Ambiguity Aversion in Standard and Extended Ellsberg Frameworks: $\alpha$-Maxmin versus Maxmin Preferences

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Motivation

Maxmin and $\alpha$-maxmin expected utility models

Market Model

Standard Ellsberg framework

Extended Ellsberg frameworks
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Ambiguity

Two kinds of uncertainty (distinction due to Knight (1921))

1. **Risk**: all events are associated with obvious *probability assignments*

2. **Ambiguity** or Knightian uncertainty: some events do not have an obvious, unanimously agreeable, *probability assignment*

Facing *ambiguity*, decision makers may adjust their behaviour:

- ambiguity *averse*: take action robust to ambiguity
- ambiguity *seeking*: take action exposed to ambiguity
Ambiguity sensitive preferences and market phenomena

1. Experimental evidence:
   majority of subjects are ambiguity averse, a few ambiguity seeking

2. Theoretical financial market models:
   different attitude towards ambiguity are consistent with non participation, portfolio inertia, and excess of volatility of assets returns

⇒

Investors’ preferences: heterogeneous, well approximated by SEU and ambiguity averse preferences with different degree of ambiguity aversion

Ambiguity sensitive behaviors are inconsistent with SEU theory ⇒

Several models to model ambiguity sensitive preferences

We focus on: maxmin expected utility and α-maxmin expected utility
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Maxmin expected utility model \textit{Gilboa–Schmeidler (1989)}

maxmin \textit{pioneering and workhorse} model to study the implications of ambiguity aversion in portfolio choice and equilibrium asset prices

\[ U(w) = \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma) \pi_\sigma \]

\begin{itemize}
  \item \( \mathcal{C} \) set of priors \textit{determined jointly by agent’s information and personal taste}
  \item \textit{ambiguity averse attitude}: portfolio \( w \) evaluated by its minimum expected utility
\end{itemize}

\rightarrow \textit{tractable optimization problems:} \( u \) concave \( \Rightarrow \) \( U \) concave

\rightarrow \textit{information and attitude to ambiguity \textit{inextricably intertwined} in} \( \mathcal{C} \): smaller sets may reflect both better information and less averse ambiguity attitude
\(\alpha\)-maxmin expected utility model

\(\alpha\)-maxmin expected utility (\(\alpha\)-MEU) model

\[
U(w) = \alpha \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma) \pi_\sigma + (1 - \alpha) \max_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma) \pi_\sigma, \quad \alpha \in [0, 1]
\]

generalizes the maxmin model (\(\alpha = 1\))

→ large spectrum of ambiguity attitude: from the ambiguity aversion of the maxmin model to ambiguity seeking of the maxmax model (\(\alpha = 0\))

→ given \(\mathcal{C} = \mathcal{C}_{\text{max}} = \{ \text{all priors consistent with available information} \} \) the parameter \(\alpha\) is a measure the degree of agent’s ambiguity aversion: the greater \(\alpha\) the more ambiguity averse the agent’s preferences

→ portfolio optimization problem not concave: \(U\) in general not concave despite \(u\) concave

→ no axiomatization of the model
\(\alpha\)-MEU model in the literature

\(\alpha\)-MEU model increasingly popular

- \textit{decision theory}

- \textit{Experimental financial economics}
  \(\alpha\)-MEU for the estimation of the agent’s degree of ambiguity aversion/seeking; \textit{Ahn et al.} (2011)

- \textit{Theoretical financial economics}: just a few
  competitive financial market with SEU and \(\alpha\)-MEU agents; \textit{Bossaerts, Ghirardato Guarnaschelli and Zame} (2010) (BGGZ)

Setting of the above papers: \textit{Standard Ellsberg framework BUT} in this setting ambiguity averse \(\alpha\)-MEU preferences \textit{reduce to} maxmin preference!
Goal of this paper

Financial economics point of view:

derive the implications of $\alpha$-MEU model for

- portfolio choices
- equilibrium asset prices

as a function of the degree of ambiguity aversion $\alpha \in (0, 1)$

This allows to

- provide criteria to estimate’s agent’s ambiguity aversion
- understand the $\alpha$-MEU model and the preferences that it represents
- contrast $\alpha$-MEU and maxmin model implications
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Setting

Arrow–Debreu complete market with two dates: $t = 0$ and $t = 1$ embedded in an Ellsberg framework

- $S$ finite state space with $m + l$ states of the economy at $t = 1$ states correspond to draws from Ellsberg (1961) urn:

  $m \geq 0$ risky states and $l \geq 2$ ambiguous states

- At $t = 0$ investors face risk and ambiguity:

  neither know which state will realize at $t = 1$ (risk), nor what is probability of some states (ambiguity)
· ∀ω ∈ S, ∃ Arrow security traded in the market ⇒ agents can form:
  
  ▶ unambiguous portfolio, i.e. portfolio with no exposure to ambiguity:

  \[
  w = \left( \underbrace{w_1^R, \ldots, w_m^R}_{\text{m risky states}}, \underbrace{\tilde{w}, \ldots, \tilde{w}}_{\text{l ambiguous states}} \right)
  \]

  ▶ ambiguous portfolios, i.e. portfolios exposed to ambiguity:

  \[
  w = \left( \underbrace{w_1^R, \ldots, w_m^R}_{\text{m risky states}}, \underbrace{w_{m+1}^R, \ldots, w_{m+l}^R}_{\text{l ambiguous states}} \right)
  \]

· \( u \) (risk aversion) strictly concave and increasing and differentiable
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Standard Ellsberg framework \((m = 1, \ l = 2)\)

\[S = \{R, G, B\}\]

\(m = 1\) risky state, \(R\), with known probability \(\pi_R \in (0, 1)\)

\(l = 2\) ambiguous states, \(G\) and \(B\), with unknown probabilities

Any set of priors consistent with the Ellsberg framework reads

\[\mathcal{D}_{a,b} = \{(\pi_R, q, 1 - q - \pi_R) : q \in [a, b]\}\]

for some given \(a\) and \(b\), \(0 \leq a \leq b \leq 1 - \pi_R\) \(\Rightarrow\)

Any \(\alpha\)-MEU utility with \(\alpha \in [0, 1]\) reads

\[U(w) = \pi_R u(w_R) + \alpha \min_{q \in [a,b]} [qu(w_G) + (1 - q - \pi_R)u(w_B)]
\]

\[+ (1 - \alpha) \max_{q \in [a,b]} [qu(w_G) + (1 - q - \pi_R)u(w_B)]\]

\(w\) is a state dependent portfolio \(w = (w_R, w_G, w_B) \in \mathbb{R}^3\)
Main findings in the Standard Ellsberg framework

- **Equivalence result**: \( \alpha\)-MEU preferences are equivalent to:
  - \( \text{maxmin} \) preferences when \( \alpha > \frac{1}{2} \)
  - \( \text{SEU} \) preferences when \( \alpha = \frac{1}{2} \)
  - \( \text{maxmax} \) preferences when \( \alpha < \frac{1}{2} \)

- **Ambiguity aversion implications on equilibrium asset prices**: ambiguity aversion does not wash out in equilibrium
When $\alpha > 1/2$:

any $\alpha$-MEU utility, $\alpha > 1/2$, set of prior $\mathcal{D}_{a,b}$ is equal to a *unique maxmin* utility with set of priors $\mathcal{C}$ *smaller* than $\mathcal{D}_{a,b}$, and univocally characterized by $\alpha$ and $\mathcal{D}_{a,b}$:

$$U(w) = \min_{q \in [c, d]} [\pi_R u(w_R) + q u(w_G) + (1 - q - \pi_R) u(w_B)]$$

$$\mathcal{C} = \{ (\pi_R, q, 1 - q - \pi_R) : q \in [c, d] \} \subset \mathcal{D}_{a,b}$$

$$c := \alpha a + (1 - \alpha) b \text{ and } d := (1 - \alpha) a + \alpha b$$

*decreasing* $\alpha$ from 1 to 1/2 in the $\alpha$-MEU representation, is equivalent to *shrinking symmetrically* $\mathcal{C}$ in the corresponding *maxmin* representation.
Implications of the equivalence result:

- **Standard Ellsberg framework is not the right setting to study the $\alpha$-MEU model:** $\alpha$-MEU preferences cannot be distinguished from maxmin, or maxmax or SEU preferences.

- **Experimental studies:**
  - It clarifies recent works in the standard Ellsberg framework that uses the $\alpha$-MEU as generalization of the maxmin model to document a substantial heterogeneity in aversion to ambiguity; e.g. BGGZ (2010), and Ahn *et al.* (2011).
  - *Same conclusion by using maxmin* varying the size of the set of priors instead of $\alpha$ to measure the degrees of aversion to ambiguity.
Implications of ambiguity aversion on equilibrium asset prices

Financial market populated by $L + M$ investors

- $L$ with SEU preferences with prior $\pi = (\pi_R, \pi_G, \pi_B)$
- $M$ with ambiguity averse preferences (maxmin or equivalently $\alpha$-MEU $\alpha > 1/2$)

1. we derive equilibrium asset prices and show through which channels ambiguity aversion impacts equilibrium asset prices
2. theoretical equilibrium asset prices perfectly match the experimental findings in BGGZ (2010)

$\Rightarrow$ Ambiguity aversion does not wash out in equilibrium
Portfolio choice of SEU and ambiguity averse agents

Well known facts:

- **SEU agent** optimal portfolio $y = (y_R, y_G, y_B)$ state dependent wealth is ranked opposite to the state-price/state-probability ratios:
  
  $$y_\sigma > y_\nu \iff \frac{p_\sigma}{\pi_\sigma} < \frac{p_\nu}{\pi_\nu}, \quad \sigma, \nu \in \{R, B, G\}$$

- **ambiguity averse agent** with utility
  
  $$U(w) = \pi_R u(w_R) + \min_{q \in [c, d]} (q u(w_G) + (1 - q - \pi_R) u(w_B)),$$

  optimal portfolio $w = (w_R, w_G, w_B)$:

  $$\begin{cases} 
  w_G > w_B & \iff \frac{p_G}{p_B} < \frac{c}{1 - \pi_R - c} \\
  w_G < w_B & \iff \frac{p_G}{p_B} > \frac{d}{1 - \pi_R - d} \\
  w_G = w_B \text{ (unambiguous)} & \iff \frac{p_G}{p_B} \in \left[\frac{c}{1 - \pi_R - c}, \frac{d}{1 - \pi_R - d}\right]
  \end{cases}$$

  The larger the set of priors, the more likely $w_B = w_G$
Market equilibrium: theoretical findings

State-price/state-probability ratios in equilibrium depend on the market total endowment $W = (W_R, W_G, W_B)$

Case of interest:
- $W_G \neq W_B$, for instance $W_G > W_B$
- ambiguity averse agents take unambiguous portfolio, i.e. $w_G = w_B$

SEU agents have to clear the supply difference $W_G - W_B \Rightarrow \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$

Intuition: to induce SEU agents to clear $W_G - W_B$ $p_G$ will be lower and $p_B$ be higher than in a market populated only by SEU agents sharing the same prior
Market equilibrium: theoretical findings (cont.)

Proposition A:

Suppose \( W_G > W_B > W_R \) ⇒ two rankings are possible in equilibrium:

1. \( \frac{P_B}{\pi_B} > \frac{P_R}{\pi_R} > \frac{P_G}{\pi_G}, y_G > y_R > y_B \) and \( w_R < w_G = w_B \)

2. \( \frac{P_R}{\pi_R} > \frac{P_B}{\pi_B} > \frac{P_G}{\pi_G}, y_G > y_B > y_R \) and \( w_R < w_G = w_B \)

When ranking 1. realizes (\( W_G - W_B \) is ”large enough”) ⇒

- the optimal portfolios of the \( L \) SEU agents do not rank as the total endowment

- the total endowment is the optimal portfolio of the SEU-representative agent rationalizing the market equilibrium

⇒ SEU-representative agent and SEU agents in the market will rank the state-price/state-probability ratio differently
CARA utility $u(z) = 1 - e^{-\delta z}/\delta$

- $\delta = a$ risk aversion of the $L$ SEU agents
- $\delta = b = $ risk aversion of the $M$ ambiguity averse agents

The equilibrium prices resulting is the same as if, with the $L$ SEU,
$\rightarrow$ instead of the $M$ ambiguity averse agents we would have
$\rightarrow M$ SEU agents with prior $(\pi_R, q, 1 - \pi_R - q)$:

$$q := \pi_G \frac{\pi_G + \pi_B}{\pi_G + \pi_B e^{\frac{a}{L}(W_G - W_B)}}$$

Dependence of $q$ on $\frac{a}{L}(W_G - W_B)$: channel through which ambiguity aversion impacts asset prices

The $M$ ambiguity averse agents do not hold the imbalance $W_G - W_B$, which is left to the $L$ SEU agents to clear:

$$\frac{a}{L}(W_G - W_B) \uparrow \Rightarrow q \downarrow \text{ and } (1 - \pi_R - q) \uparrow \Rightarrow p_G \downarrow \text{ and } p_B \uparrow$$
Matching theoretical and BGGZ’s experimental findings

BGGZ (2010): experimental sessions in which a competitive financial market is embedded in the standard Ellsberg framework.

Their findings:

▶ individuals are divided between SEU and ambiguity averse

▶ experimental evidence that ambiguity aversion matters for equilibrium prices

Our theoretical equilibrium prices fully explain and theoretically justify their experimental findings

BGGZ interpret their experimental findings using market with SEU and ambiguity averse $\alpha$-MEU investors

BGGZ do not derive the equilibrium asset prices, only provide conjectures
Empirical distribution functions of state-price/state-probability ratios from BGGZ (2010) when \(W_G = 272\), \(W_B = 162\), and \(W_R = 81\). 
\(\frac{p_R}{\pi_R}; \frac{p_B}{\pi_B}; \frac{p_G}{\pi_G}\) (Figure 8, right panel, in BGGZ (2010))

Two experimental rankings: \(\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G}\) and \(\frac{p_R}{\pi_R} > \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}\)

These are exactly the theoretical rankings 1. and 2. in Proposition A
BGGZ (2010) only expect ranking 1. $p_B/\pi_B > p_R/\pi_R > p_G/\pi_G$

Proposition A shows that Ranking 2. prevails when $W_B - W_R$ is large enough to imply an SEU optimal portfolios $y_G > y_B > y_R$.

Proposition A suggests a possible explanation of why in the experiment prices do not settle in favor of Ranking 1. or Ranking 2.

$W_R$, $W_G$, and $W_B$ in the experiment are close to the point at which the change from Ranking 1. or Ranking 2 takes place.

Proposition A predicts: to observe a clean separation of the two rankings, the aggregate wealth $W_B$ should be chosen closer to $W_R$ or $W_G$, respectively.
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Extended Ellsberg framework \((m \geq 0, \ l \geq 3)\)

\(m = 1\) risky state, \(R\), with known probability \(\pi_R \in (0, 1)\)

\(l \geq 3\) ambiguous states; \(A := S \setminus \{R\}\) set of ambiguous states

Equivalence result does not hold anymore: \(\alpha\)-MEU preference do not reduce to maxmin/SEU/maxmax \(\Rightarrow\) we study the \(\alpha\)-MEU model

To study the \(\alpha\)-MEU as function of \(\alpha\) we have to fix the set of priors of the \(\alpha\)-MEU model \(\Rightarrow\) we choose

\[
C_{\text{max}} = \{\text{all priors consistent with the uncertainty of the setting}\} = \{\text{all priors } \pi : \pi(R \text{ realizes}) = \pi_R\}
\]

because this choice:

\(\alpha\) is a measure the agent’s degree of aversion toward ambiguity

makes meaningful the comparison between \(\alpha\)-\(C_{\text{max}}\)-MEU and maxmin preferences by providing a utility specification common to the two classes of models
\( \alpha - C_{\text{max}} \)-MEU model

With set of priors \( C_{\text{max}} \), the \( \alpha \)-MEU utility reduces to

\[ U(w) = \pi_R u(w_R) + (1 - \pi_R) \left[ \alpha u(w_{\min}^A) + (1 - \alpha) u(w_{\max}^A) \right] \]

\( w_{\min}^A := \min_{\sigma \in A} w_\sigma \) smallest wealth allocated among the \( l \) ambiguous states in the portfolio \( w \in \mathbb{R}^{m+l} \)

\( w_{\max}^A := \max_{\sigma \in A} w_\sigma \) largest wealth allocated among the \( l \) ambiguous states in the portfolio \( w \in \mathbb{R}^{m+l} \)

**Remark:** \( \alpha - C_{\text{max}} \)-MEU is concave if and only if \( \alpha = 1 \)
Main findings: Extended Ellsberg framework

- Portfolio choice of the $C_{\text{max}}$-MEU as function of $\alpha$
  - pin down the different attitude (seeking and averse) towards ambiguity expressed by the $C_{\text{max}}$-MEU
  - disentangle ambiguity seeking from ambiguity averse agents, and among the last ones, $\alpha$-$C_{\text{max}}$-MEU from maxmin agents

- ambiguity seeking attitudes may prevent the existence of the market equilibrium
\( \alpha C_{\text{max}} \)-MEU portfolios choice

\[ p \in \mathbb{R}^{1+} \] state price vector

\[ p^{A}_{\text{min}} := \min_{\eta \in A} p_{\eta} \] lowest price among the ambiguous states prices

Characteristic of the \( \alpha C_{\text{max}} \)-MEU portfolios choice:

\[ \rightarrow \text{only two types of optimal portfolios, portfolio inertia at both portfolios:} \]

1. if \( \alpha \geq 1 - \frac{p^{A}_{\text{min}}}{1 - p} \Rightarrow \text{optimal portfolio unambiguous and unique} : \)

\[ w = (w_{R}, w, \ldots, w, \ldots, w) \text{ for some } w \in \mathbb{R} \]

2. if \( \alpha < 1 - \frac{p^{A}_{\text{min}}}{1 - p} \Rightarrow \text{optimal portfolio ambiguous and} \)

\[ w = (w_{R}, \overline{w}, \ldots, \overline{w}, \ldots, w) \text{ for some } \overline{w} > w \]

\[ w = \text{unambiguous} + \text{bet of } (\overline{w} - w) \text{ on a cheapest ambiguous states} \]

ambiguous optimal portfolio might be not unique: number equals number of ambiguous states with cheapest price
Exposure to ambiguity of the ambiguous portfolio

- Exposure to ambiguity of the ambiguous portfolio
  
  \[ w = (w_R, \overline{w}, \ldots, \overline{w}, \ldots, w) \text{, is equal to } \overline{w} - w > 0 \]

  \[ \rightarrow \text{ the larger } \alpha \]
  
  - the smaller \( \overline{w} - w \)
  
  - the smaller demand for the ambiguous portfolio
    
    \( \alpha = 1 \Rightarrow \text{the optimal portfolio is unambiguous} \)

  \[ \rightarrow \text{ the more } u \text{ is concave, the smaller } \overline{w} - w \]

- Allocation between risky and the ambiguous states:
  
  the larger \( \alpha \) the smaller \( \overline{w} - w_R \)

  When \( \alpha \uparrow \frac{l-1}{l} \), optimal portfolio tends to the unambiguous and optimal allocation is the same of an SEU with prior \( \tilde{\pi} = (\pi_R, \frac{1-\pi_R}{l}, \ldots, \frac{1-\pi_R}{l}) \)
Impact of ambiguity aversion $\alpha$ on portfolio choice, $l = 4$

$S = \{R\} \cup A$, $A = \{G, B, Y, Z\}$.

$w = (w_R, w_G, w_B, w_Y, w_Z) = (w_R, \bar{w}, w, w, w)$

State dependent wealth impacted by the degree of ambiguity aversion $\alpha$
Impact of risk aversion on portfolio choice, $l = 4$

$$ S = \{ R \} \cup A, \ A = \{ G, B, Y, Z \}. $$

$$ w = ( w_R, w_G, w_B, w_Y, w_Z ) = ( w_R, w, w, w, w ) $$

State dependent wealth impacted by the *degree of risk aversion* $\delta$, CARA
\(\alpha-\mathcal{C}_{\text{max}}\)-MEU and maxmin attitude towards ambiguity

\(I \geq 3\)

\begin{itemize}
  \item {\textit{ambiguity seeking}:} \(\alpha-\mathcal{C}_{\text{max}}\)-MEU agents with \(\alpha \in (0, \frac{l-1}{l})\)
  \item {not} {ambiguity seeking}
    \begin{itemize}
      \item any \(\alpha-\mathcal{C}_{\text{max}}\)-MEU agents with \(\alpha \in \left[\frac{l-1}{l}, 1\right)\)
      \item any maxmin agent with a set of priors including \(\tilde{\pi} := \text{prior assigning equal probability to all ambiguous states}\)
    \end{itemize}
\end{itemize}

Remark: \(\alpha-\mathcal{C}_{\text{max}}\)-MEU agent with \(\alpha = \frac{l-1}{l}\) is not equivalent – not even observationally – to an ambiguity neutral SEU agent
Lack of equilibrium with ambiguity seeking investors

\[ S = \{ R, G, B, Y \}, \ A = \{ G, B, Y \} \ (m = 1, \ l = 3) \]

- SEU agent with prior \( \tilde{\pi} \)
- maxmin agent with set of priors \( \mathcal{C} \), \( \tilde{\pi} \in \mathcal{C} \)
- \( \alpha\)-\( \mathcal{C}_{\text{max}}\)-MEU agent with \( \alpha \in (0, 1) \)

Market with 2 agents. Total endowment \( W \in \mathbb{R}^{1+3} \), \( W_G = W_B = W_Y \)

1. Market agents: SEU + ambiguity averse (e.g. \( \alpha\)-\( \mathcal{C}_{\text{max}}\)-MEU \( \alpha \in [\frac{l-1}{l}, 1] \) or maxmin) \( \Rightarrow \) equilibrium exists:
   - \( p \in \mathbb{R}^{1+3} \) with \( p_G = p_B = p_Y = \frac{1-p_R}{3} \)
   - SEU optimal portfolio \( y \in \mathbb{R}^{1+3} \): \( y_G = y_B = y_Y \)
   - ambiguity averse optimal portfolio \( w \in \mathbb{R}^{1+3} \): \( w_G = w_B = w_Y \)
Lack of equilibrium with ambiguity seeking investors

2. Market agents: SEU + ambiguity seeking $\alpha$-C$_{\text{max}}$-MEU agent ($\alpha \in (0, \frac{l-1}{l})$) \(\Rightarrow\) equilibrium does not exists:

\[\cdot\text{ambiguity seeking agent:}\]
always chooses portfolio exposed to ambiguity with a strictly larger wealth on one cheapest ambiguous states:

\[w \in \mathbb{R}^{1+3}: w_\sigma = \overline{w} > w, w_\eta = w \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\}\]

where $\sigma$ cheapest ambiguous state in equilibrium, i.e.

\[p_\sigma \leq p_\eta, \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\} \quad \Rightarrow \quad (5.1)\]

\[\cdot\text{SEU agent: to clear the market has to hold a portfolio } y \in \mathbb{R}^{1+3}:\]
\[y_\sigma < y_\eta = y_\nu \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\}\]

SEU agent chooses this portfolio only if equilibrium prices satisfy

\[p_\sigma > p_\eta = p_\nu, \text{ for any } \eta \in A = \{G, B, Y\} \setminus \{\sigma\} \quad (5.2)\]

which is incompatible with (5.1)
Conclusion

\(\alpha\)-MEU model increasing popular generalization of the maxmin model

▶ Standard Ellsberg framework (two ambiguous states):
  ▶ we show that *ambiguity averse \(\alpha\)-MEU and maxmin preferences are equivalent*
  ▶ we derive theoretical implications of ambiguity aversion on equilibrium asset prices. These are strikingly consistent with experimental findings in BGGZ (2010) \(\Rightarrow\) *ambiguity aversion matters does not wash out in equilibrium*

▶ Extended Ellsberg framework (at least three ambiguous states):
  ▶ equivalence result does not hold anymore
  ▶ we characterize optimal portfolio of \(\alpha-C_{\text{max}}\)-MEU agent. Several differences with the maxmin optimal portfolio (only two types of portfolios, portfolio inertia, demand not unique, \ldots)
  ▶ pin down ambiguity attitude of \(\alpha-C_{\text{max}}\)-MEU agent
  ▶ show that ambiguity seeking behaviour may prevent existence of equilibrium