Intermediary Leverage Cycles and Financial Stability

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Outline

Introduction

The Model

Solution

Distortions and Amplification
  Solvency
  Amplification
Questions about Financial Stability Policy

- Systemic distress of financial intermediaries raises questions about financial stability policies:
  - How does capital regulation affect the tradeoff between the pricing of credit and the amount of systemic risk?
  - How does macroprudential policy function in equilibrium?
  - What are the welfare implications of capital regulation?

- We develop a theoretical framework to address these questions
Our Approach

- We use a standard macro model with a financial sector and add one key assumption:
  - Funding constraints of financial intermediaries are risk based, so intermediaries have to hold more capital when the riskiness of their assets increases.

- This assumption is empirically motivated and it allows us to capture stylized facts about:
  - Procyclical leverage of intermediary balance sheets
  - Procyclical share of intermediated credit
  - Relationship between asset risk premia and intermediary leverage
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The Model
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Economy Structure

Producers
random dividend stream, \( A_t \), per unit of project financed by direct borrowing from intermediaries and households

Intermediaries
financed by households against capital investments

Households
solve portfolio choice problem between holding intermediary debt, physical capital and risk-free borrowing/lending

\[ A_t k_{ht} \]
Production

- Aggregate amount of capital $K_t$ evolves as
  \[ dK_t = (I_t - \lambda_k)K_t dt \]

- Total output evolves as
  \[ Y_t = A_t K_t \]

- Stochastic productivity of capital \( \{A_t = e^{at}\}_{t \geq 0} \)
  \[ da_t = \bar{a} dt + \sigma_a dZ_{at} \]

- $p_{kt}A_t$ denotes the price of one unit of capital in terms of the consumption good
Households

- Household preferences are:

\[
E \left[ \int_{0}^{+\infty} e^{-(\xi_t + \rho h t)} \log c_t \, dt \right]
\]

- Liquidity preference shocks (as in Allen and Gale (1994) and Diamond and Dybvig (1983)) are \( \exp(-\xi_t) \)

\[
d\xi_t = \sigma_{\xi} \rho_{\xi,a} dZ_{at} + \sigma_{\xi} \sqrt{1 - \rho_{\xi,a}^2} dZ_{\xi t}
\]

- Households do not have access to the investment technology

\[
dk_{ht} = -\lambda_k k_{ht} \, dt
\]
The Model

Households' Optimization

\[
\max \{c_t, k_{ht}, b_{ht}\} \mathbb{E} \left[ \int_0^{+\infty} e^{-(\xi_t + \rho_t)} \log c_t \, dt \right]
\]

subject to

\[
dw_{ht} = r_{ft} w_{ht} \, dt + p_{kt} A_t k_{ht} (dR_{kt} - r_{ft} \, dt) + p_{bt} A_t b_{ht} (dR_{bt} - r_{ft} \, dt) - c_t \, dt
\]

and no-shorting constraints

\[
k_{ht} \geq 0, \quad b_{ht} \geq 0
\]
Intermediaries

- Financial intermediaries create new capital
  \[ dk_t = (\Phi(i_t) - \lambda_k) k_t dt \]

- Investment carries quadratic adjustment costs (Brunnermeier and Sannikov (2012))
  \[ \Phi(i_t) = \phi_0 \left( \sqrt{1 + \phi_1 i_t} - 1 \right) \]

- Intermediaries finance investment projects through inside equity and outside risky debt giving the budget constraint
  \[ p_{kt} A_t k_t = p_{bt} A_t b_t + w_t \]
Intermediaries’ Risk Based Capital Constraint

- Risk based capital constraint (Danielsson, Shin, and Zigrand (2011))

\[ \alpha \sqrt{\frac{1}{dt} \langle k_t d (p_{kt} A_t) \rangle^2} = w_t \]

- Implies a time-varying leverage constraint

\[ \theta_t = \frac{p_{kt} A_t k_t}{w_t} = \frac{1}{\alpha \sqrt{\frac{1}{dt} \langle d(p_{kt} A_t) \rangle^2}} \]

- Note that the constraint is such that intermediary equity is proportional to the Value-at-Risk of total assets
- This will imply that default probabilities vary over time
Risk-based Capital Constraints

VaR is the potential loss in value of inventory positions due to adverse market movements over a defined time horizon with a specified confidence level. We typically employ a one-day time horizon with a 95% confidence level.

Source: Goldman Sachs 2011 Annual Report
The Model

Commercial Bank Lending Standards

![Graph showing VIX and Credit Tightening over time]

\[ \rho = 0.68013 \]
Systemic Distress

- Solvency risk defined by

\[ \tau_D = \inf_{t \geq 0} \{ w_t \leq \bar{\omega} p_{kt} A_t K_t \} \]

- Term structure of systemic distress

\[ \delta_t (T) = \mathbb{P} (\tau_D \leq T \mid (w_t, \theta_t)) \]

In distress

- Management changes
- Intermediary leverage reduced to \( \theta \approx 1 \) by defaulting on debt
- Intermediary instantaneously restarts with wealth

\[ w_{\tau_D^+} = \frac{\theta_{\tau_D}}{\theta} w_{\tau_D} \]
Intermediaries’ Optimization

Intermediary maximizes equity holder value to solve

$$\max_{\{k_t, \beta_t, i_t\}} \mathbb{E} \left[ \int_0^\tau D e^{-\rho t} w_t dt \right]$$

subject to the dynamic intermediary budget constraint

$$dw_t = k_t p_{kt} A_t \left( dR_{kt} + (\Phi (i_t) - i_t/p_{kt}) dt \right) - b_t p_{bt} A_t dR_{bt}$$

and the risk based capital constraint

$$\alpha \sqrt{\frac{1}{dt} \left< k_t d (p_{kt} A_t) \right>^2} = w_t$$
**Equilibrium**

An equilibrium in this economy is a set of price processes \( \{ p_{kt}, p_{bt}, C_{bt} \}_{t \geq 0} \), a set of household decisions \( \{ k_{ht}, b_{ht}, c_t \}_{t \geq 0} \), and a set of intermediary decisions \( \{ k_t, \beta_t, i_t, \theta_t \}_{t \geq 0} \) such that:

1. Taking the price processes and the intermediary decisions as given, the household's choices solve the household's optimization problem, subject to the household budget constraint.
2. Taking the price processes and the household decisions as given, the intermediary's choices solve the intermediary optimization problem, subject to the intermediary wealth evolution and the risk based capital constraint.
3. The capital market clears:
   \[
   K_t = k_t + k_{ht}.
   \]
4. The risky bond market clears:
   \[
   b_t = b_{ht}.
   \]
5. The risk-free debt market clears:
   \[
   w_{ht} = p_{kt} A_t k_{ht} + p_{bt} A_t b_{ht}.
   \]
6. The goods market clears:
   \[
   c_t = A_t (K_t - i_t k_t).
   \]
Related Literature

- **Leverage Cycles**: Geanakoplos (2003), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009)

- **Amplification in Macroeconomy**: Bernanke and Gertler (1989), Kiyotaki and Moore (1997)

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Solution Strategy

- Equilibrium is characterized by two state variables, leverage $\theta_t$ and relative intermediary net worth $\omega_t$

$$\omega_t = \frac{w_t}{w_t + w_{ht}} = \frac{w_t}{p_{kt}A_tK_t}$$

- Represent state dynamics as

$$\frac{d\omega_t}{\omega_t} = \mu_{\omega t} dt + \sigma_{\omega a,t} dZ_{at} + \sigma_{\omega \xi,t} dZ_{\xi t}$$

$$\frac{d\theta_t}{\theta_t} = \mu_{\theta t} dt + \sigma_{\theta a,t} dZ_{at} + \sigma_{\theta \xi,t} dZ_{\xi t}$$

- Risk-based capital constraint implies

$$\alpha^{-2}\theta_t^{-2} = \sigma_{ka,t}^2 + \sigma_{k\xi,t}^2$$
Volatility Risk

\[ y = 0.15 - 0.0067x \]

\[ R^2 = 0.088 \]

\( \alpha \uparrow \)

\( \text{Leverage Growth} \)

\( \text{Volatility} \)

\( \text{Lagged VIX} \)

\( \text{Leverage Growth} \)
Intermediary Balance Sheets

\[ y = -0.071 + 0.76x \]

\[ R^2 = 0.46 \]
Optimal Household Choices

Denote by $\pi_{kt} = (p_{kt}A_t k_{ht}) / w_{ht}$ and $\pi_{bt} = (p_{bt}A_t b_{ht}) / w_{ht}$

Lemma 3.1

The household’s optimal consumption choice satisfies:

$$c_t = \left( \frac{\sigma^2}{2} \right) w_{ht}.$$

In the unconstrained region, the household’s optimal portfolio choice is given by:

$$\begin{bmatrix} \pi_{kt} \\ \pi_{bt} \end{bmatrix} = \left( \begin{bmatrix} \sigma_{ka,t} & \sigma_{k\xi,t} \\ \sigma_{ba,t} & \sigma_{b\xi,t} \end{bmatrix} \right) \left( \begin{bmatrix} \sigma_{ka,t} & \sigma_{ba,t} \\ \sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mu R_{k,t} - r_{ft} \\ \mu R_{b,t} - r_{ft} \end{bmatrix} - \sigma_{\xi} \left( \begin{bmatrix} \sigma_{ka,t} & \sigma_{ba,t} \\ \sigma_{k\xi,t} & \sigma_{b\xi,t} \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{\rho_{\xi,a}}{\sqrt{1 - \rho_{\xi,a}^2}} \end{bmatrix}. $$
Equilibrium Expected Returns

Expected return to capital

\[
\mu_{Rk,t} - r_{ft} = \left( \sigma_{ka,t}^2 + \sigma_{k\xi,t}^2 \right) \frac{1 - \theta_t \omega_t}{1 - \omega_t} + \left( \sigma_{ka,t} \sigma_{ba,t} + \sigma_{k\xi,t} \sigma_{b\xi,t} \right) \frac{\omega_t (\theta_t - 1)}{1 - \omega_t}
\]

+ \sigma_{\xi} \left( \sigma_{ka,t} \rho_{\xi,a} + \sigma_{k\xi,t} \sqrt{1 - \rho_{\xi,a}^2} \right)

compensation for own risk

compensation for risk of correlated asset

compensation for liquidity risk

Expected return to intermediary debt

\[
\mu_{Rb,t} - r_{ft} = \left( \sigma_{ba,t}^2 + \sigma_{b\xi,t}^2 \right) \frac{\omega_t (\theta_t - 1)}{1 - \omega_t} + \left( \sigma_{ka,t} \sigma_{ba,t} + \sigma_{k\xi,t} \sigma_{b\xi,t} \right) \frac{1 - \theta_t \omega_t}{1 - \omega_t}
\]

+ \sigma_{\xi} \left( \sigma_{ba,t} \rho_{\xi,a} + \sigma_{b\xi,t} \sqrt{1 - \rho_{\xi,a}^2} \right)

compensation for own risk

compensation for risk of correlated asset

compensation for liquidity risk
Excess Returns

\[ y = 0.12 - 0.31x \]
\[ R^2 = 0.17 \]

\[ y = 0.011 - 0.049x \]
\[ R^2 = 0.063 \]
Equilibrium Prices of Risk I

Shocks

\[ d\hat{y}_t = \sigma_a^{-1} \left( d \log Y_t - \mathbb{E}_t [d \log Y_t] \right) = dZ_{at} \]

\[ d\hat{\theta}_t = \left( \sigma_{\theta a, t}^2 + \sigma_{\theta \xi, t}^2 \right)^{-\frac{1}{2}} \left( \frac{d\theta_t}{\theta_t} - \mathbb{E}_t \left[ \frac{d\theta_t}{\theta_t} \right] \right) \]

\[ = \frac{\sigma_{\theta a, t}}{\sqrt{\sigma_{\theta a, t}^2 + \sigma_{\theta \xi, t}^2}} dZ_{at} + \frac{\sigma_{\theta \xi, t}}{\sqrt{\sigma_{\theta a, t}^2 + \sigma_{\theta \xi, t}^2}} dZ_{\xi t}. \]
Equilibrium Prices of Risk II

Price of risk of leverage

\[ \eta_{\theta t} = \sqrt{1 + \frac{(\sigma_{ka,t} - \sigma_a)^2}{\sigma^2_{k\xi,t}} \left( -\frac{2\theta_t \omega_t p_{kt}}{\beta (1 - \omega_t)} \sigma_{k\xi,t} + \sigma_{\xi} \sqrt{1 - \rho^2_{\xi,a}} \right)} \]

- Price of risk of leverage is always positive (Adrian, Etula, and Muir (2011)), and depends on leverage growth in a nonmonotonic fashion (Adrian, Moench, and Shin (2010) find a negative relationship)
Equilibrium Prices of Risk III

Figure: Source: Adrian, Etula, and Muir (2011)
Equilibrium Prices of Risk IV

Price of risk of output

\[ \eta_{yt} = \sigma_a + \sigma_\xi \left( \rho_{\xi,a} - \frac{\sigma_{ka,t} - \sigma_a}{\sigma_{k\xi,t}} \sqrt{1 - \rho_{\xi,a}^2} \right) \]

- Switches sign, consistent with insignificant estimates of price of output risk
- Usually becomes negative when exposure to liquidity shock is small
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Term Structure of Systemic Risk
Volatility Paradox

![Graph showing the relationship between Local volatility and Distress probability on the left. The graph on the right displays the relationship between Price of leverage risk and Distress probability.](image)
Constant Leverage Benchmark

- Constant expected output and consumption growth
- But lower level of output and consumption growth
- Constant investment and price of capital
- Liquidity shocks have no impact on real activity
A Sample Path of the Economy

Financial Crisis Triggers Recession

Benign Financial Crisis
Household Welfare

\[ \alpha \]

\[ \text{Welfare} \]

\[ \text{α} \]

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Conclusion

- Dynamic, general equilibrium theory of financial intermediaries’ leverage cycle with endogenous amplification and endogenous systemic risk
- Conceptual basis for policies towards financial stability
- Systemic risk return tradeoff: tighter intermediary capital requirements tend to shift the term structure of systemic downward, at the cost of increased prices of risk today

Model captures important stylized facts:
1. Procyclical intermediary leverage
2. Procyclicality of intermediated credit
3. Financial sector equity return and intermediary leverage growth
4. Exposure to intermediary leverage shocks as pricing factor


