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A Question Awaiting Formalization
What is a Bid–Ask Symmetrical Market Impact?

Is it an even function?
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- Is it an even function?

Wow, nice plot! Great idea!
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- But why even?
What is a Bid–Ask Symmetrical Market Impact?

- Is it an even function?

![Graph showing even impact function]

- Wow, nice plot! Great idea!
- But why even?
- Does it represent any financial symmetry?
The Large Scale Picture Cannot Be Even

- Bid prices are floored at zero. Ask prices are not capped.
- Bid impact is capped. Ask impact may be unbounded.
The Large Scale Picture Cannot Be Even

- Bid prices are floored at zero. Ask prices are not capped
- Bid impact is capped. Ask impact may be unbounded.

- Parity can’t be a global fundamental symmetry
How to Formalize the Symmetry?

- What is the symmetry behind equivalence of supply and demand?
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- Not so obvious for – say – equities or bonds
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Buy/Sell symmetry ⇔ relativity under change of base currency
How to Formalize the Symmetry?

- What is the symmetry behind equivalence of supply and demand?
- Not so obvious for—say—equities or bonds

- But what about forex? Buying one currency is selling another one
- Buy/Sell symmetry $\iff$ relativity under change of base currency

- First we need an appropriate formalism
Basic Formalism
Price Impact of a General Security

Definition (Average price impact)

Expected price impact $\bar{\mu}(s, T)$ of an order of $s$ contracts, executed through a time horizon $T$. Best execution assumed.
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Definition (Marginal price impact)

Expected price impact $\mu(s, T)$ of additional $ds$ contracts traded.
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$$\bar{\mu}(s, T) = \frac{1}{s} \int_0^s \mu(z, T) \, dz$$
## Price Impact of a General Security

### Definition (Average price impact)

Expected price impact $\overline{\mu}(s, T)$ of an order of $s$ contracts, executed through a time horizon $T$. Best execution assumed.

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Expected price impact $\mu(s, T)$ of additional $ds$ contracts traded.

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\overline{\mu}(s, T) = \frac{1}{s} \int_0^s \mu(z, T) \, dz
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### Convention

Buy trade if $s < 0$; sell trade if $s > 0$
Supply–Demand Curves

Let $m$ be the fair price

**Definition (Supply–Demand Curve (SDC))**

Expected price of the entire order $s$

$$\overline{m}(s, T) = m - \text{sgn}(s) \overline{\mu}(s, T)$$
Supply–Demand Curves

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**Definition (Marginal Supply–Demand Curve (MSDC))**

Expected price of additional \( ds \) contracts

\[
m(s, T) = m - \text{sgn}(s) \mu(s, T)
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'Effective Order Book' Interpretation

We interpret a couple $\{ds, m(s, T)\}$ as a *quote* available within $T$
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‘Effective Order Book’ Interpretation

We interpret a couple \( \{ds, m(s, T)\} \) as a *quote* available within \( T \)

**Definition (Bid and Ask price)**

\[
m^\pm \equiv m(0^\pm, \forall T) = \bar{m}(0^\pm, \forall T)
\]
Example: Piecewise Constant MSDC

MSDC and SDC

Marginal and Average Impact
Example: Piecewise Constant MSDC

Is it symmetrical?
Definition (Liquidation operator)

Expected order proceedings

\[ L(s, T) = \bar{m}(s, T) \cdot s = \int_0^s m(z, T) \, dz \]

Cash in if \( L > 0 \), cash out if \( L < 0 \)
Regular Market Hypothesis

The only fundamental hypotheses we make

**Regular Market Hypothesis**

The MSDC $m(s, T)$ is non-increasing in $s$, for all $T$. 
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- Or equivalently, that every quote in the market can be filled partially, for arbitrarily small sizes (no block quotes)
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- Or equivalently, that every quote in the market can be filled partially, for arbitrarily small sizes (no block quotes)

**Corollary**

*The liquidation operator $L$ is concave*
Forex: Just a Special Case of Security

- One unit of foreign currency is just one particular security
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- Fair exchange rate: $X_f^d$
  - $d$: ‘domestic’ currency $\text{CCY}_d$
  - $f$: ‘foreign’ currency $\text{CCY}_f$
  - $X_f^d$ expressed in $\text{CCY}_d$ per unit $\text{CCY}_f$ traded
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- MSDC: \( X_d^f(s, T) \)
  - \( s \): number of 'foreign' currency units traded
- Similar convention for all other equivalent functions
- All the introduced functions admit a dual representation in the two currencies
Notation: Dropping Time Dependence

- In what follows, we fix some horizon $T$ and we stop indicating it.
Forex: Basic Facts
Forex–Duality of the Framework

The order sizes $s_a$ and $s_b$ of a trade in the two currencies are related by

$$s_a = - L_b^a(s_b)$$
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\[ s_a = -L^a_b(s_b) \quad \text{but also} \quad s_b = -L^b_a(s_a) \]
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**Proposition**

*Dual liquidation operators are related by*

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$$ X_a^b(s_a)X_b^a(s_b) = 1 $$

Dual SDCs are related by

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Forex–Duality of the Framework

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$$\overline{X}_b^a(s_a)\overline{X}_a^b(s_b) = 1$$

*far less obvious!*
Proof. By definition of inverse

\[-L^b_a \circ [-L^a_b](s_b) = s_b\]

Differentiating both sides by \(s_b\) we obtain

\[X^b_a(-L^a_b(s_b)) X^a_b(s_b) = 1\]
Proof.

By definition of inverse

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Differentiating both sides by \( s_b \) we obtain

\[ X_a^b(-L_b^a(s_b)) \cdot X_b^a(s_b) = 1 \]

\[ \Box \]

Proof.

Applying twice the definition of SDC: \( L(s) = \overline{X}(s) s \)

\[ [-L_a^b] \circ [-L_b^a](s_b) = \overline{X}^b_a(-L_b^a(s_b)) \cdot L_b^a(s_b) = \overline{X}^b_a(-L_b^a(s_b)) \cdot \overline{X}^a_b(s_b) \cdot s_b = s_b \]

\[ \Box \]
Supply–Demand Symmetry for Forex
To impose supply–demand symmetry, we require that the two dual forex impact functions look identical to two investors with opposite base currency.
Invariance Under Change of Base Currency

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- ... up to a constant rescaling to account for notional disparity of the two currency units
Invariance Under Change of Base Currency

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- ... up to a constant rescaling to account for notional disparity of the two currency units

Example

Suppose the ¥/€ rate is 100 ¥/€. In a symmetrical market, we expect that the relative impact of liquidating €100 and the relative impact of liquidating ¥10’000 should be the same
Imposing Supply–Demand Symmetry for Small Forex Trades

- Impose that the dual relative bid–offer spreads are identical

\[
\frac{X^- - X^+}{X} = \frac{1/X^+ - 1/X^-}{1/X}
\]

Solving for \( X \) yields \( \frac{20}{57} \)
Imposing Supply–Demand Symmetry for Small Forex Trades

- Impose that the dual relative bid–offer spreads are identical

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\frac{X^- - X^+}{X} = \frac{1/X^+ - 1/X^-}{1/X}
\]

Solving for \( X \) yields

**Proposition**

*In a symmetrical forex LS, the fair rate is the geometric average of the bid rate and the offer rate.*

\[
X_b^a = \sqrt{X_b^{a+} X_b^{a-}}
\]
Definition (Forex Supply–Demand Symmetry)

We say that a forex market is symmetrical, if there exists a constant $\alpha > 0$ such that the mapping $s_a/\alpha \leftrightarrow s_b$

$$\frac{s_a}{\alpha} = -\frac{1}{\alpha} L^a_b(s_b)$$

is an involution

$$-\frac{1}{\alpha} L^a_b = \left(-\frac{1}{\alpha} L^a_b\right)^{[-1]}$$
Imposing Supply–Demand Symmetry for Forex, in General

**Definition (Forex Supply–Demand Symmetry)**

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**Proposition**

*If such $\alpha$ exists, it’s the fair rate*

$$\alpha = X^a_b = \sqrt{X^a_b^+ X^a_b^-}$$
Definition (Forex Supply–Demand Symmetry)

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is an involution

\[
-\frac{1}{\alpha} L^a_b = \left( -\frac{1}{\alpha} L^a_b \right)^{-1}
\]

Proposition

*If such \( \alpha \) exists, it’s the fair rate ... as you may have guessed*

\[
\alpha = X^a_b = \sqrt{X^{a+}_b X^{a-}_b}
\]
Classification of Forex Symmetrical Markets

Theorem

A forex market displays supply–demand symmetry if and only if the liquidation operator $s_b \mapsto L^a_b(s_b)$ can be expressed as

$$L^a_b(s_b) = -X^a_b \phi(s_b)$$

where the function $\phi : D^a_b \rightarrow D^a_b$

1. is an involution $\phi = \phi^{-1}$$
2. is convex and strictly decreasing
3. $\phi(0) = 0$
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**Corollary**

In a symmetrical forex market the MSDC and the SDC satisfy

\[
X^a_b(s)X^a_b(\tilde{s}) = (X^a_b)^2
\]

\[
\overline{X}^a_b(s)\overline{X}^a_b(\tilde{s}) = (X^a_b)^2
\]

at conjugated points \( s \) and \( \tilde{s} = \phi(s) \).
But Then: Why Only Forex?
A Currency is Just One Security Among All Others

- If you say

  “the euro for a yen based investor is as liquid an asset as the yen is for a euro based investor”

  it might seem you’re speaking of a forex symmetry only
A Currency is Just One Security Among All Others

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- you could have been speaking of a stock, a gold bullion, an oil gallon, ...
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- similarities with “change of numeraire” type of symmetry
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- similarities with “change of numeraire” type of symmetry

Definition (Supply–Demand Symmetry for General Securities)

A security’s market is symmetrical if it has the same properties of a forex symmetrical market
Classification of Symmetrical Markets for General Securities

**Theorem**

A security’s market displays supply–demand symmetry if and only if the liquidation operator $s \mapsto L(s)$ can be expressed as

$$L(s) = -m \phi(s)$$

where the function $\phi : \mathcal{D} \to \mathcal{D}$

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**Corollary**

In a security’s symmetrical market the MSDC and the SDC satisfy

\[
m(s) m(\tilde{s}) = m^2
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\overline{m}(s) \overline{m}(\tilde{s}) = m^2
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Geometrical Interpretation of Supply–Demand Symmetry
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- \( L(x)/m \): concave, symmetrical wrt \( y = -x \), increasing, zero in zero
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- the curve is forced to live in the white area of the plane
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Geometrical Interpretation of Supply–Demand Symmetry

- \( L(x)/m \): concave, symmetrical wrt \( y = -x \), increasing, zero in zero

- The curve is forced to live in the white area of the plane
- Two extremes: perfectly liquid and perfectly illiquid market
- Natural notion of partial ordering of liquidity among different \( L \)’s
Supply–Demand Symmetry for Stocks, in Words

\[ \tilde{s} = -L(s)/m \quad \text{and} \quad s = -L(\tilde{s})/m \]

Proposition (A Market is Symmetrical iff)

If \(|\tilde{s}|\) stocks correspond in fair value to the liquidation of \(s\) stocks, then \(s\) stocks correspond in fair value to the cost of buying \(|\tilde{s}|\) stocks, \(\forall s\)
Example: Exponentially Decaying MSDC

MSDC and SDC

Marginal and Average Impact
Example: Exponentially Decaying MSDC
Example: Exponentially Decaying MSDC with Spread

MSDC and SDC

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MSDC and SDC

Marginal and Average Impact
Example: Asymptotically Finite MSDC with Spread
Example: Linear Ask MSDC

**MSDC and SDC**

- $m$
- $\bar{m}$

**Marginal and Average Impact**

- $\mu$
- $\bar{\mu}$

$\frac{37}{57}$
Example: Linear Ask MSDC with Spread

MSDC and SDC

Marginal and Average Impact

$\bar{m}$

$\bar{m}$

$\mu$

$\bar{\mu}$
Example: Linear Ask MSDC with Spread
Example: Piecewise Constant MSDC

**MSDC and SDC**

**Marginal and Average Impact**
Example: Piecewise Constant MSDC

It was symmetrical!
Example: Piecewise Constant MSDC
Example: Another Piecewise Constant MSDC

MSDC and SDC

Marginal and Average Impact

\[ \mu \bar{\mu} \]
Example: Another Piecewise Constant MSDC
Some Results
An Even Impact Always Corresponds to Excess of Supply

- Given any market ‘wing’, there exists one and only one symmetrical wing that completes a symmetrical market.
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Therefore, we can build sound notions of

- ‘more liquid market’
- ‘more liquid wing’
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  - ‘excess of demand’ (resp. ‘of supply’): sell side more (resp. less) liquid than buy side
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Proposition

*If the average impact is an even function, the market has an excess of supply*

Proof.

Not so obvious
Even Impact Always Corresponds to Excess of Supply

Figure: Illustration of the proposition. $L_+$ represents the bid wing of a LS. The plot compares the ask wing $L_-$ obtained assuming that impact is even and the ask wing $\tilde{L}_+$ assuming a symmetrical market. The former, is always more liquid.
Proposition

Consider a symmetrical market. Express the bid wing MSDC as

\[ m(s) = m_+ - m_+ \psi(s) \quad s > 0 \]

with \( \lim_{s \to 0} \psi(s) = 0 \). Then, the opposite ask wing MSDC can be approximated as an expansion in powers of \( \psi \), to give

\[ m(s) = m_- + m_- \psi(-sm_-/m) + \mathcal{O}(\psi^2) \quad s < 0 \]
Proposition

Consider a symmetrical market. Express the bid wing MSDC as

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Proof.

An application of the Lagrange inversion theorem \( \square \)
Even Impact as Small Size Limit of Supply–Demand Symmetry

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\[ m(s) = m_- + m_- \psi(-sm_-/m) + O(\psi^2) \quad s < 0 \]

**Proof.**

An application of the Lagrange inversion theorem

**Corollary**

At small impact regimes, a symmetrical market can be approximated by an even impact function iff the bid–offer spread is zero, in which case

\[ \mu(s) = m\psi(|s|) + O(\psi^2) \quad \forall s \]
Figure: A symmetrical (power-law) marginal impact with no bid–ask spread, zoomed at small impact scale. The function is very close to an even one.
Figure: A symmetrical (power-law) marginal impact with finite bid–ask spread. The function cannot be approximated by an even one at any scale.
Zooming at Low Impact Scale with Spread

Figure: Even if we compute impact from mid price instead of fair price, to offset the central gap, the ask wing remains steeper.
Comparing with the Literature

- All models in the literature (with a massive amount of empirical evidence) assume even functions to describe supply–demand equilibrium.
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All models in the literature (with a massive amount of empirical evidence) assume even functions to describe supply–demand equilibrium. Are they all wrong?
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- No. The last proposition tells us that they may be just looking into small impact regimes, neglecting bid–offer spread. Which is in fact the typical assumption in most models.
Comparing with the Literature

- All models in the literature (with a massive amount of empirical evidence) assume even functions to describe supply–demand equilibrium. Are they all wrong?
- No. The last proposition tells us that they may be just looking into small impact regimes, neglecting bid–offer spread. Which is in fact the typical assumption in most models.
- Our notion of symmetry makes testable predictions at all size scales and impact regimes, that are supposed to extend previous findings.
Conclusions
Summing Up

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- Supply–demand symmetry should represent the equilibria points of no market imbalance in all market impact models.
Thanks!
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