>
<td>SD(NI/A)</td>
<td>0.067</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ϕ</td>
<td>0.004</td>
<td>Mean(M′/A)</td>
<td>0.089</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Other Moments

<table>
<thead>
<tr>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean(L+/A)</td>
<td>n.a.</td>
</tr>
<tr>
<td>SD(D/A)</td>
<td>0.036</td>
</tr>
<tr>
<td>0.032</td>
<td>0.032</td>
</tr>
<tr>
<td>Corr(ΔB′/A, Y/A)</td>
<td>−0.333</td>
</tr>
</tbody>
</table>

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Table 3
Matching Moments for the 1995-2006 Period

The observed moments are computed using a sample of North American data from COMPUSTAT for the sample period 1995 to 2006, except for the used line of credit which is taken from Sufi (2009). The simulated moments are computed using 5 simulated panels of 6,772 firms over 10 years. I denotes investment, A total assets, Y revenues, B debt level, K capital stock, OI operating income, NI net income, M cash holdings, D dividends, ∆B’ debt issues, and primed variables refer to time $t + 1$ values rather than time $t$ values. The model is solved using a finite-element method. The parameters are estimated using a just identified system of moment matching. The numbers in parenthesis are the standard deviations of the estimated parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Targeted Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.176</td>
<td>Mean(I/A)</td>
<td>0.060</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_K$</td>
<td>0.673</td>
<td>SD(I/A)/SD(Y/A)</td>
<td>0.121</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>(0.420)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B/\bar{A}$</td>
<td>0.280</td>
<td>Mean(B’/A)</td>
<td>0.280</td>
<td>0.280</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>0.012</td>
<td>SD(B’/K’)/SD(Y’/K’)</td>
<td>0.215</td>
<td>0.215</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{F}/\bar{A}$</td>
<td>−0.1584</td>
<td>Mean(OI/A)</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_F/\bar{A}$</td>
<td>0.276</td>
<td>SD(NI/A)</td>
<td>0.180</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>(0.109)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean(M’/A)</td>
<td>0.175</td>
<td>0.171</td>
</tr>
<tr>
<td>Mean(L’+/A)</td>
<td>0.094</td>
<td>0.047</td>
</tr>
<tr>
<td>SD(D/A)</td>
<td>0.141</td>
<td>0.105</td>
</tr>
<tr>
<td>Corr(∆B’/A,Y/A)</td>
<td>−0.123</td>
<td>−0.188</td>
</tr>
</tbody>
</table>
Table 4
Sensitivity Analysis

The simulated moments are computed using 5 simulated panels of 5,307 firms over 10 years. For each parameter, we report the cash holdings and credit card usage obtained from changing the first period parameter value to its second period value, holding all other parameters constant.

<table>
<thead>
<tr>
<th></th>
<th>1971-1982</th>
<th>1995-2006</th>
<th>Mean((M'/A))</th>
<th>Mean((L^+/A))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark Calibration</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971-1982</td>
<td></td>
<td></td>
<td>0.089</td>
<td>0.022</td>
</tr>
<tr>
<td>1995-2006</td>
<td></td>
<td></td>
<td>0.175</td>
<td>0.094</td>
</tr>
<tr>
<td><strong>Liquidity Policy Parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>interest rate on cash (%) (\nu)</td>
<td>-5.319</td>
<td>0.352</td>
<td>0.223</td>
<td>0</td>
</tr>
<tr>
<td>interest rate on debt (%) (r)</td>
<td>0.585</td>
<td>1.609</td>
<td>0.105</td>
<td>0.049</td>
</tr>
<tr>
<td>interest rate on credit line (%) (\xi)</td>
<td>1.422</td>
<td>3.109</td>
<td>0.105</td>
<td>0.009</td>
</tr>
<tr>
<td>corporate tax rate (\tau_c)</td>
<td>0.473</td>
<td>0.350</td>
<td>0.096</td>
<td>0.034</td>
</tr>
<tr>
<td>interest income tax rate (\tau_r)</td>
<td>0.276</td>
<td>0.244</td>
<td>0.089</td>
<td>0.022</td>
</tr>
<tr>
<td>dividend tax rate (\tau_D)</td>
<td>0.395</td>
<td>0.233</td>
<td>0.064</td>
<td>0.030</td>
</tr>
<tr>
<td>average expense (\bar{F}/\bar{A})</td>
<td>0.020</td>
<td>0.161</td>
<td>0.091</td>
<td>0.022</td>
</tr>
<tr>
<td>additive shock volatility (\sigma_F/\bar{A})</td>
<td>0.123</td>
<td>0.274</td>
<td>0.159</td>
<td>0.079</td>
</tr>
<tr>
<td><strong>Debt Policy Parameters</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>debt adjustment cost (\omega_B)</td>
<td>0.015</td>
<td>0.011</td>
<td>0.085</td>
<td>0.023</td>
</tr>
<tr>
<td>debt target (\bar{B}/\bar{A})</td>
<td>0.305</td>
<td>0.278</td>
<td>0.089</td>
<td>0.022</td>
</tr>
<tr>
<td><strong>Capital Policy Parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>depreciation rate (\delta)</td>
<td>0.170</td>
<td>0.166</td>
<td>0.091</td>
<td>0.022</td>
</tr>
<tr>
<td>capital adjustment cost (\omega_K)</td>
<td>0.971</td>
<td>0.522</td>
<td>0.085</td>
<td>0.023</td>
</tr>
<tr>
<td>productivity scale (\Gamma)</td>
<td>2.273</td>
<td>3.180</td>
<td>0.206</td>
<td>0</td>
</tr>
<tr>
<td>capital intensity (\alpha)</td>
<td>0.579</td>
<td>0.423</td>
<td>0.051</td>
<td>0.058</td>
</tr>
<tr>
<td>revenue shock persistence (\rho_z)</td>
<td>0.245</td>
<td>0.206</td>
<td>0.090</td>
<td>0.022</td>
</tr>
<tr>
<td>revenue shock volatility (\sigma_z)</td>
<td>0.210</td>
<td>0.309</td>
<td>0.099</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Table 5
Extensions

The simulated moments are computed using 5 simulated panels of 5,307 firms over 10 years for the first period calibration and 5 simulated panels of 6,772 firms over 10 years for the last period calibration. \( M \) denotes cash holdings and \( A \) total assets. Primed variables refer to time \( t+1 \) values rather than time \( t \) values. For all three extensions, the moments are computed from versions of the model that hold all other parameters to their benchmark calibrated values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>0.089</td>
<td>n.a.</td>
<td>0.171</td>
<td>0.047</td>
</tr>
<tr>
<td>Benchmark Model</td>
<td>0.089</td>
<td>0.022</td>
<td>0.175</td>
<td>0.094</td>
</tr>
<tr>
<td>Introducing Asymmetric Costs</td>
<td>0.093</td>
<td>0.021</td>
<td>0.178</td>
<td>0.093</td>
</tr>
<tr>
<td>Using ( \phi ) to Match Payout Volatility</td>
<td>0.100</td>
<td>0.020</td>
<td>0.225</td>
<td>0.071</td>
</tr>
</tbody>
</table>
Figure 1: Average North American Firm Policies

Cash M'/A
Capital K'/A
Debt B'/A
Figure 2: Tax and Equity Issuing Cost Schedule $T(D)$

- $\tau_D=0.395$ and $\phi=0.004$
- $\tau_D=0.233$ and $\phi=0.004$
- $\tau_D=0.395$ and $\phi=0.005$
- $\tau_D=0.233$ and $\phi=0.007$
Figure 3: Cash Holdings M'/A of Simulated Firms

- 1971-1982 Calibration
- 1995-2006 Calibration
Figure 4: Used Line of Credit \( L'/A \)
Figure 5: Liquidity Constraint Multiplier $\lambda$
Figure 6: Cash Savings S/Mean(A)