Dynamic counterparty risk valuation

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Discussion by Julien Hugonnier
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Thank you Damir...
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and Congratulations on the happy event!
Counterparty risk

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  - Some of the factors that influence your counterpart’s default might also influence the pay-off of the contract or the value of the underlying at the time of default.
- To evaluate exposure (CVA) one needs both the distribution of the default time and the distribution of all state variables conditional on the occurrence of default.
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- Another example: CDS on a name whose credit quality is positively correlated to that of the protection seller.
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• **Goal(s):** calibrate the barrier and/or dynamics to observed spreads and compute the distribution of $(X_t)_{t \geq 0}$ conditional on default occurring at some given point during the life of the contract.
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\[ \sigma(t)^2 \propto H'(t) \left( 1 - H(t) \right) \]

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- The authors derive a **very neat solution**. In particular:

$$\sigma(t)^2 \propto \frac{H'(t)}{1 - H(t)}$$

so that the implied credit index becomes more volatile as the hazard rate of the default distribution increases.
2. Conditional distributions

- The theory of $h-$transforms states that

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\mathbb{P}[A|\tau = s] = \mathbb{E}\left[1(A) \frac{h_s(t, Y_t)}{h_s(0, Y_0)}\right] \equiv \hat{\mathbb{P}}_s[A]
$$

for all $t < s$ and $A \in \mathcal{F}_t$ where

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h_s(t, Y_t) \equiv 1_{\{\tau > t\}} \frac{1}{\text{d}s} \mathbb{P}[\tau \in \text{d}s|Y_t]
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- Together with Girsanov’s theorem this **allows to compute/simulate dynamics conditional on** $\{\tau = s\}$
2. Conditional distributions (cont’d)

- In particular, the dynamics of the credit index conditional on default occurring at time $s$ are given by

$$dY_t = \left( \frac{1}{Y_t} - \frac{Y_t}{v(t,s)} \right) \sigma(t)^2 dt + \sigma(t)d\hat{B}_{s,t}$$

where $\hat{B}_s$ is a Brownian motion under $\hat{P}_s$ and the function $v(t,s)$ is the integrated square volatility.
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- If $\sigma \equiv 1$ then the conditioned credit index is a 3d–Bessel bridge from the point $(0, Y_0)$ to the point $(s, 0)$.

- There are close connections to the theory of conditioned SDEs and to initial enlargements of filtrations.
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5. Can a similar approach be used in reduced–form models?