Information and Inventories in High-Frequency Trading

Johannes Muhle-Karbe

ETH Zürich and Swiss Finance Institute
Joint work with Kevin Webster

AMaMeF and Swissquote Conference, September 7, 2015
Introduction
Outline

Introduction

Model

Equilibrium

Inventory Aversion

Summary and Outlook
High-frequency trading is a game of informational advantage.
- Informational edge is small and frequent.
- Speed is necessary to take advantage.
- The only risk involved is inventory.

Simplest example: latency arbitrage.
- Regular traders observe national best bid and offer prices.
- HFTs purchase direct feeds to various electronic exchanges.
- Access to price changes before NBBO is updated.
- No long-term view on the market. Oscillating positions.
- Activity looks like white noise to traditional investors.

This paper: equilibrium model for these features.
Introduction
Model in a Nutshell

- Equilibrium between three types of agents:
  - Risk-neutral, competitive market makers.
  - HFTs with perfect one-period look-ahead filtration.
  - Low frequency “noise traders” with exogenous trading motives.
- Equilibrium determined by Stackelberg game.
  - Market makers move first by refilling order book.
  - HFTs and noise traders follow by executing market orders.
  - Same information structure as in literature on optimal price schedules (Glosten, ‘89, ‘94; Bernhardt and Hughson, ‘97; Biais, Martimort, and Rochet, ‘00).
  - Departs from double auction in Kyle’s ‘85 model and its variants (e.g., Focault, Hombert Rosu, ‘15, Rosu, ‘15).
  - Well-suited for high-frequency trading on electronic exchanges. Market makers cannot renege on posted limit orders.
- Allows to study both risk-neutral and inventory-averse HFTs.
Introduction
Results in a Nutshell

- **Risk-neutral HFTs:**
  - Hold martingale inventories. Fluctuate on same time scales as noise traders’.
  - Profits equal a fraction of the price volatility predicted.
  - Equilibrium price is conditional expectation of fundamental value.
  - Equilibrium price impact given by ratio between price and noise trading volatilities.
  - Findings consistent with sequence of one-shot Kyle models.

- **With inventory aversion:**
  - Autoregressive positions. Converge to zero in the continuous-time limit.
  - Limiting profits remain the same.
  - With sufficient trading speed: information can be monetized with almost no risk.
Model

Exogenous Inputs

- One safe asset normalized to one.
- Risky asset with fundamental value \( S_T = \int_0^T \sigma_t^S dW_t \).
- Noise trader demand driven by independent Brownian motion:
  \[
  dK_t = \mu^K_t dt + \sigma^K_t dZ_t
  \]
- To make information structure most transparent:
  - Set up model in discrete time.
  - Discretized processes \( X^N_n = X_{Tn/N} \), \( \Delta X^N_n = X^N_n - X^N_{n-1} \).
  - Pass to the limit only later to determine equilibrium.
- Market filtration \( \mathcal{F} \): generated by \( W \) and \( Z \).
- HFTs’ filtration \( \mathcal{G} \): also includes next price move.
  - E.g., latency arbitrage. But no frontrunning of low frequency traders.
Model

The trading game

- Market makers move first by posting a baseline price $P^N_n$ and a block shaped order book with height $1/\lambda^N_n$ around it.
- All trades clear together. Price impact shared equally by HFTs and noise traders.
- Risk-neutral HFTs: choose trades $\Delta L^N_{n+1}$ to maximize expected profits:

\[
\sum_{n=1}^{N-1} E \left[ (P^N_N - P^N_n) \Delta L^N_{n+1} - \frac{\lambda^N_n}{2} \left( \Delta L^N_{n+1} + \Delta K^N_{n+1} \right) \Delta L^N_{n+1} \right]
\]

- Pointwise optimization yields optimal strategy:

\[
\Delta \hat{L}^N_{n+1} = \frac{E[P^N_N|G_n] - P^N_n}{\lambda^N_n} - \frac{1}{2} E[\Delta K^N_{n+1}|G_n]
\]

- Difference between private and public forecast. Adjusted for expected noise trading activity.
Equilibrium
No Exploding Inventories

- Competitive market makers: zero expected profits.
- Market makers move first. No filtering required.
  - Short-lived information revealed anyways.
- Baseline price and price impact in equilibrium?
- Already determined by requiring that positions remain bounded in probability in the continuous-time limit.
- This minimal assumption already fixes baseline price as the martingale generated by the fundamental value:

\[ P_t = E[S_T | \mathcal{F}_t] = \int_0^t \sigma_s^S \, dW_s \]

- Corresponding price impact determined by setting expected profits of market makers equal to zero: \( \hat{\lambda}_t = \sigma_t^S / \sigma_t^K \).
Equilibrium
Comparison to Kyle ‘85

- Discrete-time quantities different because of information structure.
- Distinction vanishes in the continuous-time limit.
- Our model is consistent with a series of one-period Kyle models.
  - Equilibrium only determined by current volatilities.
  - Without long-lived information, insiders cannot time predictable trends as in Collin-Dufresne and Vos ‘14.
- But since no filtering is needed in our setting, nonlinear strategies can be treated as well.
- For example: inventory aversion.
Inventory Aversion
Criterion

- Empirical studies (Kirilenko et al. ‘14, SEC ‘10): HFTs characterized by high volume and low inventories.
- Risk-neutral case: HFTs’ and noise traders’ positions vary on the same time scales.
- As a remedy: add explicit inventory penalty $\gamma^N$:

$$E\left[\sum_{n=1}^{N-1} \left( (P^N_n - P^N_n) \Delta L^N_{n+1} - \frac{\lambda^N_n}{2} (\Delta L^N_{n+1} + \Delta K^N_{n+1}) \Delta L^N_{n+1} - \frac{\gamma^N}{2} (L^N_{n+1})^2 \right) \right]$$

- Penalty for buy-and-hold should be proportional to time held $\leadsto \gamma^N = \gamma / N$.
- Equilibrium?
- Tractable solutions?
Inventory Aversion
Equilibrium

- Inventory averse HFTs do not exploit mispricings as ruthlessly.
- No-exploding inventory condition no longer uniquely determines equilibrium price.
- But in reality, market makers do not know HFTs’ inventory aversion.
- If they are worried that at least one sufficiently risk-tolerant insider exists, they have to quote the martingale baseline price.
- Even with this choice, more complex preferences have to be tackled using dynamic programming.
  - Only possible in concrete models.
- Here: simplest specification. Brownian motions:

\[ S_T = \sigma^S W_T, \quad K_t = \sigma^K Z_t \]
Inventory Aversion

Dynamic Programming

- Quadratic ansatz as in Garleanu and Pedersen ‘13 leads to closed-form solution.
- For frequent trading, positions follow autoregressive process of order one:
  \[
  \hat{L}_{n+1}^N = -\sqrt{\frac{\gamma^N}{\lambda}} \hat{L}_n^N + \frac{\sigma^S}{\lambda} \Delta W_{n+1}^N + O(N^{-1})
  \]
- Speed of inventory management: tradeoff between inventory aversion \(\gamma^N\) and trading cost \(\lambda\).
- Same constant also shows up in other liquidation and optimization problems with linear price impact (Almgren and Chriss ‘01; Moreau, M-K, Soner ‘15).
- But here: mean reversion speed of order \(O(N^{-1/2})\) rather than \(O(N^{-1})\) as for a discretized OU process.
Consequence: “infinite trading speed” in the continuous-time limit.

HFTs’ positions converge to zero in $L^2(P)$.

But expected profits do not!
- At the leading order: same performance as without inventory management.
- Losses due to inventory management only visible in the first-order correction term of order $O(N^{-1/2})$.

Apparent contradiction to Rosu ‘15, who finds nontrivial losses for both “fast” ($O(1)$) and “slow” ($O(N^{-1})$) inventory management in a series of one-shot Kyle models.

Resolved by noticing that neither of his ad hoc policies uses the optimal trading speed of order $O(N^{-1/2})$.
Summary and Outlook

▶ Summary:
  ▶ Equilibrium model for information asymmetries in high-frequency trading.
  ▶ Risk-neutral HFTs hold martingale inventories fluctuating on the same time scales as noise traders’.
  ▶ Inventory aversion leads to vanishing positions yielding approximately the same returns in the continuous-time limit.
  ▶ Information can be monetized with very little risk.

▶ Outlook:
  ▶ Add information about noise trader order flow obtained by frontrunning. Should lead to HFT strategies alternating between market and limit orders.
  ▶ Add strategic low frequency traders, i.e., institutional investors. Do these benefit from a transaction tax?