Measuring Default Contagion and System Risk: insights from network models

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Lausanne, October 2010
Main points of the paper

- How to measure *stability* of a financial system
- How to improve stability in an *efficient* way

- The authors work on a unique set of data (Brazil Banks, June 07 - Dec 08).
- Different measures are developed and show intuitive results on this dataset.
On the methodological side, the goal is to represent the financial system as a network.

- $n$ nodes; the exposure of node $i$ to node $j$ is $E_{ij}$.
- Node $i$ has capital $c_i$ and liquidity $l_i$.
- If $c_i = 0$ then the node $i$ defaults.
- Contagion: If $i$ defaults, then node $j$ also defaults if
  \[ c_j < (1 - R_i)E_{ji}. \]

Introduce stochastic market shocks:

- Consider $\epsilon_i, \ldots, \epsilon_n$ which reduce capital to $(c_i + \epsilon_i)_+$.
- If $i$ defaults, then node $j$ defaults if
  \[ (c_j + \epsilon_j)_+ < (1 - R_i)E_{ji} \]
  or derivative payouts are larger than the liquidity
  \[ l_i + \sum_j \pi_{ij}(c + \epsilon, E) < 0. \]
This induces a default cascade: $D_0(A) \subset D_1(A) \subset \cdots \subset D_{n-1}(A)$.

- **Static**: Set $\epsilon = 0$. Leads to the default impact $DI(i, c + \epsilon)$ (loss by $D_{n-1}({i})$).
- **Stochastic**: Choose model for $\epsilon$ and define the contagion index

$$\mathbb{E}(DI(i, c + \epsilon)|c_i + \epsilon_i \leq 0).$$

For assessing systemic risk the authors only consider a subset $C \subset \{1, \ldots, n\}$ of all banks. Then they define analogously

- **Static** Set $\epsilon = 0$ and define systemic risk index $I_C$ as default index only of those nodes in $C$.
- **Stochastic** Choose model for $\epsilon$ and define the systemic risk index

$$\mathbb{E}(I_C(i, c + \epsilon)|c_i + \epsilon_i \leq 0).$$
Main assumption

- Gaussian one-factor model, $Z_0, Z_1, \ldots$ iid $N(0, 1)$ and
  \[ \epsilon_i = \sqrt{\rho}Z_0 + \sqrt{1-\rho}Z_i \]
- Heavy tailed factor model $Z_0, Z_1, \ldots$ iid $\alpha$-stable and
  \[ \epsilon_i = \rho^\alpha Z_0 + (1 - \rho)^\alpha Z_i \]
- $c_i$ are chosen such that the default probability is met.

Questions

- What are the requirements for a good distribution of $\epsilon_i$?
- What is the model risk?
- Should one incorporate feedback effects?
Target immunization

Susceptibility ratio:

$$\chi_i = \max_{j \neq i} \frac{E_{ij}}{c_i}$$

(maximal fraction of wiped out capital on default of node $i$)

- Capital requirement: Impose a cap on $\chi$ for the most systemic nodes.

- Are the results stable amongst distributions of $\epsilon$?
- Time between defaults is not taken into account.
- Relatively short interval of data (stability of the results/outcome)?
- The measures are estimates! Can you give confidence bounds?