Discussion of Good Deal Bound Pricing, with Applications to Credit Risk
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World OTC Options Outstanding Notionals

Figure: Notional amounts outstanding of OTC Options in billions of USD. Source: BIS.
Counterparty Risk

- Notional amount outstanding of OTC options Dec 2009:
  - Interest rate options: 48,808 bn USD
  - Equity-linked options: 4,762 bn USD
- Counterparty risk is substantial
- Counterparty risk implies market incompleteness (e.g. vulnerable options)
Pricing in Incomplete Markets

- Goal: value risky payoff by reference to prices of traded assets.
- Difficult if there is no perfect replicating portfolio (incompleteness).
- Challenge: find bounds on prices in this situation.
- No-arbitrage bounds (super-hedging): too large
- Ad-hoc choice: e.g. minimal martingale measure
- Cochrane and Saà Requejo (2000) impose bound on Sharpe ratios: good deal bounds

- Financial market
  \[ dS_0 = S_0 r \, dt \]
  \[ dS_i = S_i (\alpha_i \, dt + (\sigma \, dW)_i) \]

- No-arbitrage: there exists a (non-unique) market price of risk \( \lambda \)
  \[ \sigma \lambda = \alpha - r \, 1 \]

- Minimal market price of risk (pseudo-inverse)
  \[ \hat{\lambda} = \sigma^{\top} (\sigma \sigma^{\top})^{-1} (\alpha - r \, 1) \]
  satisfies
  \[ \| \hat{\lambda} \| \leq \| \lambda \| \]

- Corresponds to minimal martingale measure

- Portfolio \( \pi \), self-financing \( (\pi^\top \mathbf{1} = 1) \)

\[
dS^\pi = S^\pi \left( \pi^\top \alpha \, dt + \pi^\top \sigma \, dW \right)
\]

- Sharpe ratio \( SR(\pi) = \frac{\pi^\top \alpha - r}{\| \pi^\top \sigma \|} \) satisfies

\[
|SR(\pi)| = \left| \frac{\pi^\top \sigma \lambda}{\| \pi^\top \sigma \|} \right| \leq \| \lambda \|
\]

- Hansen–Jagannathan Bounds:

\[
|SR(\pi)| \leq \| \hat{\lambda} \| \leq \| \lambda \|
\]
Björk and Slinko (2006)

- Extend Hansen–Jagannathan bounds to jump-diffusion market model
  \[ |SR(\pi)|^2 \leq \|\lambda\|^2 + \int_{\mathcal{X}} \varphi(x)^2 \nu(x) \, dx \]

- Extend Cochrane and Saá Requejo (2000) notion of upper/lower good-deal price by bounding the HJ bounds
  \[ \|\lambda\|^2 + \int_{\mathcal{X}} \varphi(x)^2 \nu(x) \, dx \leq B^2 \]

- Good deal bound pricing becomes mathematically tractable.
Good Deal Bound Pricing

- Contingent claim \( \Phi(S_T, Z_T) \) has upper good deal bound price

\[
U_t = \sup_{\lambda, \varphi} \mathbb{E}_Q \left[ e^{-\int_t^T r_s \, ds} \, \Phi(S_T, Z_T) \mid \mathcal{F}_t \right]
\]

subject to
- NA and good deal bound \( \|\lambda\|^2 + \int x \, \varphi(x)^2 \nu(x) \, dx \leq B^2 \)
- Similar definition for lower good deal bound price

\[
L_t = \inf_{\lambda, \varphi} \mathbb{E}_Q \left[ e^{-\int_t^T r_s \, ds} \, \Phi(S_T, Z_T) \mid \mathcal{F}_t \right]
\]

- Both price processes, \( L_t \) and \( U_t \), have a Sharpe ratio bounded by \( B \), and so does (indeed?) any arbitrage-free price process

\[
L_t \leq P_t \leq U_t
\]
Risk Measurement

- Lower good deal bound price $L_t$ is coherent risk-adjusted value of $\Phi(S_T, Z_T)$.
- In other words, $-L_t$ is coherent risk measure,
- Complies with axioms of coherent risk measures.
- GDB price seems robust with respect to $\mathbb{P}$-drift and GDB $B$ specifications.
  - Good choices of $B$ seem to be around 2–4
- Can reduce model risk!
  - Has this been exploited?
Computability

- Good deal bound pricing leads to “beautiful” mathematical problems (non-standard Hamilton–Jacobi–Bellman equations, etc.).
- Challenge: can we compute the prices efficiently and accurately?
- Systematic numerical studies needed, including Markovian credit migration models à la Donnelly
Summary

- The GDB approach provides a potential alternative risk-adjusted pricing tool.
- Further studies are needed to understand the implementation and effects.
- In particular, it should also be applied to vulnerable interest rate options (see first slide)!
References

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