A threshold model for fire sales and price-mediated contagion.

7th AMaMeF and Swissquote Conference - EPFL Lausanne

Rama Cont and Eric Schaanning*

Imperial College London

9th September 2015

*supported by the Fonds National de la Recherche Luxembourg.
Overview

1. Introduction: Price-mediated contagion and endogenous risk
2. A threshold model for fire sales
3. Comparison of threshold and leverage targeting models
4. Assessing the effectiveness of policy measures
5. Indirect exposures
6. Conclusion
Fire sales and price mediated contagion

- Balance-sheet contagion, either through counterparty risk or funding relations, cannot explain the magnitude and breadth of contagion, across sectors, countries and asset classes, observed in the crisis of 2007-2008.
- Market stress can lead institutional investors to unwind positions, in order to satisfy constraints on capital requirements, leverage, liquidity ratios or investor redemptions (Shleifer 2010, Coval & Stafford 2007, Ellul et al 2011).
- Recent studies (Greenwood et al 2013, Eisenbach et al 2014) point to fire sales as a key contagion channel during the crisis.
- ‘Fire sales’ in a stress scenario, as opposed to portfolio rebalancing in normal times, are not orderly and may entail sizeable market impact, which in turn can move prices and generate more deleveraging by the same or other institutions: destabilizing feedback loop.
- These phenomena increase in magnitude during a crisis or following a sudden drop in asset values.
This leads to **price-mediated contagion** which, unlike balance sheet contagion, affects all institutions holding the asset subject to a fire sale.

Unlike direct contagion mechanisms (through counterparty risk or funding channels), price-mediated contagion due to fire sales defies 'institutional ring-fencing' barriers.

Price-mediated contagion due to fire sales leads to an externality which cannot be quantified at the level of a single portfolio: need for global risk indicators and macroprudential tools.

Regulatory constraints on portfolios -leverage ratios, capital ratios, liquidity ratios- play a role in the triggering of fire sales.
Introduction: Price-mediated contagion and endogenous risk

Feedback and contagion from fire sales: summary

1. Shocks to illiquid assets in portfolios are transmitted via fire sales to liquid assets. **Amplification due to leverage.**

2. This generates losses in other portfolios with overlapping asset positions and may result in another round of deleveraging. **Amplification due to overlaps across portfolios**

Shock to asset values

\[ \uparrow \]

Market impact

\[ \uparrow \]

Deleveraging of overlapping portfolios

Portfolio constraints

\[ \Rightarrow \]

Deleveraging

\[ \downarrow \]

Market impact

\[ \downarrow \]

Price-mediated contagion

\[ \Leftarrow \]

Correlated losses in liquid assets
Objectives

- How can one quantify system-wide exposure to fire sales?
- How does the magnitude of price-mediated contagion depend on characteristics of institutional portfolios (holdings, leverage, capital, ...)?
- What does this imply for the risk exposure of an individual institution?
- How can regulators mitigate this channel of contagion?
- How sensitive are such analyses and results to underlying modeling assumptions?

We propose a stylized, multiperiod threshold model of fire sales in a multi-asset, multi-portfolio setting which can give quantitative responses to these questions, and apply the model to portfolio holding data for European banks from EBA stress tests.
Key findings

- **Non-linearity**: due to the threshold nature of deleveraging, the response to an external shock is a non-linear, convex function of initial losses in asset value $\rightarrow$ multiplier effect. In particular, this is different from 'leverage targeting' which leads to a concave response.

- Fire sales in a multi-asset setting leads to **contagion of losses across asset classes**: even if initial shock hits only a single asset class, other asset classes can be subject to fire sales through deleveraging of diversified portfolios.

- Price-mediated contagion results in **indirect exposures**; an institution may have exposure to shocks on an asset class it does not actually hold!

- **Liquidity weighted overlaps** of institutional asset holdings quantify magnitude of spillover effects.
Relation to literature

- Shleifer & Vishny (1991), Brunnermeier & Pedersen (2005) study endogenous risk generated by fire sales on *price levels* in the case of a single portfolio and a single asset.
- Adrian & Shin (2012) study macro-level endogenous risk due to deleveraging of a representative portfolio in a single-asset framework.
- Greenwood et al. (2013), Duarte & Eisenbach (2014) study the magnitude of fire sale risk in a linear model of leverage targeting using bank holding data.
- Cont & Wagalath (2013): continuous-time model of endogenous risk and spillover effects from fire sales across multiple assets and multiple portfolios.
- Overlapping portfolios: Minca & Braverman (2014)
- Default cascades: Caccioli et al. (2013), Amini, Cont & Minca (2012)
- Agent-based model: Bookstaber et al. (2014)
1. Introduction: Price-mediated contagion and endogenous risk

2. A threshold model for fire sales

3. Comparison of threshold and leverage targeting models

4. Assessing the effectiveness of policy measures

5. Indirect exposures

6. Conclusion
A threshold model for fire sales

Step 0: Initial state

- $i = 1 \ldots N$, leveraged institutions;
- $\mu = 1 \ldots M$, classes of liquid assets: $\Pi_{i\mu}$ is the dollar value $i$ holds in the class $\mu$;
- $K$ classes of illiquid assets: $\Theta_{i\mu}$.
- Liquid assets’ value is $L_i = \sum_{\mu=1}^{M} \Pi_{i\mu}$;
- Illiquid assets’ value is $I_i = \sum_{\mu=1}^{K} \Theta_{i\mu}$;
- Core Tier 1 capital $C_i$;
- In matrix form, the initial leverage is
  \[ \Lambda = (L + I)C^{-1} \]
- Leverage constraint: $\Lambda_i \leq \Lambda_{\text{max}}$
A threshold model for fire sales

Step 1: External shock

- Some illiquid assets undergo a loss of $\epsilon_\mu \%$
- Loss for institution $i$: $\Xi_i = \sum_{\mu=1}^{K} \Theta_{i\mu} \epsilon_\mu$.
- Impulse response analysis of the system to an arbitrary external shock;
- Scenario can be a regulatory stress test with joint shocks to several asset classes;
- The resulting leverage after the shock is

$$\Lambda_1 = (L_0 + (I_0 - \Xi_1))(C_0 - \Xi_1)^{-1}$$

$$= (L_1 + I_1)C_1^{-1}$$

- Deleveraging is \textit{asymmetric} wrt loss vs gain $\neq$ 'leverage targeting'.
The critical loss triggering a fire sale is

\[ \Xi^*_1 = (\Lambda_{\text{max}} - 1_N)^{-1} C_0 (\Lambda_{\text{max}} - \Lambda_0). \]  

(1)

It measures an individual bank’s (direct) vulnerability to a given stress scenario. Banks that exceed the maximum leverage sell a portion of their liquid assets and repay debt to delever. However, the mechanism differs from “leverage targeting” models in three fundamental ways:

- Threshold based: nothing happens if the initial shock is “too small” \((\Lambda \leq \Lambda_{\text{max}})\);
- Asymmetry: no “fire-buy”;
- Rebuilding a buffer: target \(\Lambda_b < \Lambda_{\text{max}}\).
Step 2: Deleveraging

When a bank exceeds the maximum leverage $\Lambda_{\text{max}}$, it solves for the deleveraging proportion $\Gamma^i \in [0, 1]$

$$\frac{(1 - \Gamma^{i}_{t+1})L^i_t + I^i_t}{C^i_t} = \Lambda_b,$$

which yields in the **Threshold model:**

$$\Gamma^{i}_{t+1} = \left( (\Lambda^i_t - \Lambda_b) \frac{C^i_t}{L^i_t} \mathbb{1}_{\Lambda^i_t > \Lambda_{\text{max}}} \right) \wedge 1,$$

and in the **Leverage targeting model:**

$$\Gamma^{i}_{t+1} = (\Lambda^i_0 - 1) (L^i_t)^{-1} \Xi^i_t$$
Step 2: Deleveraging

This results in an aggregate supply in asset $\mu$:

$$Q_{t+1}^\mu(\Xi) = \sum_{i=1}^{N} \Gamma_{t+1}^i \Pi_t^i \mu = \sum_{i=1}^{N} \Pi_t^i \left( (\Lambda_t^i - \Lambda_b^i) \frac{C_t^i}{L_t^i} 1_{\Lambda_t > \Lambda_{\text{max}}} \right).$$

The aggregate response function is non-linear and

- Threshold based: no fire sales for small shocks $\leq \min(\epsilon_j^*)$.
- Convex: higher multiplier for larger shocks;
- Asymmetric: No “fire buy”;

The threshold model yields results that are quite different from the linear response function implied by 'leverage targeting' models (Greenwood et al 2013, Eisenbach et al 2014, Danielsson et al). Linear models of fire sales overestimate the response to shocks, in particular small ones.
Step 2: Deleveraging

**Figure:** Volume of fire sales in UK bonds for a stress scenario to Spanish commercial and residential mortgages.
Step 3: Mark to market losses and feedback effects

- The price impact is modeled in a linear and multiplicative way

\[
\Pi^{i\mu}_{t+1} = \Pi^{i\mu}_t \left( 1 - \delta^{-1}_\mu \sum_{j=1}^{N} \Gamma^j_{t+1} \Pi^{j\mu}_1 \right),
\]

where \( \delta_\mu \) is the market depth (price elasticity) in monetary units of asset \( \mu \);

- In matrix notation, the portfolio transition can be written as

\[
\Pi_{t+1} = (1_N - \Gamma_{t+1}) \Pi_t \left[ 1_M - D \text{diag} \left( \Pi^\top_t \Gamma_{t+1} 1 \right) \right],
\]

where the diag operator takes a vector to a diagonal matrix, and where \( D \) is the diagonal matrix with entries \( \delta^{-1}_\mu \).
Step 3: Mark to market losses and feedback effects

Each round, the price impact gives rise to a **mark to market loss** on the remaining assets

\[
M^i_{t+1} = (1 - \Gamma^i_{t+1}) \sum_{j=1}^N \sum_{\mu=1}^M \Pi_{i\mu}(t) \delta_{\mu}^{-1} \Pi_{j\mu}(t) \Gamma^j_{t+1}.
\]

and a **realized loss** on the liquidated assets

\[
R^i_{t+1} = \frac{1}{2} \Gamma^i_{t+1} \sum_{j=1}^N \sum_{\mu=1}^M \Pi_{i\mu}(t) \delta_{\mu}^{-1} \Pi_{j\mu}(t) \Gamma^j_{t+1}.
\]

These two components sum to the **total loss**:

\[
\Xi^i_{t+1} = \left( 1 - \frac{1}{2} \Gamma^i_{t+1} \right) \sum_{j=1}^N \sum_{\mu=1}^M \Pi_{i\mu}(t) \delta_{\mu}^{-1} \Pi_{j\mu}(t) \Gamma^j_{t+1},
\]

which is proportional to the **liquidity-weighted overlap**

\[
\omega_{ij} = \sum_{\mu=1}^M \Pi_{i\mu} \Pi_{j\mu} \delta_{\mu}^{-1}. \text{ (Cont & Wagalath (2013))}.
\]
These dynamics lead to a time inhomogeneous matrix recurrence relation for the portfolio $\Pi$:

$$
\Pi_{t+1} = (\mathbb{1}_N - \frac{1}{2} \Gamma_{t+1}) \Pi_t (\mathbb{1}_M - D \Pi_t \Gamma_{t+1})
$$

The losses yield an inhomogeneous quadratic first order matrix recurrence relation:

$$
\Xi_{t+1} = \text{diag} \left( (\mathbb{1}_N - \frac{1}{2} \Gamma_{t+1}) \Pi_t D \Pi_t^T \Gamma_{t+1} \mathbb{1} \right)
$$

Capital evolution:

$$
C_{t+1} = (C_t - \Xi_{t+1})^+, \quad t \geq 0
$$
Introduction: Price-mediated contagion and endogenous risk

A threshold model for fire sales

Comparison of threshold and leverage targeting models

Assessing the effectiveness of policy measures

Indirect exposures

Conclusion
### Table: Model balance sheet built from the EBA data.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Illiquid asset classes (by region)</strong></td>
<td>Capital</td>
</tr>
<tr>
<td>Residential mortgage exposures</td>
<td>Tier 1 capital</td>
</tr>
<tr>
<td>Commercial real estate exposures</td>
<td></td>
</tr>
<tr>
<td><strong>Other illiquid assets ($\epsilon_\mu \equiv 0$)</strong></td>
<td>Debt</td>
</tr>
<tr>
<td>Defaulted exposures</td>
<td></td>
</tr>
<tr>
<td>Residential mortgage exposures</td>
<td></td>
</tr>
<tr>
<td>Commercial real estate exposures</td>
<td></td>
</tr>
<tr>
<td>Remaining sovereign exposures</td>
<td></td>
</tr>
<tr>
<td><strong>Liquid asset classes (by region)</strong></td>
<td></td>
</tr>
<tr>
<td>Institutional client exposures</td>
<td></td>
</tr>
<tr>
<td>Corporate exposures</td>
<td></td>
</tr>
<tr>
<td>Retail: Revolving exposures</td>
<td></td>
</tr>
<tr>
<td>Retail: SME exposures</td>
<td></td>
</tr>
<tr>
<td>Retail: other exposures</td>
<td></td>
</tr>
<tr>
<td>Sovereign exposures</td>
<td></td>
</tr>
<tr>
<td>Direct sovereign exposures in derivatives</td>
<td></td>
</tr>
<tr>
<td>Indirect sovereign exp. in the trading book</td>
<td></td>
</tr>
</tbody>
</table>
Comparison of threshold and leverage targeting models

Leverage of European Banks

Figure: Reported leverage and model leverage of the 90 European banks in our sample.
A stress scenario is defined by a vector $\epsilon \in \mathbb{R}^K$ whose components $\epsilon_\mu$ are the percentage shocks to asset class $\mu$.

Gradual increase of $\epsilon$ from 0% to 20%.

Four scenarios:

1. Spanish residential and commercial real estate losses (both illiquid categories for the region ES).
2. Northern Europe housing decline (“Residential real estate exposures” for the regions GB, BE, NO, SE);
3. Southern Europe commercial real estate losses (“Commercial real estate exposures” for the regions IT, GR, ES, PT);
4. Eastern Europe commercial real estate losses (“Commercial real estate exposures” for the regions CZ, EE, HU, LV, LT, PL, RO, SK, RU, BG, E3);
The systemwide initial loss is given by $1^T \Xi_1 1$ and the systemwide fire sales loss is given by

$$L_{FS} = \sum_{t=1}^{T} 1^T \Xi_t 1.$$ 

**Proposition**

In the leverage targeting model, the fire sales loss is a concave function of the initial shock size.

Proof via direct verification of

$$\frac{d^2}{d\epsilon^2} \sum_{t=2}^{\infty} \Xi_t < 0 \quad (2)$$

**Proposition**

In the threshold model, the systemwide loss is an initially convex function of the initial shock size.
Comparison of threshold and leverage targeting models

Relative magnitude of fire sales losses

![Graph showing the comparison between threshold and leverage targeting models for fire sales losses.](image)

**Figure:** The magnitude of the fire sales losses in both models.
Comparison of threshold and leverage targeting models

(a) Scenario 2

(b) Scenario 3

(c) Scenario 4

Figure: The magnitude of the fire sales losses in both models.
**Proposition**

*In the leverage targeting model, the proportion of fire sales to the systemwide loss is a decreasing function of the initial shock size.*

Proof by direct verification that

$$\frac{d}{d\epsilon} \left( \frac{\sum_{t=2}^{\infty} \Xi_t}{\Xi_1 + \sum_{t=2}^{\infty} \Xi_t} \right) < 0,$$

(3)

using the fact that \(\sum_{t=2}^{\infty} \Xi_t\) is a concave function, and \(\frac{\partial \Xi_1}{\partial \Xi_k} = 1\) for all \(k\).
Comparison of threshold and leverage targeting models

Figure: Contribution of fire sales losses to total loss.
Sensitivity to initial stress scenario and internal model parameters:

- The leverage targeting model produces fire sales losses that are relatively insensitive to the initial stress scenario.
- We compare the scatter plots of individual bank fire sales losses in different scenarios.
- The loss is furthermore sensitive to the number of iterations used in simulating the fire sales cascade.
- In the threshold model, the number of iterations follows endogenously once the model has been calibrated to data.
Comparison of threshold and leverage targeting models

(a) Scenarios 1 and 2.

(b) Scenarios 2 and 3.

Figure: Individual bank fire sales losses in different scenarios.
We want to compare the similarity of the output as a function of the initial shock size. In order to compare the similarity of the resulting fire sales loss vectors in different scenarios \(i, j\), we define the quantity

\[
Y_{i,j}^{model} = \frac{\langle \Xi^{model}_{T,i}, \Xi^{model}_{T,j} \rangle}{\|\Xi^{model}_{T,i}\|_2 \|\Xi^{model}_{T,j}\|_2}.
\] (4)

By Cauchy-Schwarz, \(0 \leq Y_{i,j} \leq 1\) for all \(i, j\). Figure 7 compares the \(Y_{i,j}\) ratios between the fire sales loss vectors that result from different initial stress scenarios in both models.
Comparison of threshold and leverage targeting models

The (ir)relevance of different initial stress scenarios

Figure: In comparison to the threshold model and across all shock sizes, the leverage targeting model produces fire sales losses that are very similar for different stress scenarios.
Price-mediated contagion has an amplifying effect on the initial loss. This can be quantified in terms of loss multipliers.

**Individual loss multiplier:**

\[
LM_i(\epsilon) = 1 + \sum_{t=2}^{T(\epsilon)} \Xi_t^{i} \Xi_1^{i}.
\]

**System-wide loss multiplier:**

\[
LM_{system}(\epsilon) = 1 + \frac{\sum_{t=2}^{T(\epsilon)} \sum_{i=1}^{N} \Xi_t^{i} \Xi_1^{i}}{\sum_{i=1}^{N} \Xi_1^{i}}.
\]
Comparison of threshold and leverage targeting models

![Graph: System loss multiplier as a function of the initial shock](image)

**Figure:** The estimated system loss multipliers of the threshold model in 4 stress scenarios—Losses can increase by up to 40%.
Comparison of threshold and leverage targeting models

(a) Threshold model.

(b) Leverage targeting model.

Figure: Comparison of the system loss multipliers in both models. The leverage targeting model suggests that the fire sales multiplier effect is largest for the smallest shocks.
Comparison of threshold and leverage targeting models

1. Introduction: Price-mediated contagion and endogenous risk

2. A threshold model for fire sales

3. Comparison of threshold and leverage targeting models

4. Assessing the effectiveness of policy measures

5. Indirect exposures

6. Conclusion
Which policy reduces losses more effectively?

- **Uniform** increase in capital ratio for all banks = uniform increase in risk weights for all asset classes
- **Targeted risk weights**: Increase the risk weight of an asset class $\mu$ proportionally to its contribution to system-wide losses across a fixed set of stress scenarios (‘calibration set’).

The assessment of the targeted risk weights policy differs significantly in both models:

- The leverage targeting model suggests that the targeted risk weights always outperform the uniform policy.
- The threshold model yields a more nuanced picture: calibration of the risk weights is more subtle and a simplistic application of the policy does not always outperform the benchmark.
Assessing the effectiveness of policy measures

Figure: Stress scenario 1.

(a) Threshold model.  
(b) Leverage targeting model.
Assessing the effectiveness of policy measures

**Figure**: Stress scenario 3.

(a) Threshold model.

(b) Leverage targeting model.
Assessing the effectiveness of policy measures

1. Introduction: Price-mediated contagion and endogenous risk
2. A threshold model for fire sales
3. Comparison of threshold and leverage targeting models
4. Assessing the effectiveness of policy measures
5. Indirect exposures
6. Conclusion
The **apparent** exposure to asset class $\mu$ is equal to the balance sheet exposure $\Theta_{i\mu}$.

Absent contagion $\Rightarrow$ a shock $\epsilon$ gives rise to a loss $\epsilon\Theta_{i\mu}$.

But when accounting for losses through price-mediated contagion, an $\epsilon$ shock can give rise to a larger loss!

The **effective** exposure accounts for contagious losses, and is defined via

$$\tilde{\Theta}_{i\mu}^s(\epsilon) = \frac{\Delta \text{loss}}{\Delta \text{shock}} = \frac{\epsilon\Theta_{i\mu} + \sum_{t=2}^{T(\epsilon)} \Xi_t(\epsilon)}{\epsilon}.$$ 

The **indirect** exposure is the difference between the two:

$$\hat{\Theta}_i^s = \tilde{\Theta}_i^s - \Theta_{i\mu}.$$ 

So, through price mediated contagion, an institution may be **indirectly** exposed to an asset class it does not actually hold.
Figure: Loss with and without contagion for two banks of the EBA dataset as a function of the initial shock.
Table: The apparent, average indirect and average effective exposures of some selected European banks to Spanish commercial and residential mortgages in stress scenario 1.
Figure: Indirect exposures computed in both models as function of the initial shock size and added capital: The threshold model yields a more plausible functional form. The leverage targeting model’s estimated exposures are very sensitive to internal model parameters such as the number of iterations of the cascade.
In presence of heterogeneous portfolios, fire sales generate a non-linear feedback loop whose marginal response to an external shock increases with shock size.

Price-mediated contagion results in indirect exposure of an institution to an asset class $\neq$ apparent notional exposure.

Liquidity weighted overlaps of institutional asset holdings quantify magnitude of spillover effects.

Even a localized shock on a single asset class can lead to widespread losses across many asset classes and institutions.
Conclusions for modelling

- The full dynamics of fire sales contagion cannot be captured by one step, one asset or one investor models.
- Leverage targeting and threshold models differ significantly in their output.
- Leverage targeting models display a reduced sensitivity to the initial stress scenario, while at the same time exhibiting an increased sensitivity to internal model parameters.
- Leverage targeting models of fire sales tend to overestimate the effect of fire sales.
- Policy assessments are sensitive to what model is being used to evaluate its effectiveness.
Conclusions for modelling II

- Traditional models of default cascades seem not appropriate: Institutions do not fail and enter an absorbing “defaulted” state. They fight for survival, and it is precisely in this state where contagion to other portfolios occurs.

- The deleveraging state can be exited and re-entered multiple times: One-step models of fire sales cannot capture these dynamics. Neither do they capture the potential of losses originating in one specific segment to spread to remote market segments.

- Analytical models seem to be inappropriate to investigate these phenomena. Models like Moallemi et al. resort to assuming extreme case portfolios: mutual fund networks where everybody holds the market portfolio, or an isolated portfolio network with zero overlap.

- Need for new quantitative tools to investigate these phenomena in a realistic and data-driven manner?


Conclusion


When everyone runs for the exit.

Risk and Liquidity.
Oxford University Press.

Fire sales in finance and macroeconomics.
Journal of Economic Perspectives.

Liquidation values and debt capacity: A market equilibrium approach.
Thank you for your attention!