Fully Flexible Views: Theory and Practice

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discussion by T. Berrada

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Summary

- Non-linear views in non-linear markets
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- Stress testing, scenario analysis, ranking allocation
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- No restrictions on distributions and opinions
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Important Contributions

- No restrictions on distributions and *opinions*
- No repricing in the numerical approach
- High dimensional problems
Relative entropy

- Relative entropy: distance between $f$ and $\tilde{f}$
Relative entropy

- Relative entropy: *distance* between $f$ and $\tilde{f}$
- Also known as *Kullback-Leibler distance*: KL
Relative entropy

- Relative entropy: *distance* between \( f \) and \( \tilde{f} \)

- Also known as **Kullback-Leibler distance**: KL

- KL belongs to the Ali-Silvey class of information theoretic distance measures

\[
d(p_0, p_1) = f \left( E^{p_0} [c(\Lambda(X))] \right)
\]

where \( c \) is convex, \( \Lambda(\cdot) \) is the likelihood ratio and \( f(\cdot) \) is non decreasing
Relative entropy

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where \( c \) is convex, \( \Lambda(\cdot) \) is the likelihood ratio and \( f(\cdot) \) is non decreasing

- \( f(x) = x, \ c(x) = x \log x \quad \Rightarrow \quad \text{KL} \)
The KL distance is not symmetric

\[ KL(p_0, p_1) \neq KL(p_1, p_0) \]
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Appropriate for learning/estimation problems
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Appropriate for learning/estimation problems

Appropriate for opinion pooling?
Consistency Requirement

It would be reasonable to observe

\[ f(x) \xrightarrow{G} \tilde{f}(x) \]
\[ \tilde{f}(x) \xrightarrow{G^{-1}} f(x) \]

\( G \) is a view on the distribution: increase the variance by 1 %

\( G^{-1} \) is a view on the distribution: decrease the variance by 1 %
A simple example: non linear transformation

- Start with an initial distribution $P_0$ of the random variable $X$
A simple example: non linear transformation

- Start with an initial distribution $P_0$ of the random variable $X$
- $\mathcal{V}$ is the set of distribution such that

$$
E^{P \in \mathcal{V}} \left[ (X - E^{P \in \mathcal{V}}(X))^2 \right] = E^{P_0} \left[ (X - E^{P_0}(X))^2 \right] + 1\% 
$$
A simple example: non linear transformation

- Start with an initial distribution $P_0$ of the random variable $X$
- $\mathcal{V}$ is the set of distribution such that
  \[
  \mathbb{E}_{P \in \mathcal{V}} \left[ (X - \mathbb{E}_{P \in \mathcal{V}}(X))^2 \right] = \mathbb{E}_{P_0} \left[ (X - \mathbb{E}_{P_0}(X))^2 \right] + 1\%
  \]
- Choose $P_1$ such that
  \[
  P_1 \equiv \arg \min_{P \in \mathcal{V}} [KL(P, P_0)]
  \]
A simple example: non linear transformation

- Start with an initial distribution $P_0$ of the random variable $X$
- $V$ is the set of distribution such that
  $$E_{P \in V} \left[ (X - E_{P \in V}(X))^2 \right] = E_{P_0} \left[ (X - E_{P_0}(X))^2 \right] + 1\%$$
- Choose $P_1$ such that
  $$P_1 \equiv \arg \min_{P \in V} [KL(P, P_0)]$$
- Starting from $P_1$
A simple example: non linear transformation

- Start with an initial distribution $P_0$ of the random variable $X$
- $\mathcal{V}$ is the set of distribution such that

$$E_{P \in \mathcal{V}}[(X - E_{P \in \mathcal{V}}(X))^2] = E_{P_0}[(X - E_{P_0}(X))^2] + 1\%$$

- Choose $P_1$ such that

$$P_1 \equiv \arg \min_{P \in \mathcal{V}} [KL(P, P_0)]$$

- Starting from $P_1$
- $\mathcal{W}$ is the set of distribution such that

$$E_{P \in \mathcal{W}}[(X - E_{P \in \mathcal{W}}(X))^2] = E_{P_1}[(X - E_{P_1}(X))^2] - 1\%$$
A simple example: non linear transformation

- Start with an initial distribution $P_0$ of the random variable $X$
- $\mathcal{V}$ is the set of distribution such that

$$E_{P \in \mathcal{V}} [(X - E_{P \in \mathcal{V}}(X))^2] = E_{P_0} [(X - E_{P_0}(X))^2] + 1\%$$

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- Starting from $P_1$
- $\mathcal{W}$ is the set of distribution such that

$$E_{P \in \mathcal{W}} [(X - E_{P \in \mathcal{W}}(X))^2] = E_{P_1} [(X - E_{P_1}(X))^2] - 1\%$$

- Choose $P_2$ such that

$$P_2 \equiv \arg \min_{P \in \mathcal{W}} [KL(P, P_1)]$$
A simple example: non-linear transformation

Initial distribution $P_0$

Intermediary distribution $P_1$

Final distribution $P_2$
A simple(r) example: linear transformation

Initial distribution $P_0$

Intermediary distribution $P_1$

Final distribution $P_2$
Alternative distance measure

- Use a *symmetrized* version of KL?

An example from Johnson and Sinanovic (2000):

\[
R(p_0, p_1) \equiv \frac{1}{2} \text{KL}(p_0, p_1) + \frac{1}{2} \text{KL}(p_1, p_0)
\]

Problem: not an Ali-Silvey distance, it may not be appropriate for parameter estimation problems

Arithmetic average (similar to the \(J\)-divergence)

\[
A(p_0, p_1) \equiv \text{KL}(p_0, p_1) + \text{KL}(p_1, p_0)
\]

Seghouane and Amari (2007): the Akaike information criterion is an asymptotically unbiased estimator of

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Alternative distance measure

- Use a \textit{symmetrized} version of KL?

- An example from Johnson and Sinanovic (2000):

  Resistor-average distance $\mathcal{R}$
  \[
  \mathcal{R}(p_0, p_1) \equiv \frac{1}{KL(p_0, p_1)} + \frac{1}{KL(p_1, p_0)}
  \]
Alternative distance measure

- Use a *symmetrized* version of KL?

- An example from Johnson and Sinanovic (2000):
  
  Resistor-average distance $\mathcal{R}$

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  \frac{1}{\mathcal{R}(p_0, p_1)} \equiv \frac{1}{KL(p_0, p_1)} + \frac{1}{KL(p_1, p_0)}
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- Seghouane and Amari (2007): the Akaike information criterion is an asymptotically unbiased estimator of $\mathcal{A}$
Alternative distance measure

- Use a *symmetrized* version of KL?
- An example from Johnson and Sinanovic (2000):
  
  Resistor-average distance $R$

  
  $\frac{1}{R(p_0, p_1)} \equiv \frac{1}{KL(p_0, p_1)} + \frac{1}{KL(p_1, p_0)}$

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