A Theoretical and Empirical Comparison of Systemic Risk Measures

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Contribution to Systemic Risk

- Which financial institutions contribute the most to the risk of the system?

  - Which banks?
  - Which ETF issuers?
  - Which clearing houses?
  - Which gov.-sponsored enterprises?
  - Which insurance companies?
  - Which hedge funds?
  - Which exchanges?

- Who are the so-called Systemically Important Financial Institutions (SIFIs)?
Who are the SIFIs?

**Definition**

“SIFIs are financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity.”

<table>
<thead>
<tr>
<th>(2.5%) Citigroup, Deutsche Bank, HSBC, JP Morgan Chase</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.0%) Barclays, BNP Paribas</td>
</tr>
<tr>
<td>(1.5%) Bank of America, Bank of New York Mellon, Credit Suisse, Goldman Sachs, Mitsubishi UFJ FG, Morgan Stanley, Royal Bank of Scotland, UBS</td>
</tr>
<tr>
<td>(1.0%) Bank of China, BBVA, Groupe BPCE, Group Crédit Agricole, ING Bank, Mizuho FG, Nordea, Santander, Société Générale, Standard Chartered, State Street, Sumitomo Mitsui FG, Unicredit Group, Wells Fargo</td>
</tr>
</tbody>
</table>

FSB Financial Stability Board
November 2012
How to Measure Systemic Risk?

1. A first approach relies on proprietary data, such as cross-holdings, interbank lending, derivatives positions, or common exposures to exogeneous sources of risk
   - Gourieroux, Heam and Monfort (2012)
   - Greenwood, Landier and Thesmar (2012)

2. A second approach relies on public market data, such as stock or option prices, or CDS spreads
   - Giglio (2012)
Three prominent examples of market data-based measures are:

1. **Marginal Expected Shortfall (MES)** of Acharya, Pedersen, Philippon, and Richardson (2010)


3. **Delta Conditional Value-at-Risk (DCoVaR)** of Adrian and Brunnermeier (2011)
High-Impact Academic Papers

These papers had a high impact both in the academia and on the regulatory debate

1. Dozens of research papers have implemented these systemic risk measures

2. Regularly discussed in the financial press

3. Computed in real-time and publically disclosed on the Internet (V-Lab, New York University)

4. Used by central banks and banking regulators
Goal of this Paper

- We propose a **theoretical and empirical comparison** of the major market-data based systemic risk measures (MES, SRISK, and DCoVaR) in a **common framework**.

- **Uncover the theoretical link** between systemic risk measures and standard financial risks (systematic risk, tail risk, correlation, beta).
The Econometrics of Systemic Risk

Methodology and Definitions
Expected Shortfall

ES of the market defined with a threshold $C$ is the expected return of the market conditional on the market return being below $C$

$$ES_{m,t-1}(C) = \mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < C)$$

$$= \sum_{i=1}^{N} w_i \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C)$$
The Marginal Expected Shortfall measure proposed by Acharya, Pedersen, Philippon, and Richardson (2010) extends the ES to measure the contribution of firm $i$

**Definition**

The **MES** is the expected equity loss per dollar invested in a particular financial institution $i$ if the overall market return $r_m$ declines by at least $C$

\[
MES_{it}(C) = \frac{\partial ES_{m,t-1}(C)}{\partial w_i} = \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C)
\]
Systemic Risk (SRISK)

The SRISK measure proposed by Acharya, Engle and Richardson (2012) and Brownlees and Engle (2012) extends the MES to take into account the market cap and the liabilities of the firm.

**Definition**

The SRISK corresponds to the expected capital shortfall of a given financial institution, conditional on a crisis affecting the whole financial system.
Systemic Risk (SRISK)

The Systemic Risk captures the expected capital shortage of a firm given its liabilities and MES:

\[
\text{SRISK} = k \cdot D_{it} - (1 - k) \cdot W_{it} \cdot (1 - LRMES_{it})
\]

- \( SRISK \) is the Systemic Risk.
- \( k \) is the Prudential Ratio.
- \( D_{it} \) is the Liability.
- \( W_{it} \) is the Market Value.
- \( LRMES_{it} \) is the Liability-to-Market Value Ratio.
The third systemic risk measure is the CoVaR introduced by Adrian and Brunnermeier (2011)

**Definition**

The CoVaR corresponds to the VaR of the market return obtained conditionally on an event for firm $i$. 


CoVaR

CoVaR is the Value-at-Risk of the market return given a specific event on the firm's returns.

\[
\Pr \left( r_{mt} \leq \text{CoVaR}_t^m \mid r_{it} = \text{VaR}_t(\alpha) \right) = \alpha
\]

\(\Delta\text{CoVaR}

The contribution of the institution to systemic risk is the difference between its CoVaR and the CoVaR calculated at the median state.

\[
\Delta\text{CoVaR}_t(\alpha) = \text{CoVaR}_t^m \mid r_{it} = \text{VaR}_t(\alpha) - \text{CoVaR}_t^m \mid r_{it} = \text{Median}(r_{it})
\]
How to Compare Systemic Risk Measures

1. Compare actual risk contributions to estimated risk contributions [NO]

2. See which measure can best reproduce the FSB list of SIFIs [NO]

3. Derive the risk measures in a common framework and try to uncover the driving forces of each systemic risk measure [YES]
A Theoretical Comparison of Systemic Risk Measures
A Theoretical Comparison of Systemic Risk Measures

We consider the linear market model of Brownlees and Engle (2012):

\[
\begin{align*}
    r_{mt} &= \sigma_{mt}\varepsilon_{mt} \\
    r_{it} &= \sigma_{it}\rho_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{1 - \rho_{it}^2}\zeta_{it} \\
    (\varepsilon_{mt}, \zeta_{it}) &\sim D
\end{align*}
\]

where \( \nu_t = (\varepsilon_{mt}, \zeta_{it})' \) satisfies \( \mathbb{E}(\nu_t) = 0 \) and \( \mathbb{E}(\nu_t\nu_t') = I_2 \), and \( D \) denotes the bivariate distribution of the standardized innovations.
A Theoretical Comparison of Systemic Risk Measures

Proposition 1

In our framework, we have:

\[ MES_{it}(\alpha) = \rho_{it} \frac{\sigma_{it}}{\sigma_{mt}} \mathbb{E}_{t-1}\left(r_{mt} \mid r_{mt} < \text{VaR}_{mt}(\alpha)\right) \]

\[ = \beta_{it} \text{ES}_{mt}(\alpha) \]

Interpretation

MES is proportional to the **systematic risk** and the coefficient corresponds to the **expected shortfall of market returns**
A corollary of Proposition 1 is that $\text{SRISK}$ can be expressed as a linear function of the beta, liabilities, and market capitalization:

$$\text{SRISK}_{it} \approx k \ D_{it} - (1 - k) \ W_{it} \ \exp\left[18 \times \beta_{it} \times \text{ES}_{mt}(\alpha)\right]$$
A Theoretical Comparison of Systemic Risk Measures

Proposition 2

Under A1, we have

$$\Delta \text{CoVaR}_{it}(\alpha) = \left( \frac{\rho_{it} \sigma_{mt}}{\sigma_{it}} \right) \times \left( \text{VaR}_{it}(\alpha) - \text{VaR}_{it}(0.5) \right)$$

and if the marginal distribution is symmetric around zero:

$$\Delta \text{CoVaR}_{it}(\alpha) = \tilde{\gamma}_{it} \text{VaR}_{it}(\alpha)$$

Interpretation

$\Delta \text{CoVaR}$ is proportional to the **linear projection coefficient** of the market return on the firm return and to the **firm tail risk**.
A Theoretical Comparison of Systemic Risk Measures

Proposition 3

Under A1, the **ratio is not homogenous across financial institutions** and is the product of two terms:

\[
\frac{\Delta \text{CoVaR}_{it}(\alpha)}{\text{MES}_{it}(\alpha)} = \frac{\text{firm-specific}}{\frac{\text{common}}{\frac{\text{VaR}_{it}(\alpha)}{\sigma_{it}^2}} \times \frac{\sigma_{mt}^2}{\text{ES}_{mt}(\alpha)}}
\]

Interpretation

The fact that this ratio is firm-specific implies that the systemic risk rankings based on the two measures may not be the same.
A Theoretical Comparison of Systemic Risk Measures

Proposition 4

A financial institution $i$ is more systemically risky than an institution $j$ according to the MES and the $\Delta$CoVaR measures, $MES_{it}(\alpha) \leq MES_{jt}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$, if:

$$\rho_{it} \geq \max \left( \rho_{jt}, \frac{\rho_{jt} \sigma_{jt}}{\sigma_{it}} \right)$$
Proposition 5

A financial institution $i$ is more systemically risky than a financial institution $j$ (with the same level of liabilities) according to the SRISK and the $\Delta$CoVaR measures, $SRISK_{it}(\alpha) \geq SRISK_{jt}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$, if:

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \leq W_{jt} \times \exp\left[18 \times ES_{mt}(\alpha) \times \left(\beta_{jt} - \beta_{it}\right)\right]$$

where $W_{it}$ and $W_{jt}$ denote the market capitalizations of both firms.
Summary of the Main Theoretical Findings

1. Strong link between MES and beta (Prop. 1)
2. Strong link between $\Delta$CoVaR and beta (Prop. 2)
3. Rankings of systemic firms can be different (Prop. 3)
4. Rankings of systemic firms are likely to be different (Prop. 4&5)
An Empirical Comparison of Systemic Risk Measures
An Empirical Comparison of Systemic Risk Measures

In practice, the dependence between firm returns can be richer than in our framework. Let’s relax it.

A panel of US financial institutions

- Unbalanced panel of 94 companies with daily returns from 01/03/2000 to 12/31/2010.
- Same dataset as in Acharya et al. (2010) and Brownlees and Engle (2012).
- Same estimation techniques as in the original papers.
Outside our framework, the conditional expectation

\[ \mathbb{E}_{t-1} \left( \xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}} \right) \]

is not null and has to be estimated

\[
MES_{it} (C) = \sigma_{it} \rho_{it} \mathbb{E}_{t-1} \left( \varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}} \right) \\
+ \sigma_{it} \sqrt{1 - \rho_{it}^2} \mathbb{E}_{t-1} \left( \xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}} \right)
\]

We use a \textbf{nonparametric estimator} (see Scaillet, 2005 for more details) of the tail expectation.
An Empirical Comparison of Systemic Risk Measures

SIFI or not SIFI?
Different Systemic Risk Measures, Different SIFIs

<table>
<thead>
<tr>
<th>Rank</th>
<th>MES</th>
<th>SRISK</th>
<th>ΔCoVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MBI</td>
<td>BAC</td>
<td>HRB</td>
</tr>
<tr>
<td>2</td>
<td>AIG</td>
<td>C</td>
<td>MI</td>
</tr>
<tr>
<td>3</td>
<td>MI</td>
<td>JPM</td>
<td>BEN</td>
</tr>
<tr>
<td>4</td>
<td>CBG</td>
<td>MS</td>
<td>CIT</td>
</tr>
<tr>
<td>5</td>
<td>RF</td>
<td>AIG</td>
<td>WU</td>
</tr>
<tr>
<td>6</td>
<td>LM</td>
<td>MET</td>
<td>AIZ</td>
</tr>
<tr>
<td>7</td>
<td>JNS</td>
<td>PRU</td>
<td>AXP</td>
</tr>
<tr>
<td>8</td>
<td>HRB</td>
<td>HIG</td>
<td>JNS</td>
</tr>
<tr>
<td>9</td>
<td>BAC</td>
<td>SLM</td>
<td>NYB</td>
</tr>
<tr>
<td>10</td>
<td>UNM</td>
<td>LNC</td>
<td>MTB</td>
</tr>
</tbody>
</table>
Different Systemic Risk Measures, Different SIFIs

- Percentage of concordant pairs between MES and SRISK
- Percentage of concordant pairs between SRISK and ΔCoVaR
- Percentage of concordant pairs between MES and ΔCoVaR
An Empirical Comparison of Systemic Risk Measures

MES and SRISK
Systemic or Systematic Risk?

Figure: Strong Relationship between MES and beta
Systemic or Systematic Risk?

![Percentage of concordant pairs between MES and Beta](chart1)

![Percentage of concordant pairs between SRISK and Beta](chart2)

![Percentage of concordant pairs between SRISK and Leverage](chart3)

![Percentage of concordant pairs between SRISK and Liabilities](chart4)

![Percentage of concordant pairs between ΔCoVaR and VaR](chart5)
An Empirical Comparison of Systemic Risk Measures

DCoVaR
An Empirical Comparison of Systemic Risk Measures

Figure: $\Delta \text{CoVaR}$ is not Equivalent to $\text{VaR}$ in the Cross-Section
An Empirical Comparison of Systemic Risk Measures

Figure: \( \Delta \text{CoVaR} \) is Equivalent to \( \text{VaR} \) in Time Series
An Empirical Comparison of Systemic Risk Measures

Regression Analysis
Regression Analysis

\[ MES_{i,t} = a + b \cdot X_{i,t} + e_{i,t} \]

<table>
<thead>
<tr>
<th>MES</th>
<th>MV</th>
<th>LTQ</th>
<th>LVG</th>
<th>beta</th>
<th>VaR</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>average ( R^2 )</td>
<td>0.0071</td>
<td>0.0403</td>
<td>0.2591</td>
<td>0.9571</td>
<td>0.7968</td>
<td>0.9837</td>
</tr>
<tr>
<td>min ( R^2 )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0137</td>
<td>0.7198</td>
<td>0.3972</td>
<td>0.9433</td>
</tr>
<tr>
<td>max ( R^2 )</td>
<td>0.0452</td>
<td>0.1852</td>
<td>0.7883</td>
<td>0.9946</td>
<td>0.9785</td>
<td>0.9986</td>
</tr>
<tr>
<td>std ( R^2 )</td>
<td>0.0086</td>
<td>0.0416</td>
<td>0.1477</td>
<td>0.0319</td>
<td>0.1100</td>
<td>0.0105</td>
</tr>
</tbody>
</table>
Regression Analysis

\[ SRISK_{i,t} = a + b \cdot X_{i,t} + e_{i,t} \]

<table>
<thead>
<tr>
<th>SRISK</th>
<th>MV</th>
<th>LTQ</th>
<th>LVG</th>
<th>beta</th>
<th>VaR</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>average $R^2$</td>
<td>0.3197</td>
<td>0.8341</td>
<td>0.1840</td>
<td>0.1173</td>
<td>0.0592</td>
<td>0.9932</td>
</tr>
<tr>
<td>min $R^2$</td>
<td>0.0085</td>
<td>0.2569</td>
<td>0.0110</td>
<td>0.0034</td>
<td>0.0022</td>
<td>0.9807</td>
</tr>
<tr>
<td>max $R^2$</td>
<td>0.5759</td>
<td>0.9952</td>
<td>0.4103</td>
<td>0.3331</td>
<td>0.2269</td>
<td>0.9995</td>
</tr>
<tr>
<td>std $R^2$</td>
<td>0.1073</td>
<td>0.1279</td>
<td>0.0757</td>
<td>0.0661</td>
<td>0.0445</td>
<td>0.0036</td>
</tr>
</tbody>
</table>
Regression Analysis

\[ \Delta \text{CoVaR}_{it} = a + b \cdot X_{i,t} + e_{i,t} \]

<table>
<thead>
<tr>
<th>( \Delta \text{CoVaR} )</th>
<th>MV</th>
<th>LTQ</th>
<th>LVG</th>
<th>beta</th>
<th>VaR</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>average ( R^2 )</td>
<td>0.3235</td>
<td>0.1870</td>
<td>0.3642</td>
<td>0.2645</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>min ( R^2 )</td>
<td>0.0022</td>
<td>0.0000</td>
<td>0.0001</td>
<td>0.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>max ( R^2 )</td>
<td>0.8478</td>
<td>0.7876</td>
<td>0.7453</td>
<td>0.9799</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>std ( R^2 )</td>
<td>0.2244</td>
<td>0.1766</td>
<td>0.2178</td>
<td>0.2339</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
We have shown that:

- Different systemic risk measures identify different SIFIs
- Firm rankings based on systemic risk estimates mirror rankings obtained by sorting firms on market risk or liabilities
- One-factor linear models explain between 83% and 100% of the variability of the systemic risk estimates
- Standard systemic risk measures fail to capture the multiple facets of systemic risk
Given the very nature of systemic risk, future research on systemic risk should combine various sources of information, including balance-sheet data, proprietary data on positions, and market data (hybrid approach).

Future research on systemic risk should also consider the definition of the perimeter of the financial system.