Taming the Leverage Cycle

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An anecdote about a leverage cycle

Figure: Leverage of US Broker- Dealers (solid black line), S&P500 index (dashed blue line), VIX S&P500 (red dash-dotted line).
Intuition for leverage cycle

Basic idea:
In good times, when perceived risk is low, leverage goes up. In bad times, when risk is high, leverage goes down. Prices respond to leverage and risk responds to prices.

\[
\text{Leverage} = F(\text{Perceived risk}), \\
\text{Prices} = G(\text{Leverage}), \\
\text{Perceived risk} = H(\text{Prices}).
\]
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What do we need to model this?

- Leveraged investor (bank).
- An unleveraged fundamentalist investor.
- A risky and a risk free asset.
Stochastic discrete time model of leverage cycles

- underleveraged → new borrowing
- overleveraged → liquidate assets

- update risk estimate
- adjust balance sheet
- detect mispricing
- update decision
- fund
- exogenous shock
- equity flow
- asset revaluation
- regulatory constraint

- bank
Outline for the remainder of this talk

1. A model of a leveraged bank and a fund investor.

2. Emergence of endogenous risk $\rightarrow$ leverage cycles.

3. Optimal leverage policy in the presence of both exogenous and endogenous risk.
Bank leverage regulation

**Motivation:** VaR constraint with normal returns

\[ \lambda(t) \leq \bar{\lambda}(t) = F_{\text{VaR}}(\sigma^2(t)) = \frac{1}{\sigma(t)\Phi^{-1}(a)} \propto \frac{1}{\sigma(t)}. \]

**Our model:** 3 parameter leverage constraint

\[ \lambda(t) \leq \bar{\lambda}(t) = F_{(\alpha, \sigma_0^2, b)}(\sigma(t)) := \alpha(\sigma^2(t) + \sigma_0^2)^b. \]

**Note:**

- Due to profit maximization: \( \lambda(t) \approx \bar{\lambda}(t) := \text{target leverage}, \)
- \( \alpha: \) bank risk level (leverage at a given level of risk),
- \( b < 0: \) procyclical w.r.t \( \sigma(t), \)
- \( b > 0: \) countercyclical w.r.t \( \sigma(t), \)
- \( \sigma_0: \) lower/upper bound on leverage.
Cyclicality parameter $b$: procyclical vs. countercyclical policies

- **Procyclical**: $b = -0.5$
- **Constant**: $b = 0.0$
- **Countercyclical**: $b = 0.5$

The diagram illustrates the relationship between $\lambda$ (target leverage) and $\sigma^2$ (perceived risk) for different values of $b$. The horizontal lines represent $\alpha/\sigma_0$ and $\alpha\sigma_0$, with the corresponding lines for each type of policy.

- **Procyclical** line: $b = -0.5$
- **Constant** line: $b = 0.0$
- **Countercyclical** line: $b = 0.5$
Risk estimation and portfolio adjustment

Historical estimation of volatility

Let $p(t)$ be the price of the risky asset at time $t$. Then the bank’s **perceived risk** evolves as

$$
\sigma^2(t + \tau) = (1 - \tau \delta)\sigma^2(t) + \tau \delta \left( \log \left[ \frac{p(t)}{p(t - \tau)} \right] \frac{t_{\text{VaR}}}{\tau} \right)^2.
$$

Balance sheet

Adjust size of balance sheet to meet target leverage:

$$
\Delta B(t) = \tau \theta \{ \bar{\lambda}(t)(A_B(t) - L_B(t)) - A_B(t) \}.
$$

Adjust equity to meet equity target:

$$
\kappa_B(t) = \tau \eta \{ \bar{E} - (A_B(t) - L_B(t)) \}.
$$
The fund stabilizes the price dynamics of the risky asset

Fund characteristics:

- Not leveraged.
- Fund has a notion of a fundamental value $\mu$ of the risky asset.
- Dynamics of portfolio weight for risky asset:

$$\Delta \omega_F(t + \tau) \sim \rho(\mu - p(t)) + \sqrt{\tau} s(t) \xi(t),$$

where $\xi(t) \sim \mathcal{N}(0, 1)$ and $s(t)$ follow GARCH(1,1).

Note:

- Fund stabilizes prices (buys if price below fundamental, sells above).
- For $s = 0$ we obtain deterministic system.
- Fund is source of “clustered” exogenous volatility.
Market mechanism for risky asset

1. Bank and fund demand function:

\[
D_B(t + \tau) = \frac{1}{p(t + \tau)} w_B(n(t)p(t + \tau) + c_B(t) + \Delta B(t)),
\]

\[
D_F(t + \tau) = \frac{1}{p(t + \tau)} w_F(t + \tau)((1 - n(t))p(t + \tau) + c_F(t)).
\]

2. Compute \( p(t + \tau) \) by market clearing:

\[
1 = D_B(t + \tau) + D_F(t + \tau)
\]

3. Compute new ownership of risky asset for bank \( n(t + \tau) \) and fund \( 1 - n(t + \tau) \)
We can collect full model in 6D map

Map:

\[ x(t + \tau) = g(x(t)) \]

State vector:

\[ x(t) = [p(t), \sigma^2(t), n(t), L_B(t), w_F(t), p'(t)]^T, \]

where:

- \( p \): Price of risky asset.
- \( \sigma^2 \): Perceived risk.
- \( n \): Amount of asset owned by bank.
- \( L_B \): Liabilities of bank.
- \( w_F \): Investment into risky asset by fund.
- \( p' \): Past price of risky asset.
Guiding principles for choice of main parameters

1. Properties of the leverage cycle:
   - Peak-to-trough ratio $\approx 2$,
   - Period of cycles $\approx 10$ years,

   determines $\alpha$ (bank risk level), $\bar{E}$ (bank equity target).

2. Timescale for risk estimation:
   - $t_\delta = 1/\delta \approx 2$ years (based on RiskMetrics).
Examples of leverage cycles: we consider four parameter scenarios

(i) Deterministic, small bank (weak endogenous risk): $\bar{E} = 10^{-5}$ and $s = 0$,

(ii) Deterministic, large bank (strong endogenous risk): $\bar{E} = 2.27$ and $s = 0$,

(iii) Stochastic, small bank (weak endogenous risk): $\bar{E} = 10^{-5}$ and $s > 0$.

(iv) Stochastic, large bank (strong endogenous risk): $\bar{E} = 2.27$ and $s > 0$, 
Deterministic: (i) small bank vs. (ii) large bank

\[ \bar{E} = 10^{-5}, \alpha = 0.01 \]

\[ \bar{E} = 2.27, \alpha = 0.01 \]
Stochastic: (iii) small bank vs. (iv) large bank

\[ \bar{E} = 10^{-5}, \alpha = 0.075 \]

\[ \bar{E} = 2.27, \alpha = 0.075 \]
How do leverage cycles depend on the model parameters?

**Figure:** Deterministic model (eigenvalues)
How do leverage cycles depend on the model parameters?

Figure: Critical leverage for emergence of leverage cycles: deterministic/stochastic (Lyapunov exponents)
Which cyclicality parameter ($b$) minimizes bank losses for a given leverage: intuition

Case 1: Small bank, strong exogenous volatility clustering

1. No endogenous volatility due to leverage cycles.
2. Expect Value-at-Risk policy to be optimal ($b = -0.5$).

Case 2: Large bank, weak exogenous volatility clustering

1. Strong endogenous volatility due to leverage cycles.
2. Exogenous volatility is roughly constant.
3. Expect constant leverage policy to be optimal ($b = 0$).
Which cyclicality parameter \((b)\) minimizes bank losses for a given leverage: results

Figure: Realized shortfall at constant leverage.
Conclusions

1. Feedback between risk management, risk estimation and asset prices can lead to **endogenous volatility**.
2. Endogenous volatility increases with **bank leverage and size**.
3. **Bank losses can be reduced** by an appropriate leverage policy but choice of parameter depends crucially on levels of endogenous and exogenous volatility.
Back up

BACK UP
Recall:

\[ x(t) = [\sigma^2(t), w_F(t), p(t), n(t), L_B(t), p'(t)]^T, \]  

Definitions:

- Bank assets \( A_B(t) = p(t)n(t)/w_B, \)
- Target leverage \( \bar{\lambda}(t) = \alpha(\sigma^2(t) + \sigma_0^2)^b, \)
- Balance sheet adjustment \( \Delta B(t) = \tau \theta(\bar{\lambda}(t)(A_B(t) - L_B(t)) - A_B(t)), \)
- Equity redistribution \( \kappa_B(t) = -\kappa_F(t) = \tau \eta(\bar{E} - (A_B(t) - L_B(t))), \)
- Bank cash \( c_B(t) = (1 - w_B)n(t)p(t)/w_B + \kappa_B(t), \)
- Fund cash \( c_F(t) = (1 - w_F(t))(1 - n(t))p(t)/w_F(t) + \kappa_F(t). \)
Dynamical system:

\[ x(t + \tau) = g(x(t)) \]  

where the function \( g \) is the following 6-dimensional map:

\[
\begin{align*}
\sigma^2(t + \tau) &= (1 - \tau \delta)\sigma^2(t) + \tau \delta \left( \log \left[ \frac{p(t)}{p'(t)} \right] \frac{t_{\text{VaR}}}{\tau} \right)^2, \\
w_F(t + \tau) &= w_F(t) + \frac{w_F(t)}{p(t)} \left[ \tau \rho(\mu - p(t)) + \sqrt{\tau} s \xi(t) \right], \\
p(t + \tau) &= \frac{w_B(c_B(t) + \Delta B(t)) + w_F(t + \tau)c_F(t)}{1 - w_B n(t) - (1 - n(t))w_F(t + \tau)}, \\
n(t + \tau) &= \frac{w_B(n(t)p(t + \tau) + c_B(t) + \Delta B(t))}{p(t + \tau)}, \\
L_B(t + \tau) &= L_B(t) + \Delta B(t), \\
p'(t + \tau) &= p(t).
\end{align*}
\]