A pricing measure for non-tradable assets with mean-reverting dynamics

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AMAMEF & Swiss Quote Conference
EPFL, Laussane
7-10 September 2015

Support from the projects EMMOS and MAWREM is gratefully acknowledged
Non tradable assets

- An asset is non tradable if we cannot build a portfolio with it.
- Energy related examples: non-storable commodities, weather indices, electricity...
- Interested in pricing forwards contracts on these assets.
- There is no buy and hold strategy $\implies$ classical non-arbitrage arguments break down.
- Any probability measure $Q$ equivalent to the historical measure $P$ is a valid risk neutral pricing measure.
- The forward price with time to delivery $0 < T < T^*$ at time $0 < t < T$ is given by

$$F_Q(t, T) = \mathbb{E}_Q[S(T)|\mathcal{F}_t]$$

where $\mathcal{F}_t$ is the information in the market up to time $t$. Assuming deterministic interest rates and $r = 0$. 
The risk premium for forward prices is defined by

\[ R^F_Q(t, T) \triangleq \mathbb{E}_Q [S(T) | \mathcal{F}_t] - \mathbb{E}_P [S(T) | \mathcal{F}_t]. \]

**Goal**: To be able to obtain more realistic risk premium profiles. For instance:

\[
\begin{array}{ccccccccc}
50 & 100 & 150 & 200 & 250 & 300 & 350 \\
4 & 2 & 2 & 4 & & & \\
\end{array}
\]

where \( \tau = T - t \) is the time to delivery.
Mathematical model

- Let \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \in [0, T^*]}, P)\) be a filtered probability space satisfying the usual hypothesis, where \(T^* > 0\) is a fixed finite time horizon.

- Consider a standard Brownian motion \(W\) and a pure jump Lévy subordinator

\[
L(t) = \int_0^t \int_0^\infty zN^L(ds, dz), \quad t \in [0, T^*],
\]

where \(N^L(ds, dz)\) is a Poisson random measure with Lévy measure \(\ell\) satisfying \(\int_0^\infty z\ell(dz) < \infty\).

- Let

\[
\kappa_L(\theta) \triangleq \log \mathbb{E}_P[e^{\theta L(1)}],
\]

and

\[
\Theta_L \triangleq \sup\{\theta \in \mathbb{R}_+ : \mathbb{E}_P[e^{\theta L(1)}] < \infty\}.
\]

- A minimal assumption is that \(\Theta_L > 0\).
Mathematical model

- Consider the Ornstein-Uhlenbeck processes $X$ with Barndorff-Nielsen & Shephard stochastic volatility $\sigma$

\[
X(t) = X(0) - \alpha \int_0^t X(s) ds + \int_0^t \sigma(s) W(t),
\]
\[
\sigma^2(t) = \sigma^2(0) - \rho \int_0^t \sigma^2(s) ds + L(t)
\]
\[
= \sigma^2(0) + \int_0^t (\kappa_L'(0) - \rho \sigma^2(s)) ds + \int_0^t \int_0^\infty z \tilde{N}_L(ds, dz),
\]
with $t \in [0, T^*]$, $\alpha, \rho > 0$, $X(0) \in \mathbb{R}$, $\sigma^2(0) > 0$ and
\[
\tilde{N}_L(ds, dz) = N_L(ds, dz) - \ell(dz) ds.
\]

- We model the spot price process by

\[
S(t) = \Lambda g \exp(X(t)), \quad t \in [0, T^*].
\]
The change of measure

- Let $\bar{\beta} = (\beta_1, \beta_2) \in [0, 1]^2$ and $\bar{\theta} = (\theta_1, \theta_2) \in \bar{D}_L \triangleq \mathbb{R} \times D_L$, where $D_L \triangleq (-\infty, \Theta_L/2)$.
- Consider the following family of kernels
  \[ G_{\theta_1, \beta_1}(t) \triangleq \sigma^{-1}(t)(\theta_1 + \alpha \beta_1 X(t)), \quad t \in [0, T^*], \]
  \[ H_{\theta_2, \beta_2}(t, z) \triangleq e^{\theta_2 z} \left( 1 + \frac{\rho \beta_2}{\kappa_2''(\theta_2)} z \sigma^2(t-) \right), \quad t \in [0, T^*], z \in \mathbb{R}_+. \]
- Next, define the following family of Wiener and Poisson integrals
  \[ \tilde{G}_{\theta_1, \beta_1}(t) \triangleq \int_0^t G_{\theta_1, \beta_1}(s)dW(s), \quad t \in [0, T^*], \]
  \[ \tilde{H}_{\theta_2, \beta_2}(t) \triangleq \int_0^t \int_0^\infty (H_{\theta_2, \beta_2}(s, z) - 1) \tilde{N}^L(ds, dz), \quad t \in [0, T^*], \]
  associated to the kernels $G_{\theta_1, \beta_1}$ and $H_{\theta_2, \beta_2}$, respectively.
The change of measure

- The family of measure changes is given by $Q_{\tilde{\theta}, \tilde{\beta}} \sim P$, $\tilde{\beta} \in [0, 1]^2$, $\tilde{\theta} \in \tilde{D}_L$, with

$$\frac{dQ_{\tilde{\theta}, \tilde{\beta}}}{dP} \bigg|_{\mathcal{F}_t} \triangleq \mathcal{E}(\tilde{G}_{\theta_1, \beta_1} + \tilde{H}_{\theta_2, \beta_2})(t), \quad t \in [0, T^*],$$

where $\mathcal{E}(\cdot)$ denotes the stochastic exponential.

- Recall that, if $M$ is a semimartingale, the stochastic exponential of $M$ is the unique strong solution of

$$d\mathcal{E}(M)(t) = \mathcal{E}(M)(t-)^dM(t), \quad t \in [0, T^*],$$

$\mathcal{E}(M)(0) = 1,$

which is given by

$$\mathcal{E}(M)(t) = \exp \left( M(t) - \frac{1}{2} \langle M^c, M^c \rangle(t) \right) \prod_{0 \leq s \leq t} \left( 1 + \Delta M(s) \right) e^{-\Delta M(s)}.$$
The change of measure

- **Yor’s Addition Formula**: Let $M_1$ and $M_2$ two semimartingales starting at 0. Then,

$$
\mathcal{E}(M_1 + M_2 + [M_1, M_2])(t) = \mathcal{E}(M_1)(t)\mathcal{E}(M_2)(t), \quad 0 \leq t \leq T^*,
$$

where

$$
[M_1, M_2](t) = \langle M_1^C, M_2^C \rangle(t) + \sum_{s \leq t} \Delta M_1(s)\Delta M_2(s).
$$

- As $[\tilde{G}_{\theta_1, \beta_1}, \tilde{H}_{\theta_2, \beta_2}] \equiv 0$, we can write

$$
\mathcal{E}(\tilde{G}_{\theta_1, \beta_1} + \tilde{H}_{\theta_2, \beta_2})(t) = \mathcal{E}(\tilde{G}_{\theta_1, \beta_1})(t)\mathcal{E}(\tilde{H}_{\theta_2, \beta_2})(t), \quad t \in [0, T^*].
$$

- Conditioning on $\mathcal{F}^L_T$, we have

$$
\mathbb{E}_P[\mathcal{E}(\tilde{G}_{\theta_1, \beta_1} + \tilde{H}_{\theta_2, \beta_2})(T^*)] = \mathbb{E}_P[\mathbb{E}_P[\mathcal{E}(\tilde{G}_{\theta_1, \beta_1})(T^*)|\mathcal{F}^L_T]\mathcal{E}(\tilde{H}_{\theta_2, \beta_2})(T^*)].
$$

- Hence the problem is reduced to show that $\mathcal{E}(\tilde{G}_{\theta_1, \beta_1})$ and $\mathcal{E}(\tilde{H}_{\theta_2, \beta_2})$ are true martingales with respect to appropriate filtrations.
Sketch of the proof that $\mathcal{E}(M)$ is a martingale

- $M$ can be $\tilde{G}_{\theta_1, \beta_1}$ or $\tilde{H}_{\theta_2, \beta_2}$.
- Localize $\mathcal{E}(M)$ using a reducing sequence $\{\tau_n\}_{n \geq 1}$.
- Check that $1 = \lim_{n \to \infty} \mathbb{E}_P[\mathcal{E}(M)^{\tau_n}(T^*)] = \mathbb{E}_P[\mathcal{E}(M)(T^*)]$.
- Test the uniform integrability of $\{\mathcal{E}(M)^{\tau_n}(T^*)\}_{n \geq 1}$ with $F(x) = x \log(x)$, i.e.
  \[
  \sup_n \mathbb{E}_P[F(\mathcal{E}(M)^{\tau_n}(T^*))] < \infty. \tag{1}
  \]
- For any $n \geq 1$, $\{\mathcal{E}(M)^{\tau_n}(t)\}_{t \in [0, T^*]}$ is a true martingale and induces a change of measure $\frac{dQ^n}{dP} \bigg|_{\mathcal{F}_t} = \mathcal{E}(M)^{\tau_n}(t)$.
- Condition (1) can be rewritten as
  \[
  \sup_n \mathbb{E}_{Q^n}[\log(\mathcal{E}(M)^{\tau_n}(T^*))] < \infty.
  \]
- We can get rid of the ordinary exponential in $\mathcal{E}(M)^{\tau_n}(T^*)$.
- The problem is reduced to finding a uniform bound for the second moment of $X$ and $\sigma^2$ under $Q^n$. 
By Girsanov’s theorem for semimartingales, we can write

\[ X(t) = X(0) + B^{X}_{Q,\tilde{\theta},\tilde{\beta}}(t) + \int_{0}^{t} \sigma(s) dW^{Q}_{\tilde{\theta},\tilde{\beta}}(t), \quad t \in [0, T]^*, \]

\[ \sigma^2(t) = \sigma^2(0) + B^{\sigma^2}_{Q,\tilde{\theta},\tilde{\beta}}(t) + \int_{0}^{t} \int_{0}^{\infty} \tilde{N}^{L}_{Q,\tilde{\theta},\tilde{\beta}}(ds, dz), \quad t \in [0, T^*], \]

where

\[ B^{X}_{Q,\tilde{\theta},\tilde{\beta}}(t) = \int_{0}^{t} (\theta_1 - \alpha(1 - \beta_1)X(s)) ds, \quad t \in [0, T^*], \]

and

\[ B^{\sigma^2}_{Q,\tilde{\theta},\tilde{\beta}}(t) = \int_{0}^{t} \left( \kappa'_L(\theta_2) - \rho(1 - \beta_2)\sigma^2(s) \right) ds, \quad t \in [0, T^*]. \]
The dynamics under the new pricing measure

- The $Q_{\tilde{\theta}, \tilde{\beta}}$-compensator measure of $\sigma^2$ is given by

\[
v_{Q_{\tilde{\theta}, \tilde{\beta}}}^{\sigma^2}(dt, dz) = e^{\theta_2 z} \left( 1 + \frac{\rho \beta_2}{\kappa_L''(\theta_2)} z \sigma^2(t-) \right) \ell(dz) dt.
\]

- Using integration by parts, we get

\[
X(T) = X(t) e^{-\alpha(1-\beta_1)(T-t)} + \frac{\theta_1}{\alpha(1-\beta_1)} (1 - e^{-\alpha(1-\beta_1)(T-t)})
\]
\[
+ \int_t^T \sigma(u) e^{-\alpha(1-\beta_1)(T-u)} dW_{Q_{\tilde{\theta}, \tilde{\beta}}}(u),
\]

\[
\sigma^2(T) = \sigma^2(t) e^{-\alpha(1-\beta_2)(T-t)} + \frac{\kappa_L'(\theta_2)}{\alpha(1-\beta_2)} (1 - e^{-\alpha(1-\beta_2)(T-t)})
\]
\[
+ \int_t^T \int_0^\infty e^{-\rho(1-\beta_2)(T-u)} z \tilde{N}_{Q_{\tilde{\theta}, \tilde{\beta}}}^L(du, dz),
\]

where $0 \leq t \leq T \leq T^*$. 
Moment condition under the historical measure

- A sufficient condition for $S$ to have finite expectation under $P$ is the following:

Assumption ($\mathcal{P}$)

We assume that $\alpha, \rho > 0$ and $\Theta_L$ satisfy

$$\frac{1}{2\rho} \left( \frac{\rho}{2\alpha} \right)^{\frac{1}{1-\frac{\rho}{2\alpha}}} \leq \Theta_L - \delta,$$

for some $\delta > 0$.

- If $\Theta_L = \infty$ then assumption $\mathcal{P}$ is satisfied.
- If $\Theta_L < \infty$, then if we choose $\rho$ close to zero the value of $\alpha$ must be bounded away from zero, and vice versa, for assumption $\mathcal{P}$ to be satisfied.
Forward price formula

Proposition

The forward price \( F_{Q_{\bar{\theta}, \bar{\beta}}}(t, T) \) is given by

\[
F_{Q_{\bar{\theta}, \bar{\beta}}}(t, T) = \Lambda_g(T) \exp \left( X(t) e^{-\alpha(1-\beta_1)(T-t)} + \sigma^2(t) e^{-\rho(1-\beta_2)(T-t)} \frac{1 - e^{-(2\alpha-\rho(1-\beta_2))(T-t)}}{2(2\alpha - \rho)} \right) \
\times \exp \left( \frac{\kappa'(\theta_2)}{2\rho(1-\beta_2)} \left( \frac{1 - e^{-2\alpha(T-t)}}{2\alpha} - e^{-\rho(1-\beta_2)(T-t)} \frac{1 - e^{-(2\alpha-\rho(1-\beta_2))(T-t)}}{2(2\alpha - \rho(1-\beta_2))} \right) \right) \
\times \exp \left( \frac{\theta_1}{\alpha(1-\beta_1)} (1 - e^{-\alpha(1-\beta_1)(T-t)}) \right) \
\times \mathbb{E}_{Q_{\bar{\theta}, \bar{\beta}}} \left[ \exp \left( \frac{e^{-2\alpha T}}{2} \int_t^T e^{(2\alpha-\rho(1-\beta_2))s} \left( \int_t^s \int_0^\infty e^{\rho(1-\beta_2)u} \tilde{N}_Q(du, dz) \right) ds \right) \mid \mathcal{F}_t \right] 
\]

In the particular case \( Q_{\bar{\theta}, \bar{\beta}} = P \), it holds that

\[
F_P(t, T) = \Lambda_g(T) \exp \left( X(t) e^{-\alpha(T-t)} + \sigma^2(t) e^{-\rho(T-t)} \frac{1 - e^{-(2\alpha-\rho)(T-t)}}{2(2\alpha - \rho)} \right) \
\times \exp \left( \int_0^{T-t} \kappa_L \left( e^{-\rho s} \frac{1 - e^{-(2\alpha-\rho)s}}{2(2\alpha - \rho)} \right) ds \right). 
\]
Theorem

Let \( \bar{\beta} = (\beta_1, \beta_2) \in [0, 1]^2, \bar{\theta} = (\theta_1, \theta_2) \in \bar{D}_L \) and \( T > 0 \). Suppose there exist functions \( \Psi_{i, \bar{\beta}}^{\bar{\theta}} \), \( i = 0, 1, 2 \) belonging to \( C^1([0, T]; \mathbb{R}) \), satisfying the generalised Riccati equation

\[
\frac{d}{dt} \Psi_{0, \bar{\beta}}^{\bar{\theta}}(t) = \theta_1 \Psi_{2, \bar{\beta}}^{\bar{\theta}}(t) + \kappa L \left( \Psi_{1, \bar{\beta}}^{\bar{\theta}}(t) + \theta_2 \right) - \kappa L(\theta_2),
\]

\[
\frac{d}{dt} \Psi_{1, \bar{\beta}}^{\bar{\theta}}(t) = -\rho \Psi_{1, \bar{\beta}}^{\bar{\theta}}(t) + \frac{(\Psi_{2, \bar{\beta}}^{\bar{\theta}}(t))^2}{2} + \frac{\rho \beta_2}{\kappa'' L(\theta_2)} \left( \kappa' L \left( \Psi_{1, \bar{\beta}}^{\bar{\theta}}(t) + \theta_2 \right) - \kappa' L(\theta_2) \right),
\]

\[
\frac{d}{dt} \Psi_{2, \bar{\beta}}^{\bar{\theta}}(t) = -\alpha (1 - \beta_1) \Psi_{2, \bar{\beta}}^{\bar{\theta}}(t),
\]

with initial conditions \( \Psi_{0, \bar{\beta}}^{\bar{\theta}}(0) = \Psi_{1, \bar{\beta}}^{\bar{\theta}}(0) = 0 \) and \( \Psi_{2, \bar{\beta}}^{\bar{\theta}}(0) = 1 \).

Moreover, suppose that the integrability condition

\[
\sup_{t \in [0, T]} \kappa'' L \left( \theta_2 + \Psi_{1, \bar{\beta}}^{\bar{\theta}}(t) \right) < \infty,
\]

holds.
Affine transform formula

Theorem (Continue)

Then,

\[ \mathbb{E}_{Q_{\tilde{\theta}, \tilde{\beta}}}[\exp(X(T))|\mathcal{F}_t] = \exp \left( \Psi_0^{\tilde{\theta}, \tilde{\beta}} (T - t) + \Psi_1^{\tilde{\theta}, \tilde{\beta}} (T - t)\sigma^2(t) + \Psi_2^{\tilde{\theta}, \tilde{\beta}} (T - t)X(t) \right), \]

and

\[ R_{Q_{\tilde{\theta}, \tilde{\beta}}}^F(t, T) = \mathbb{E}_P[S(T)|\mathcal{F}_t] \]

\[ \times \left\{ \exp \left( \Psi_0^{\tilde{\theta}, \tilde{\beta}} (T - t) - \int_0^{T-t} \kappa_L \left( e^{-\rho s} \frac{1 - e^{-(2\alpha - \rho)s}}{2(2\alpha - \rho)} \right) ds \right. \right. \]

\[ + \left( \Psi_1^{\tilde{\theta}, \tilde{\beta}} (T - t) - e^{-\rho(T-t)} \frac{1 - e^{-(2\alpha - \rho)(T-t)}}{2(2\alpha - \rho)} \right) \sigma^2(t) \]

\[ + \left( \Psi_2^{\tilde{\theta}, \tilde{\beta}} (T - t) - e^{-\alpha(T-t)} \right) X(t) \left. \right) - 1 \}, \]

for \( t \in [0, T] \).
Some remarks on the theorem

- The proof follows by applying a result by Kallsen and Muhle-Karbe (2010).
- Limited applicability as it is stated.
- Reduction to a one dimensional non autonomous ODE.
  - We have that for any \( \bar{\theta} \in \bar{D}_L, \bar{\beta} \in [0, 1]^2 \), the solution of the last equation is given by \( \Psi_{2, \bar{\theta}, \bar{\beta}}(t) = \exp(-\alpha(1 - \beta_1)t) \).
  - Plugging this solution to the first equation we get the following equation to solve for \( \Psi_{1, \bar{\theta}, \bar{\beta}}(t) \)

\[
\frac{d}{dt} \Psi_{1, \bar{\theta}, \bar{\beta}}(t) = -\rho \Psi_{1, \bar{\theta}, \bar{\beta}}(t) + \frac{e^{-2\alpha(1-\beta_1)t}}{2} + \frac{\rho \beta_2}{\kappa''_L(\theta_2)} (\kappa'_L(\Psi_{1, \bar{\theta}, \bar{\beta}}(t) + \theta_2) - \kappa'_L(\theta_2)),
\]

with initial condition \( \Psi_{1, \bar{\theta}, \bar{\beta}}(0) = 0 \).
- The equation for \( \Psi_{0, \bar{\theta}, \bar{\beta}}(t) \) is solved by integrating \( \Lambda_{0, \bar{\theta}, \bar{\beta}}(\Psi_{1, \bar{\theta}, \bar{\beta}}(t), \Psi_{2, \bar{\theta}, \bar{\beta}}(t)) \), i.e.,

\[
\Psi_{0, \bar{\theta}, \bar{\beta}}(t) = \int_0^t \left\{ \theta_1 \Psi_{2, \bar{\theta}, \bar{\beta}}(s) + \kappa_L(\Psi_{1, \bar{\theta}, \bar{\beta}}(s) + \theta_2) - \kappa_L(\theta_2) \right\} ds
= \theta_1 \frac{1 - e^{-\alpha(1 - \beta_1)t}}{\alpha(1 - \beta_1)} + \int_0^t \left\{ \kappa_L(\Psi_{1, \bar{\theta}, \bar{\beta}}(s) + \theta_2) - \kappa_L(\theta_2) \right\} ds.
\]
Sufficient conditions for existence of global solutions

- Study of the equation $\frac{d}{dt} \Psi_{\theta_2, \beta_2}(t) = \Lambda_{\theta_2, \beta_2}(\Psi_{\theta_2, \beta_2})$, where
  \[
  \Lambda_{\theta_2, \beta_2}(u) = -\rho u + \frac{1}{2} + \frac{\rho \beta_2}{\kappa_L''(\theta_2)}(\kappa_L'(u + \theta_2) - \kappa_L'(\theta_2)).
  \]

- Let
  \[
  D_b = \{(\theta_2, \beta_2) \in D_L \times (0, 1) : \exists u \in [0, \Theta_L - \theta_2) \text{ s.t. } \Lambda_{\theta_2, \beta_2}(u) \leq 0\}.
  \]

**Theorem**

If $(\theta_2, \beta_2) \in D_b$ and $(\theta_1, \beta_1) \in \mathbb{R} \times [0, 1)$ then $\Psi_{0, \bar{\beta}}(t)$, $\Psi_{1, \bar{\beta}}(t)$ and $\Psi_{2, \bar{\beta}}(t)$ are $C^1([0, T]; \mathbb{R})$ for any $T > 0$. Moreover,

\[
\Psi_{0, \bar{\beta}}(t) \longrightarrow \frac{\theta_1}{\alpha(1 - \beta_1)} + \int_0^\infty \left\{ \kappa_L \left( \Psi_{1, \bar{\beta}}(s) + \theta_2 \right) - \kappa_L(\theta_2) \right\} \, ds, \quad t \to \infty,
\]

\[
(\Psi_{1, \bar{\beta}}(t), \Psi_{2, \bar{\beta}}(t)) \longrightarrow (0, 0), \quad t \to \infty,
\]

and

\[
t^{-1} \log \left\| (\Psi_{1, \bar{\beta}}(t), \Psi_{2, \bar{\beta}}(t)) \right\| \to \gamma, \quad t \to \infty,
\]

for some negative constant $\gamma$. 
Risk premium analysis in the geometric BNS model

Lemma
If \((\theta_2, \beta_2) \in D_b\) and \((\theta_1, \beta_1) \in \mathbb{R} \times [0, 1)\), the sign of the risk premium \(R^F_{Q_{\bar{\theta}, \bar{\beta}}}(t, T)\) is the same as the sign of the function

\[
\Sigma(t, T) \triangleq \Psi^\bar{\theta}, \bar{\beta}_0(T - t) - \Psi^0,0_0(T - t) + (\Psi^\bar{\theta}, \bar{\beta}_1(T - t) - \Psi^0,0_1(T - t))\sigma^2(t) + (\Psi^\bar{\theta}, \bar{\beta}_2(T - t) - \Psi^0,0_2(T - t))X(t).
\]

Moreover,

\[
\lim_{T \to \infty} \Sigma(t, T) = \frac{\theta_1}{\alpha(1 - \beta_1)} + \int_0^\infty \int_0^1 \kappa'_L \left( \lambda \Psi^\bar{\theta}, \bar{\beta}_1(s) + \theta_2 \right) d\lambda \Psi^\bar{\theta}, \bar{\beta}_1(s) ds
\]

\[
- \int_0^\infty \int_0^1 \kappa'_L \left( \lambda e^{-\rho s} 1 - e^{-2(\alpha - \rho)s} \right) d\lambda e^{-\rho s} \frac{1 - e^{-(2\alpha - \rho)s}}{2(2\alpha - \rho)} ds,
\]

and

\[
\lim_{T \to t} \frac{d}{dT} \Sigma(t, T) = \theta_1 + \alpha \beta_1 X(t).
\]

Conclusions

- A pricing measure for the BNS model for non-tradable assets with mean reversion extending Esscher’s transform.
- Preserves affine structure of the model.
- Gives control on the speed and level of mean reversion.
- Provides more realistic risk premium profiles.

Further work

- Study the BNS model with jumps

\[ X(t) = X(0) - \alpha \int_0^t X(s) ds + \int_0^t \sigma(s) W(t) + \eta L(t), \]
\[ \sigma^2(t) = \sigma^2(0) - \rho \int_0^t \sigma^2(s) ds + L(t). \]

- Calibration.
- Pricing more complex derivatives.
- Other models with mean reversion.
References


