Short-Term Debt and Incentives for Risk-Taking*

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Abstract

We challenge the view that short-term debt curbs moral hazard and analytically show that, in a world with financing frictions and fair debt pricing, short-term debt does not decrease but instead increases incentives for risk-taking. To do so, we develop a model in which firms are financed with equity and short-term debt and cannot freely optimize their default decision because of financing frictions. Using this model, we show that short-term debt can give rise to a “rollover trap,” a scenario in which firms burn revenues and cash reserves to absorb severe rollover losses. In this rollover trap, shareholders find it optimal to increase cash flow risk in an attempt to avoid inefficient closure. These risk-taking incentives do not arise when debt maturity is sufficiently long.

Keywords: Short-term debt financing; rollover risk; risk-taking.

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1 Introduction

A central result in corporate finance is that equity holders in levered firms have incentives to increase asset risk, as they benefit from successful outcomes of high-risk activities while the losses from unsuccessful outcomes are borne by debtholders (see Jensen and Meckling (1976)).\(^1\) As argued in much of the corporate finance literature, this “potential agency cost can be substantially reduced or eliminated by using shorter-term debt” (Leland and Toft (1996)).\(^2\) Similarly, following Calomiris and Kahn (1991), much of the banking literature argues that short-term debt disciplines management because the fragility induced by short-term debt prevents managerial moral hazard.

The view that short-term debt disciplines management and curbs moral hazard does not accord well, however, with the available evidence. In their survey of corporate managers, Graham and Harvey (2001) find little evidence that short-term debt reduces the chance that shareholders take on risky projects. Admati and Hellwig (2013), Admati, DeMarzo, Hellwig, and Pfleiderer (2013), and Eisenbach (2017) also question this theory by observing that the increasing reliance on short-term debt in the years before the financial crisis of 2007-2009 went hand in hand with exceedingly risky activities. Admati, DeMarzo, Hellwig, and Pfleiderer (2013) further note that “in addition to recent history, there are conceptual reasons to doubt the effectiveness of “debt renewal” as an optimal disciplining mechanism. Absent solvency or market failure, debt can always be renewed at a sufficient yield. In that case, the only potential disciplining effect can come from the information that is provided when the debt is repriced.”

In this paper, we develop a model that can rationalize this evidence using two important features of firms’ real world environments: Financial frictions and fair debt

\(^{1}\)See Eisdorfer (2008) and Favara, Morellec, Schroth, and Valta (2017) for empirical evidence on this “asset substitution” or “risk-shifting” problem.

\(^{2}\)This view was first expressed in Barnea, Haugen and Senbet (1980). Besides Leland and Toft (1996), contributions to this literature include Leland (1998), Cheng and Milbradt (2012), or Huberman and Repullo (2015). For example, Cheng and Milbradt write: “Although short-term debt can lead to freezes, it mitigates the risk-shifting problem by imposing a punishment in the form of liquidation.”
pricing. Notably, we show that, in a world with financing frictions and fair debt pricing, short-term debt does not decrease but, instead, increases incentives for risk-taking. To demonstrate this result and examine its implications for corporate policies, we formulate a dynamic model in which firms are financed with equity and short-term debt and cannot freely optimize their default decisions because of financing frictions. In this model, debt is repriced continuously to reflect changes in firm performance. Firms operate risky assets and have the option to invest in risk-free, liquid assets such as cash reserves. Firms maximize shareholder value by choosing their precautionary buffers of liquid assets as well as their payout, financing, risk management, and (constrained) default policies.

As in Leland and Toft (1996), Leland (1998), He and Xiong (2012a), and much of the literature on short-term debt and rollover risk, we consider that when a short-term bond matures, the firm issues a new bond with the same face value, coupon rate, and maturity at market price. When the market price of the new bond is lower than the principal of the maturing bond, the firm bears rollover losses. To avoid default and liquidation, shareholders need to absorb these losses. A fundamental difference between our work and prior contributions is that we do not assume that outside equity can be issued instantly and at no cost to absorb rollover losses. Rather, firms face financing frictions, which may lead to forced, inefficient liquidations. This in turn provides shareholders with incentives to build up liquidity buffers that can be used to absorb operating or rollover losses and reduce expected refinancing costs and the risk of inefficient closure.

A central result of the paper is to show that combining short-term debt with financing frictions and fair debt pricing provides incentives for shareholders to increase asset risk, thereby rationalizing the evidence discussed above. To understand this result, consider first the effects of financing frictions on shareholders’ risk taking incentives. As shown by previous models, shareholders in a solvent firm facing financing frictions behave in a risk-averse fashion to avoid inefficient closure (see e.g. Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011)). Similarly, Leland (1994a) and Toft and Prucyk (1997) show that equity value can become a concave function of asset value when firms cannot freely choose their default policy, either because debt
contracts include protective covenants—like net-worth covenants—or because firms face leverage requirements—like in banks. In these environments, shareholders cannot freely optimize the timing of default and, if the firm is liquidity constrained but fundamentally solvent, default is suboptimal to shareholders. In such instances, the equity value function becomes concave and shareholders are effectively risk-averse.

In models with financing frictions, as well as in Leland (1994a) and Toft and Prucyk (1997), debt is either absent or has infinite maturity. The main contribution of our paper is to show that allowing for short-term debt financing yields radically different implications. Notably, when a firm experiences negative operating shocks, default risk increases, leading to a drop in the price of newly issued debt and to an increase in rollover losses. Rollover losses therefore compound operating losses, draining the firm’s liquid reserves and increasing default risk. Because firms issuing debt with shorter maturity need to roll over a larger fraction of their debt, this amplification mechanism is more important when debt maturity is shorter.

Through this mechanism, we show that when firms are close to distress and debt maturity is short enough, rollover losses can become larger than expected net income. We call this scenario, in which the firm “burns” cash reserves and expected cash flows to shareholders are negative because of severe rollover losses, the rollover trap. In the rollover trap, the concavity generated by the threat of forced liquidation (i.e., shareholders’ effective risk aversion) is more than offset by the convexity generated by transitory losses. That is, shareholders have incentives to increase asset volatility in an attempt to improve firm performance and interim debt repricing and reduce the risk of inefficient liquidation. As shown in the paper, this is also true in Leland-type models in which default decisions are constrained by debt covenants or leverage requirements. In other

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3 This is also the case in the Black and Scholes (1973) model, in which maximum leverage ratio or minimum interest coverage ratio requirements imply that equity is akin to a down-and-out call option on the firm’s assets (see e.g. Black and Cox (1976)). In this case, shareholders do not have incentives to shift risk when firms fundamental worsen and asset value approaches the “knock-out” barrier corresponding to the protective covenant or regulatory requirement (see Derman and Kani (1996)).
words, our result that short-term debt increases risk-taking incentives when debt is fairly
priced is fundamentally driven by the presence of frictions and the ensuing inability of
shareholders to freely optimize their default decision.4

These risk-taking incentives in the presence of financing frictions decrease as debt
maturity increases and do not arise when debt maturity is sufficiently long or when firms
are all-equity financed. In such cases, debt needs to be rolled over less often (or never),
rollover losses are small (or absent), and expected net cash flows are always positive,
implying that the main effect of financing frictions is to expose shareholders to the risk
of an inefficient liquidation, so that shareholders do not want to increase risk.

An important question is whether shareholders’ risk-taking incentives constitute a
source of agency conflicts. We show in the paper that risk-taking strategies can be
value-enhancing for both shareholders and debtholders when in the rollover trap.5
Indeed, while making negative cash flow shocks potentially more harmful, increasing asset
volatility in distress—or “gambling for resurrection”—also makes it more likely that
positive cash flow shocks will allow the firm to escape the rollover trap. Nonetheless,
because shareholders capture all the returns above those required to service debt, and
therefore benefit disproportionately from risk-taking, a conflict between debtholders and
shareholders still exists. Notably, debtholders may only have incentives to increase asset
risk at the very brink of distress, when their promised cash flows are at stake. That is,
we find that while risk-taking may increase debt value when default is imminent, it also
leads to a disproportionate decrease (increase) in debt value (yields) outside of distress.

4In this paper, we follow prior models on financing frictions (e.g. Décamps et al. (2011) or Bolton et
al. (2011)) by assuming that the firm cash flows are governed by an arithmetic Brownian motion. This
differs from Leland-type models in which cash flows are governed by an geometric Brownian motion.
As shown in the paper, our result that short-term debt increases risk-taking incentives does not rest on
specific assumptions about the stochastic process governing the firm’s cash flows.
5Cheng and Milbradt (2012) also show that an increase in asset risk can increase debt value. In their
model, this arises because increasing asset risk enhance creditors’ confidence that future creditors will
not run. In our model, increasing asset risk has a positive impact on the price of newly-issued debt in
distress, which reduces rollover losses and effectively reduces credit spreads.
Thus, an agency problem exists in the region where equity value is convex and debt value is concave. We also find that firms financed with shorter-term debt, with lower profitability, and with more volatile cash flows are more likely to face such agency problems. Our predictions differ from Jensen and Meckling (1976) as debt maturity plays an key role in determining risk-taking incentives. In addition, risk-taking may be optimal for both shareholders and debtholders at the very brink of distress.

We show the robustness of our results to a number of extensions of our baseline setup. First, we consider the possibility for the firm to acquire additional debt via a credit line. We show that when credit lines are senior to market debt (as is typically the case), rollover losses are larger when the firm approaches distress, which strengthens the amplification mechanism described above and shareholders’ incentives for risk-taking. That is, when short-term debt is subordinated to other claims (as is the case in banks, where deposits are usually senior), shareholders have stronger risk-taking incentives. Second, we demonstrate that our results are not driven by the specific way financing frictions are modeled. In fact, our results hold in the extreme case in which the firm does not have access to the equity market (and, thus, financing frictions are the largest) as well as when assuming that the cost of raising equity is time-varying. Third, we show that our results are not driven by the specific assumption about the stochastic process governing firm cash flows, but rather by the shareholders’ inability to optimize their default decisions. To do so, we relax the assumption that shareholders have deep pockets in a setup à la Leland (1994b, 1998). In this setup, we confirm our result that short-term debt generates risk-taking incentives.

Our work is related to the recent papers that incorporate financing frictions into dynamic models of corporate financial decisions. These include Asvanunt, Broadie, and Sundaresan (2011), Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011, 2013), Hugonnier, Malamud and Morellec (2015), or Décamps, Gryglewicz, Morellec, and Villeneuve (2017). In this literature, it is generally assumed that firms are all-equity financed. Notable exceptions are Gryglewicz (2011), Bolton, Chen, and Wang (2015), and Hugonnier and Morellec (2017), in which firms and/or
financial institutions are financed with equity and long-term (infinite maturity) debt. In these models, firms have good fundamentals and financing frictions introduce the risk of forced liquidations, leading shareholders to behave as if they were risk-averse to preserve equity value. That is, convexity in equity value and risk-taking incentives do not arise in these models.\textsuperscript{6} Our paper advances this literature by characterizing the interaction between debt maturity and corporate policies and by showing that short-term debt and rollover losses can encourage risk taking when firms are close to financial distress.

Our paper also relates to the growing literature that examines the relation between short-term debt financing and credit risk using dynamic models with roll-over debt structure. Starting with Leland (1994b, 1998) and Leland and Toft (1996), these models show that short-term debt generally leads to an increase in default risk via rollover losses. Important contributions in this literature include Hilberink and Rogers (2002), Eom, Helwege, and Huang (2004), Ericsson and Renault (2006), Hackbarth, Miao, and Morellec (2006), He and Xiong (2012a), He and Milbradt (2014), Dangl and Zechner (2016), DeMarzo and He (2016), or Chen, Cui, He, and Milbradt (2018).

All of these models assume that shareholders have deep pockets and can inject liquidity in the firm at no cost (i.e. there are no financing frictions), or just do not allow firms to hoard precautionary cash reserves. In our model, firms face financing frictions and optimally retain part of their earnings to build up liquid reserves that they can use to absorb rollover losses. Consistent with this modeling, Harford, Klasa, and Maxwell (2014) document that refinancing risk due to short-term debt financing represents a key motivation for why non-financial firms hoard cash on their balance sheets.

Our paper is also related to the early studies of Diamond (1991) and Flannery (1986, 1994), in which short-term debt can be repriced given interim news. Debt repricing implies that the yield on corporate debt changes over time to reflect the firm’s operating performance. A central difference with these papers is that, in our dynamic model, there

\textsuperscript{6}Two notable exceptions are Hugonnier, Malamud and Morellec (2015) and Babenko and Tserlukievich (2017) in which lumpy investment can make the equity value function locally convex \textit{away} from distress. In these models, firms are all-equity financed.
are always creditors who are willing to buy debt at a sufficient yield and debt repricing
does not lead short-term debt to discipline shareholders.

Lastly, our paper also relates to the banking literature on the disciplining role of
short-term debt; see e.g. Calomiris and Kahn (1991), Diamond and Rajan (2001), or
Eisenbach (2017). In this literature, the fragility induced by short-term debt financing
prevents managerial moral hazard. The experience leading up to the financial crisis of
2007-2009 calls into question the effectiveness of short-term debt as a disciplining device.
Admati and Hellwig (2013) note, for example, that “in light of this experience, the claim
that reliance on short-term debt keeps bank managers “disciplined” sounds hollow,” as
the heavy reliance on short-term debt was accompanied by overly risky activities. Our
paper shows that short-term debt financing exacerbates incentives for risk-taking when
shareholders’ cannot freely optimize their default decision because of financing frictions,
regulatory constraints, or debt covenants.\footnote{In Eisenbach (2017), short-term debt if effective as a disciplining device only if firms face purely idiosyncratic shocks. Otherwise, good aggregate states lead firms to take excessive risks while bad aggregate states suffer costly fire-sales.}

The paper is organized as follows. Section 2 presents the model. Section 3 demonstra-
tes the effects of short-term debt on risk-taking and discusses the key implications
of the model. Section 4 shows the robustness of our results to alternative model specifi-
cations. Section 5 concludes. Technical developments are gathered in the Appendix.

2 Model and assumptions

Throughout the paper, time is continuous and all agents are risk neutral and discount
cash flows at a constant rate $r > 0$. The subject of study is a firm held by shareholders
that have limited liability. As in He and Xiong (2012a), one may interpret this firm as
any firm, either financial or non-financial. However, our model is perhaps more appealing
\footnote{Since most deposits are insured and depositors are not likely to run, Calomiris (1999) has suggested that banks should issue additional debt to fulfill the monitoring role that depositors fail to supply. Our paper shows that as long as debt is fairly priced, debt is unlikely to discipline management.}
for financial firms because of their heavy reliance on short-term debt financing.\footnote{A number of intermediaries, such as insurance companies, hedge funds, brokers and dealers, special purpose vehicles, and government-sponsored enterprises like Fannie Mae and Freddie Mac, do not take deposits directly from households, but in many ways behave like banks in debt markets (see Krishnamurthy (2010)).}

Specifically, we consider a firm that owns a portfolio (or operates a set) of risky, illiquid assets as well as cash reserves and is financed with equity and short-term debt. Risky assets generate after-tax cash flows given by \( dY_t \) and governed by the process:

\[
dY_t = (1 - \theta) (\mu dt + \sigma dZ_t),
\]

where \( \mu \) and \( \sigma \) are positive constants representing respectively the mean and the volatility of pre-tax cash flows from risky investments, \( (Z_t)_{t \geq 0} \) is a standard Brownian motion representing random shocks to cash flows, and \( \theta \in (0,1) \) is the corporate tax rate. Equation (1) implies that over any time interval \( (t, t + dt) \), the after-tax cash flows from risky assets are normally distributed with mean \( (1 - \theta)\mu dt \) and volatility \( (1 - \theta)\sigma \sqrt{dt} \). This in turn implies that the firm can make profits as well as losses. This cash flow specification is similar to that used for example in DeMarzo and Sannikov (2006), Bolton, Chen, and Wang (2011), DeMarzo, Fishman, He, and Wang (2012), or Hugonnier, Malamud, and Morellec (2015).

Because it pays corporate income tax and interest payments are tax deductible, the firm has an incentive to issue debt. To make our results comparable with prior contributions in the literature, we consider finite-maturity debt structures in a stationary environment as in Leland (1998), Hackbarth, Miao, and Morellec (2006), He and Xiong (2012b), or Cheng and Milbradt (2012). Notably, we assume that the firm has issued debt with constant principal \( S \) and paying a constant total coupon \( C < \mu \). At each moment in time, the firm rolls over a fraction \( m \) of its total debt. That is, the firm continuously retires outstanding debt principal at a rate \( mS \) and replaces it with new debt vintages of identical coupon, principal, and seniority. In the absence of default, average debt maturity equals \( M \equiv 1/m \).
Management acts in the best interest of shareholders and chooses not only the firm’s financing policy but also its payout and default policies. Notably, we allow management to retain earnings inside the firm and denote by $W_t$ the firm’s cash/liquid reserves at time $t \geq 0$. Liquid reserves earn a rate of interest $r - \lambda$ and can be used to cover operating and rollover losses if other sources of funds are costly or unavailable. The wedge $\lambda > 0$ represents a carry cost of liquidity.\footnote{The cost of holding cash includes the lower rate of return on these assets because of a liquidity premium and tax disadvantages (Graham (2000) finds that cash retentions are tax-disadvantaged because corporate tax rates generally exceed tax rates on interest income). This cost of carrying cash may also be related to a free cash flow problem within the firm, as in Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011), or Hugonnier, Malamud, and Morellec (2015).} When choosing its target level of cash reserves, the firm balances this carry cost with the benefits of liquidity.

The firm can increase its cash reserves either by retaining earnings or by raising funds in the capital markets. As in Bolton, Chen, and Wang (2013), the firm operates in an environment characterized by time-varying financing opportunities. Specifically, the firm can be in one of two observable states of the world, that we denote by $i = G, B$. In the good state $G$, the firm can raise funds at any time by incurring a fixed cost $\phi > 0$. In the bad state $B$, the firm has no access to outside funds or, equivalently, funding costs are too high. The state switches from $G$ to $B$ (resp. from $B$ to $G$) with probability $\pi_G dt$ (resp. $\pi_B dt$) on any time interval $(t, t + dt)$. As we show below, financing frictions provide incentives for the firm to retain earnings and build up liquid reserves.

We denote by $D_i(w; C, m, S)$ the market value of short-term debt in state $i = G, B$ for a level of cash reserves $w$. Debt rollover implies that short-term debt of a new vintage is issued at market price and has principal value and coupon payment given by $mS$ and $mC$, respectively. The market value of newly issued debt—which represents a firm inflow—may differ from the principal repayment $mS$ of maturing—which represents an outflow to the firm. When the market value of newly issued debt is lower than the principal, the firm bears rollover losses. Otherwise, it enjoys rollover gains. Over any time interval $(t, t + dt)$, the rollover gains or losses are given by $m(D_i(w; C, m, S) - S) dt$
and the dynamics of cash reserves satisfy

\[ dW_t = (1 - \theta)[(r - \lambda)W_t \, dt + (\mu - C)dt + \sigma dZ_t] \]

\[ + m \left( D_t(W_t; C, m, S) - S \right) dt - dP_t + dH_t - dX_t, \tag{2} \]

where \( P_t, H_t, \) and \( X_t \) are non-decreasing, adapted processes representing respectively the cumulative dividends paid to shareholders, the firm’s cumulative external financing, and the firm’s cumulative issuance costs until time \( t \). Equation (2) shows that liquid reserves grow with earnings net of taxes, with outside financing, with rollover gains, and with the interest earned on cash holdings. Liquid reserves decrease with payouts to shareholders, with the coupon paid on outstanding debt, with the cost of outside funds, and with rollover losses.

The firm can be forced into default if its cash reserves reach zero following a series of negative shocks and it is not possible/optimal to raise outside funds. The liquidation value of risky assets represents a fraction of their first best value and is given by

\[ \ell \equiv (1 - \varphi) \left( 1 - \theta \right) \mu \frac{1}{r}, \]

where \( \varphi \in [0,1] \) represents a haircut related to default costs. We denote by \( \tau \) the stochastic default time of the firm.

As in Leland (1998), Hackbarth, Miao, and Morelec (2006), He and Xiong (2012b), or Chen, Cui, He, and Milbradt (2018), the firm commits to a stationary debt structure \((C, m, S)\). Given this debt structure, management chooses the firm’s payout \((P)\), financing \((H)\), and default \((\tau)\) policies to maximize the present value of future dividends to shareholders. That is, given \((C, m, S)\), management solves:

\[ E_i(w; C, m, S) \equiv \sup_{(P,H,\tau)} \mathbb{E}_{w,i} \left[ \int_0^\tau e^{-rt} (dP_t - dH_t) + e^{-r\tau} \max \{0; \ell + W_\tau - S\} \right]. \tag{3} \]

The first term on the right-hand side of equation (3) represents the flow of dividends accruing to incumbent shareholders, net of the claim of new shareholders on future cash flows. The second term represents the present value of the cash flow to shareholders in
default. In the following, we focus on the case in which the liquidation value of assets is lower than the face value of outstanding short-term debt, i.e. \( \ell < S \). We will show that since \( W_\tau = 0 \) in default, this implies that short-term debt is risky. Also, in most of our analysis we take the debt structure \((C, m, S)\) as given. We discuss the initial debt structure choice (maturity and leverage) in Section 3.6.

**Discussion of assumptions**

Firms in our model have the same debt structure as firms in Leland (1994b, 1998), Hackbarth, Miao, and Morellec (2006), or Chen, Cui, He, and Milbradt (2018). As in these models, firm cash flows are stochastic and debt is repriced continuously to reflect changes in firm fundamentals. As a result, debt is always fairly priced and debtholders have no incentives to run. A key difference with our setup is that firms in these models do not face financing frictions and/or regulatory constraints. As a result, there is no role for cash holdings, the timing of default maximizes shareholder value, and shortening debt maturity decreases shareholders’ incentives to increase asset risk.

Introducing financial or regulatory constraints in a setup à la Leland (1994b, 1998) implies that the firm can be forced into liquidation at a time that does *not* maximize equity value. In such instances, shortening debt maturity does not decrease but, instead, increases shareholders’ incentives for risk-taking (see Section 4.3). That is, our main result is robust to different assumptions regarding the stochastic process governing the firm cash flows. In the baseline version of our model, we focus on a setup featuring precautionary cash reserves and cash flows following an arithmetic Brownian motion as in Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011, 2013), because financing frictions are a key ingredient of our model. Consistently, Harford, Klasa, and Maxwell (2014) document that firms facing refinancing risk due to short-term debt financing have greater cash reserves.

The models of He and Xiong (2012b) and Cheng and Milbradt (2012) also share the debt structure described above. However, these models assume that firms deliver a
constant cash flow through time that is all paid out to debtholders. Since the firm’s assets may be terminated at a random time and their liquidation value is assumed to fluctuate over time (and may fall below the face value of debt), debtholders have incentives to run if the liquidation value of assets falls below some endogenous threshold. By contrast, our model allows periodic cash flows to vary randomly, and debt is repriced on any time interval to reflect time-varying operating performance. Debt is fairly priced, and debtholders have no incentives to run.

3 The rollover trap: Short-term debt and risk-taking

In the model, management chooses the firm’s payout, financing, savings, and default policies to maximize shareholder value. Because creditors have rational expectations, the price at which maturing short-term debt is rolled over reflects these policy choices and feeds back into the value of equity by determining the magnitude of rollover imbalances. The policy choices of the firm and the values of equity and short-term debt are therefore the solution to a fixed point problem.

To aid in the intuition of the model, we focus in this section on an environment in which the firm only raises new funds by rolling over short-term debt and does not have access to outside equity. This is the case when the cost of equity financing is too high (due, e.g., to a liquidity crisis). Since there is only one financing state, we omit the subscript \( i \). In Section 4.1, we give the firm access to a credit line and show that this reinforces the economic mechanism underlying our results and therefore the model’s empirical predictions. In Section 4.2, we analyze a model in which the firm can raise outside equity and faces time-varying financing conditions (as described above) and show that all of our results hold in this more general model.

3.1 Valuing corporate securities

We start our analysis by deriving the value of equity. In our model, financing frictions lead the firm to value inside equity and, therefore, to retain earnings. Keeping cash inside
the firm, however, entails an opportunity cost $\lambda$ on any dollar saved. For sufficiently large cash reserves, the benefit of an additional dollar retained in the firm is decreasing. Since the marginal cost of holding cash is constant, we conjecture that there exists some target level $W^*$ for cash reserves where the marginal cost and benefit of cash reserves are equal and it is optimal to start paying dividends.

To solve for equity value, we first consider the region in $(0, \infty)$ over which it is optimal for shareholders to retain earnings. In this region, the firm does not deliver any cash flow to shareholders and equity value satisfies (where we omit the arguments $(C, m, S)$):

$$rE(w) = \left[ (1-\theta)(r-\lambda)w + \mu - C + \frac{m}{Rollover gains/losses} (D - S) \right] E'(w) + \frac{1}{2} ((1-\theta)\sigma)^2 E''(w). \tag{4}$$

The left-hand side of this equation represents the required rate of return for investing in the firm’s equity. The right-hand side is the expected change in equity value in the earnings retention region. The first term on this right-hand side captures the effects of cash savings and reflects debt rollover. That is, one important aspect of this equation is that the value of short-term debt feeds back into the value of equity via rollover imbalances. The second term captures the effects of cash flow volatility.

Equation (4) is solved subject to the following boundary conditions. First, when cash reserves exceed the target level $W^*$, the firm places no premium on internal funds and it is optimal to make a lump sum payment $w - W^*$ to shareholders. We thus have

$$E(w) = E(W^*) + w - W^*$$

for all $w \geq W^*$. Subtracting $E(W^*)$ from both sides of this equation, dividing by $w - W^*$, and taking the limit as $w$ tends to $W^*$ yields the condition:

$$E'(W^*) = 1.$$  

The equity-value-maximizing payout threshold $W^*$ is then the solution to the high-contact condition (see Dumas (1991)):

$$E''(W^*) = 0.$$
When the firm makes losses, its cash buffer decreases. If its cash buffer decreases sufficiently, the firm may be forced to raise new equity or to default. When the firm has no access to outside equity, it defaults as soon as its liquid reserves are depleted. As a result, the condition

$$E(0) = \max\{\ell - S; 0\} = 0$$

holds at zero and the liquidation proceeds are used to partially repay debtholders.

Consider next the value of short-term debt. Denote by $D^0(w; C, m, S, t)$ the date-$t$ value of short-term debt issued at time 0. Since a fraction $m$ of this original debt is retired continuously, these original debtholders receive a payment rate $e^{-mt} (C + mS)$ at any time $t \geq 0$ as long as the firm is solvent. Now define the value of total outstanding short-term debt by $D(w; C, m, S) \equiv e^{mt} D^0(w; C, m, S, t)$. Because $D(w; C, m, S)$ receives a constant payment rate $C + mS$, it is independent of $t$. In the following, we only derive the function $D(w; C, m, S)$, i.e. the value of total short-term debt. From this value, we can also derive the value of newly issued short-term debt, denoted by $d(w; C, m, S, 0)$. In the Appendix, we show that it satisfies: $d(w; C, m, S, 0) = mD(w; C, m, S)$.

To solve for the value of total short-term debt $D(w)$ (where we again omit the arguments $(C, m, S)$), we first consider the region in $(0, \infty)$ over which the firm retains earnings. In this region, the value of total short-term debt satisfies:

$$
(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w)
$$

$$+ \frac{(1 - \theta)^2 \sigma^2}{2} D''(w) + C + mS.
$$

The left-hand side of equation (5) is the return required by short-term debtholders. The right-hand side represents the expected change in the value of total short-term debt on any time interval. The first and second terms capture the effects of a change in cash reserves and in cash flow volatility on debt value. The third and fourth terms are the coupon and principal payments to short-term debtholders.

This equation is solved subject to the following boundary conditions. First, the firm defaults the first time that its cash buffer is depleted. The value of short-term debt at
this point is equal to the liquidation value of assets:

\[ D(0) = \min\{\ell, S\} = \ell. \]

Second, the value of short-term debt does not change when dividends are paid out, because dividend payments accrue exclusively to shareholders. We thus have:

\[ D'(W^*) = 0. \]

3.2 The economic mechanism

Before proceeding with the analysis of the model, we provide some intuition on the economic mechanism driving our results—in particular, how short-term debt can generate risk-taking incentives. Our model incorporates two important features of real world environments: Financing frictions and short-term debt financing. Consider first the effects of financing frictions on shareholders’ risk taking incentives. As shown by previous dynamic cash management models (see Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011)), shareholders in a solvent firm facing financing frictions behave in a risk-averse fashion to preserve equity value and avoid inefficient closure. Similarly, Leland (1994a) and Toft and Prucyk (1997) show that equity value can become a concave function of asset value in Leland-type models when the possibility of inefficient liquidation is introduced, e.g. via protective debt covenants (see also Section 4.3). In these environments, because shareholders cannot optimize the timing of default and the firm is fundamentally solvent (implying that the default option has a negative payoff), shareholders never want to exercise the option to default and the convexity in the shareholder value function disappears.

In previous cash management models, as well as in Leland (1994a) and Toft and Prucyk (1997), debt is either absent or has infinite maturity. The main contribution of our paper is to show that allowing for short-term debt financing in the presence of financing frictions yields radically different implications. Notably, when debt has finite maturity, it needs to be rolled over. If firm cash flows deteriorate, the market value of
newly-issued debt drops, leading to rollover losses. If rollover losses become sufficiently large, expected net cash flows may flip from positive to negative. When this is the case, shareholders have incentives to increase asset risk and “gamble for resurrection” to improve firm performance and avoid inefficient closure.

To single out this economic mechanism, consider a counterfactual firm financed with equity and infinite maturity debt (as in Leland (1994a), Bolton, Chen, and Wang (2015), or Hugonnier and Morellec (2017)). Since this firm does not need to roll over debt, its equity value \( E_{\infty}(w) \) satisfies the following equation

\[
r E_{\infty}(w) = (1 - \theta) [(r - \lambda) w + \mu - C] E'_{\infty}(w) + \frac{1}{2} ((1 - \theta) \sigma)^2 E''_{\infty}(w),
\]

in the earnings retention region. This equation is solved subject to the following boundary conditions: \( E_{\infty}(0) = E'_{\infty}(W^{*}_{\infty}) - 1 = E''_{\infty}(W^{*}_{\infty}) = 0 \), where \( W^{*}_{\infty} \) is the optimal payout trigger for shareholders. The value of risky, infinite-maturity debt in turn satisfies:

\[
r D_{\infty}(w) = (1 - \theta) [(r - \lambda) w + \mu - C] D'_{\infty}(w) + \frac{1}{2} ((1 - \theta) \sigma)^2 D''_{\infty}(w) + C,
\]

in the earnings retention region, which is solved subject to \( D_{\infty}(0) - \ell = D'_{\infty}(W^{*}_{\infty}) = 0 \).

Three important features differentiate a firm financed with infinite-maturity debt from a firm financed with finite-maturity debt. First, while the value of debt reflects the equity value-maximizing dividend and saving policies (\( W^{*}_{\infty} \) enters the debt’s boundary conditions), the market value of infinite-maturity debt does not directly affect the market value of equity, because it does not need to be rolled over. By contrast, when maturity is finite, the repricing of debt affects the market value of equity via debt rollover.

Second, expected net cash flows are given by \((1 - \theta)(\mu - C)dt > 0\) in the infinite-maturity case, i.e. they are time-invariant and positive. As a result, the expected change in cash reserves on each interval of length \( dt \) is given by

\[(1 - \theta)[(r - \lambda)w + \mu - C]dt > 0,
\]

and is always positive because \( \mu > C \) and \( w \geq 0 \). By contrast, expected net cash flows are given by \([((1 - \theta)(\mu - C) + m(D(w) - S)]dt \) in the finite-maturity case, which can flip
from positive to negative if rollover losses are sufficiently large. As a result, the expected change in cash reserves on each interval of length $dt$,

$$
[(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]dt,
$$

(6)
can also become negative if rollover losses are sufficiently large.

Third, because the firm is always solvent in the infinite maturity case (but may become liquidity constrained), shareholders behave in a risk-averse fashion. The reason is that shareholders want to avoid inefficient liquidation (or save on refinancing costs in the model with time-varying costs analyzed in Section 4.2) and have no incentives to increase asset risk, even when the firm is levered. By contrast, expected net cash flows as well as the expected change in cash reserves can be negative in the finite debt maturity case because of rollover losses. In these instances, the firm is temporarily unprofitable and shareholders do not fear the threat of liquidation. The value of equity then becomes convex (because of shareholders’ limited liability), and shareholders have incentives to increase the riskiness of assets in order to improve firm fundamentals and debt repricing close to distress, as we show next.

### 3.3 The “rollover trap”: Short-term debt and convexity

When a firm is financed with short-term debt (i.e. $m > 0$), it needs to roll over maturing debt. Fair pricing implies that the value of newly-issued debt may differ from the principal repayment on maturing debt, leading to rollover imbalances. Over each time interval of length $dt$, rollover imbalances are given by

$$
R(w)dt \equiv m(D(w) - S)dt,
$$

and depend on the firm’s cash reserves, debt maturity, and leverage. Since the probability of liquidation decreases with cash reserves $w$, the value of debt is monotonically increasing in $w$ in the earnings retention region (see Section 3.4). Thus, there exists at most one threshold $\overline{W}$ at which the rollover imbalance is zero, i.e. such that:

$$
D(\overline{W}) = S.
$$
The firm bears rollover losses for any \( w < \bar{W} \), as the inequality \( D(w) < S \) holds. That is, lower cash reserves are associated with higher default risk, which reduces the value of newly-issued debt. As a result, the proceeds from newly issued debt are not sufficient to cover the principal repayment of maturing debt, and cash reserves are used to absorb the rollover losses. Conversely, for any \( w \in [\bar{W}, W^*] \), the firm is financially strong and default risk is low. The proceeds from newly issued debt exceed the principal repayment of maturing debt and rollover gains increase the firm’s cash reserves.

Figure 1 plots the firm’s rollover imbalances as a function of cash reserves. The baseline values of the model parameters are reported in Table 1. We set the risk-free rate of return to \( r = 0.035 \), the corporate tax rate to \( \theta = 0.3 \), the mean cash flow rate to \( \mu = 0.09 \), and the carry cost of cash to \( \lambda = 0.01 \). We base the volatility of cash flows on the estimates reported by Sundaresan and Wang (2017) and set \( \sigma = 0.08 \). We base the value of liquidation costs on the estimates of Glover (2016) and set \( \varphi = 0.45 \). Given these input parameter values, the liquidation value of assets is equal to \( \ell = 0.99 \). The coupon rate \( C \) is set to 0.052. The face value \( S = 1.27 \) is uniquely determined by requiring that debt is issued at par when at \( W^*/2 \) for \( M = 1 \). This face value implies a recovery rate of 78% in default (i.e., \( \frac{S}{S} = 0.78 \)).

Figure 1 shows that rollover imbalances are markedly asymmetric, as rollover losses are larger in absolute value than rollover gains. The reason is that at the target cash level, positive operating shocks are paid out to shareholders, and debt value is insensitive to these shocks (i.e., \( D'(W^*) = 0 \)). As a result, debt value is almost insensitive to changes in cash reserves if cash reserves are sufficiently large. The left panel of the figure also shows that rollover losses are more severe when debt maturity is shorter, because the fraction of debt that needs to be rolled over on each time interval is greater. The right panel shows that rollover losses are increasingly larger as the firm’s profitability/solvency declines.
(i.e., $\mu$ decreases). If profitability deteriorates, the market value of debt decreases and rollover imbalances become more negative, all else equal.

As we show next, severe rollover losses due to short-term debt financing lead to convexity in equity value and to risk-taking incentives when firms face financing frictions. The reason is that as the firm approaches financial distress, the price of newly-issued debt decreases and rollover losses increase, as illustrated by Figure 1. As a result, when the firm is sufficiently close to default, the expected change in cash reserves (i.e., expression (6)) can be negative. This leads to the following proposition:

**Proposition 1 (Short-term debt and incentives for risk-taking)** When a firm is financed with short-term debt, equity value is locally convex when rollover losses are sufficiently large so that

$$M [(1 - \theta)((r - \lambda)w + \mu - C)] + D(w) \leq S$$

holds, where $M = \frac{1}{m}$ is the average debt maturity. In such instances, short-term debt financing provides shareholders with risk-taking incentives.

A direct implication of Proposition 1 is that, in the presence of financing frictions and short-term debt financing, fair debt pricing implies that shareholders have risk-taking incentives if expected net cash flows are negative so that condition (7) is satisfied. The reason is the following. As long as (7) is satisfied, the sum of the expected net cash flows, the interest earned on cash holdings, and the proceeds from newly issued debt (left-hand side of (7)) is lower than the repayment of maturing debt (right-hand side of (7)). In other words, rollover losses are larger than net income. As a result, the value of an additional unit of cash to shareholders is low because it plays a minor role in helping the firm escape financial distress. Indeed, that unit of cash will be used to repay maturing debt and not to rebuild cash reserves. In expectation, the firm keeps on making rollover losses, further reducing its cash reserves and increasing the risk of inefficient liquidation. In such instances, shareholders want to improve firm fundamentals and interim debt repricing to turn cash flows from negative to positive,
which provides them with incentives to increase risk. Moreover, as shown by (7), risk-taking incentives decrease as debt maturity $M$ increases (because of lower rollover losses) and do not arise with infinite maturity debt.

We call this scenario, in which the firm “burns” cash and expected net cash flows are negative because of severe rollover losses, “the rollover trap.” When a firm is in the rollover trap, the marginal value of cash progressively increases as the firm approaches the break-even point at which (6) becomes positive. In this region, the value of equity is convex and shareholders have incentives to increase asset risk. The marginal value of cash to shareholders only starts decreasing—and equity value becomes concave—when expected cash flows become sufficiently large to guarantee that an additional unit of cash helps increase cash reserves rather than cover rollover losses.$^{11}$

Figure 2 plots the value of equity $E(w)$ and the marginal value of cash to shareholders $E'(w)$ as functions of cash reserves for $w \in [0,W^*]$. Figure 2 shows that the value of equity is increasing in cash reserves. However, the top panel of the figure also shows that the relation between value of equity, debt maturity, and cash reserves is non-trivial and reflects the potential losses generated by debt rollover. A shorter debt maturity decreases (respectively, increases) the value of equity when cash reserves are small (large) due to rollover losses (gains). Equity value is concave and shareholders are quasi risk-averse for any $w$ for long debt maturities. Equity value can be locally convex close to liquidity distress if debt maturity $M = \frac{1}{m}$ is sufficiently short.

To understand when short-term debt is more likely to induce a risk-loving behavior, Figure 2 also plots the value of equity $E(w)$ and the marginal value of equity $E'(w)$ as functions of cash reserves for varying levels of asset profitability $\mu$ and bankruptcy costs $\varphi$ when $M = 5$. The figure shows that an increase in default costs increases the region

$^{11}$In some of our numerical examples, the marginal value of cash to shareholders can fall below one because we do not allow the firm to pay a liquidating dividend to shareholders. The model predictions are robust to relaxing this assumption.
over which equity value is convex. A lower recovery rate makes debt more risky and increases rollover losses, which in turn widens the rollover trap and makes it more likely that risk-taking increases equity value. An increase in asset profitability decreases the region of convexity for equity value. That is, as firms become more profitable, potential rollover losses decrease and so do shareholders’ incentives to increase risk.

Our result that short-term debt is associated with larger risk-taking incentives stands in contrast with previous models of rollover risk in which shareholders have deep pockets and can optimally choose the timing of default, such as Leland and Toft (1996) or Leland (1998). Section 4.3 shows that relaxing the assumption that shareholders can freely default at the time that maximizes equity value—for instance, because they face regulatory or financing constraints—implies that short-term debt increases risk-taking incentives in these models.

Also, it is worth noting that the principal and the coupon payment on outstanding aggregate debt is fixed in our model, as in Leland and Toft (1996), Leland (1998), or He and Xiong (2012a), among many others. This assumption does not trim the generality of our results. Suppose indeed that shareholders are allowed to take on more debt when close to distress. As the face value of debt increased, rollover losses would widen with respect to the case in which shareholders keep leverage constant. All else equal, our results would be magnified. Section 3.6 illustrates this point by allowing the firm to take on more debt via credit line drawdowns.

### 3.4 Incentive compatibility problems

We next turn to the effects of the rollover trap on the value of short-term debt and debtholders’ risk preferences. The dynamics of the value of short-term debt in the earnings retention region are given by equation (5). Now, consider a firm for which (6) is negative (or equivalently such that (7) is satisfied). This condition is necessary but not sufficient for convexity in debt value to arise. Indeed, while the value of short-term debt increases with \( w \) in that \( D'(w) \geq 0 \), equation (5) shows that debtholders receive the
periodic payments $C + mS > 0$ in the earnings retention region. These payments imply that the level of cash reserves that separates the convexity and concavity regions is not the same for equityholders and debtholders. In particular, the region of convexity for the value of risky debt is smaller than the region of convexity for equity value or may not exist. That is, because debtholders want to receive the periodic payment $C + mS > 0$, they only have incentives to increase asset volatility at the very brink of distress, when this periodic payment is at stake. As a result, an incentive compatibility problem exists for the range of cash reserves for which the value of equity is convex but the value of debt is concave. This leads to the following proposition.

**Proposition 2 (Agency conflicts and risk taking)** Whenever rollover losses are sufficiently large, the value of debt can be locally convex. The region of convexity in debt value is smaller than the region of convexity in equity value, giving rise to agency problems between shareholders and debtholders.

Figure 3 helps illustrate this result. When debt maturity is sufficiently long, both shareholders and debtholders are effectively risk averse, so agency conflicts do not arise (top panel). When debt maturity is sufficiently short, two scenarios can arise. First, only shareholders have incentives for risk-taking, so an agency problem arises when cash reserves are close to zero (middle panel). Second, both shareholders and debtholders have incentives for risk-taking at the very brink of distress but, still, an agency problem arises for intermediate levels of cash reserves (bottom panel).

To better understand when incentive compatibility problems are likely to arise, Table 2 reports the inflection points for debt ($W_D$) and equity ($W_E$), the size of the region over which equity value is convex and debt value is concave (agency region, $AR$), as well as the target cash level ($W^*$) for different debt maturities ($M$), cash flow drift ($\mu$), cash flow volatility ($\sigma$), and liquidation costs ($\varphi$).
Table 2 shows that risk-taking incentives for both debtholders and shareholders decrease with debt maturity in that both $W_D$ and $W_E$ decrease with $M$. When debt maturity is sufficiently long, equity and debt values become concave for any level of cash reserves, and both classes of claimholders behave as if they were risk-averse. In this case, $W_D \notin (0,W^*)$ and $W_E \notin (0,W^*)$. In Table 2, we indicate these cases using “n.a.” for the value of $W_D$ and $W_E$. The second panel of Table 2 shows that both risk-taking incentives (i.e., $W_D$ and $W_E$) and agency problems (i.e., the agency region AR) decrease with profitability $\mu$. Indeed, an increase in profitability decreases rollover losses and makes it less likely that (6) becomes negative. The third panel shows that increasing volatility results in a large increase in target cash holdings and in a decrease (respectively, increase) in debtholders’ (shareholders’) risk-taking incentives. As a result, incentive compatibility problems increase with $\sigma$. The last panel shows that, because they increase rollover losses, liquidation costs increase both shareholders and debtholders’ risk-taking incentives. When liquidation costs are sufficiently low, a forced liquidation is less harmful and neither bondholders nor shareholders have risk-taking incentives.

Figure 4 plots the value of debt $D(w)$ and the marginal value of cash to debtholders $D'(w)$ as functions of cash reserves, for different debt maturities. $D(w)$ increases with maturity, as a shortening of maturity implies an increase in rollover losses and, thus, in liquidation risk. In addition, while debtholders suffer from the risk implied by a shorter debt maturity due to larger rollover losses, they do not capture the upside potential due to any rollover gains. Figure 4 also shows that the convexity is less pronounced for debtholders than for equityholders, leading to an incentive compatibility problem.

It is worth noting that our predictions are different from the standard Jensen and Meckling (1976) result that risk-shifting incentives are larger when firms are close to default. In fact, in our model, shareholders have no incentives to increase asset risk if the firm is financed with debt with sufficiently long maturity. In addition, and importantly, risk-taking incentives lead to agency conflicts for intermediate levels of cash reserves,
i.e., in the cash interval \((W_D, W_E)\), but may be optimal at the very brink of distress for both shareholders and debtholders.

### 3.5 Assessing the effect of risk-taking strategies

We have just shown that, in a world with financing frictions and fair debt pricing, short-term debt financing generates a local convexity in the value of equity and, to a lower extent, in the value of debt. In this section, we show that this convexity generated by short-term debt financing may lead shareholders to increase the riskiness of assets, for instance by engaging in zero-NPV investments with random returns. That is, we show that financing frictions imply a behavior that is in sharp contrast with the long-standing idea that short-term debt has a disciplinary role (see e.g. Barnea, Haugen, and Senbet (1980), Leland and Toft (1996), Leland (1998), or Cheng and Milbradt (2012)).

To analyze the firm’s incentives to increase asset risk, we follow Bolton, Chen, and Wang (2011), Hugonnier, Malamud, and Morellec (2015), and Décamps, Gryglewicz, Morellec, and Villeneuve (2017) and assume that the firm has access to a futures contract whose price is a Brownian motion \(B_t\), uncorrelated with the Brownian motion \(Z_t\) driving the firm cash flows. A position \(\gamma_t\) in the futures contract thus changes the dynamics of firm cash flows from \(dY_t\) to \(dY_t + (1 - \theta)\gamma_t dB_t\), i.e., it only changes the riskiness of cash flows. Futures positions are generally constrained by margin requirements. To capture these requirements, we consider that the firm’s futures position \(\gamma_t\) cannot exceed some fixed size (or collateral constraint) \(\Gamma\) and study the effects of varying \(\Gamma\) on optimal policies and equity value.

Assuming frictionless trading in the futures contract, standard arguments show that in the region over which the firm retains earnings, equity value satisfies:

\[
re(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w)
+ \max_{0 \leq \gamma \leq \Gamma} \left\{ \frac{1}{2}(1 - \theta)^2 \left(\sigma^2 + \gamma^2\right) E''(w) \right\}, \quad (8)
\]
where the last term on the right hand side captures the effects of risk-taking on equity value. By differentiating with respect to $\gamma$, we can determine the optimal risk-taking strategy. This leads to the following Proposition.

**Proposition 3 (Optimal risk-taking strategy)** For all $w$ such that $E''(w) > 0$, shareholders find it optimal to increase the volatility of assets by taking the maximum position in future contracts ($\gamma = \Gamma$). For all $w$ such that $E''(w) < 0$, shareholders behave as if they were risk-averse and take no positions in future contracts ($\gamma = 0$).

Proposition 3 reveals that the optimal risk-taking policy is of bang-bang type: If risk-taking is optimal for shareholders because equity value is convex, shareholders choose the riskiest strategy. When risk-taking is possible, the value of equity is defined over three intervals: $[0, W_\Gamma)$, $[W_\Gamma, W^*(\Gamma))$, and $[W^*(\Gamma), \infty)$, where $W_\Gamma$ represents the threshold marking off the convex and concave regions and $W^*(\Gamma)$ is the optimal payout threshold. Equity value solves equation (8) subject to boundary conditions at zero and at the target cash level $W^*(\Gamma)$, as well as continuity and smoothness conditions at $W_\Gamma$. The details are reported in Appendix A.4.

Figure 5 shows that risk-taking substantially increases the value of equity when it is convex, that is when debt maturity is short and cash reserves are low. The top panel of Figure 5 plots equity value under different risk-taking strategies and shows that the increase in equity value due to risk-taking is stronger when debt maturity is shorter, because the region of convexity is larger. Moreover, when equity value is convex, the strategy leading to the largest increase in cash flow volatility (i.e., the largest $\Gamma$) is the one that increases equity value the most.

Proposition 2 and Proposition 3 together lead to the following result.

**Corollary 4 (Effect of risk-taking on debt value)** Risk-taking policies lead to: (1) an increase in the value of debt in the region over which debt value is convex, (2) a
decrease in the value of debt in the region over which equity value is convex but debt value is concave.

Corollary 4 demonstrates that risk-taking has ambiguous effects on debt value. First, as shown in section 3.4, the value of debt can be locally convex when rollover imbalances are large and the firm is sufficiently close to distress. In such instances, risk-taking strategies increase the values of debt and equity. This result is illustrated in the bottom panel of Figure 5, which shows that increasing asset volatility leads to a modest decrease in yield spreads at the very brink of distress. Second, because the size of the region of convexity in equity value is larger than the region of convexity in debt value (by Proposition 2), shareholders have incentives to increase asset risk even when this is suboptimal for debtholders. In line with this result, Figure 5 shows that increasing asset volatility leads to an increase in yield spreads for intermediate levels of cash reserves—i.e. when gambling is not optimal to debtholders because the value of debt is concave in cash reserves. It is worth noting that the increase in yield spreads for intermediate levels of cash reserves (i.e., outside distress) is greater in absolute value than the modest decrease in spreads for small levels of cash reserves (i.e., close to distress). That is, risk-taking strategies lead to a sharp increase in the cost of debt when the firm is far from distress, which exacerbates the likelihood of entering the rollover trap. As shown in the figure, the increase in spreads is smaller when maturity is longer. In addition, the effect of risk-taking on yield spreads is amplified for larger values of $\Gamma$.

Cheng and Milbradt (2012) also show that risk-taking incentives can increase both equity and debt values close to distress. Our result differs from theirs in two important dimensions. First, in Cheng and Milbradt (2012), increasing asset risk enhances creditors’ confidence that future creditors will not run. Instead, in our model increasing asset risk has a positive impact on the price of newly-issued debt in distress, effectively reducing yield spreads on debt and decreasing the magnitude of rollover losses. Second, in Cheng and Milbradt (2012), risk-shifting incentives are minimized for an intermediate debt maturity, as long debt maturities lead to an increase in risk-taking in good times.
(i.e., when the firm is sufficiently far from default). In our model, agency problems decrease with debt maturity and do not arise if debt maturity is sufficiently long.

3.6 Optimal capital structure

We now investigate the effects of rollover imbalances on capital structure choices. To do so, we allow the debt principal to be a function of the coupon $C$ and impose that debt is issued at par at a given level of cash reserves. We use different initial levels of cash reserves to account for varying degrees of financial constraints at the time the firm is set up. As in Leland (1994), the coupon is chosen at the initial date to maximize the sum of equity and debt values as calculated in Section 3, under the constraint that debt is issued at par. That is, we look for

$$V(w_0; C, m, S) \equiv \sup_{C \in \mathbb{R}_+} \left[ E(w_0; C, m, S) + D(w_0; C, m, S) \right],$$

under the budget constraint

$$w_0 = W_0 - S - I,$$

where $I$ is the initial investment cost and $W_0$ is the initial cash endowment before financing and the constraint that short-term debt is initially issued at par

$$D(w_0; C, m, S) = S.$$

Table 3 shows the capital structure that maximizes firm value (i.e. shareholder wealth) as a function of debt maturity.

Insert Table 3 Here

When debt maturity is infinite, there are no rollover imbalances. In this case, the optimal debt level balances the tax benefits of debt with bankruptcy costs. When debt maturity is finite, two additional factors shape capital structure choices. First, a short debt maturity imposes larger rollover losses when cash reserves are low, which increases
the cost of debt and, thus, decreases the firm’s incentives to issue debt. Second, a short debt maturity increases the proceeds from debt rollover when cash reserves are large (and default risk is low), which decreases the cost of debt and, thus, creates an incentive to increase leverage ratios. Table 3 shows that when debt maturity is relatively short, the first effect dominates and the threat of large rollover losses makes the coupon that maximizes firm value smaller compared to the infinite maturity case. That is, by generating substantial rollover losses, a shorter maturity decreases the firm’s debt capacity and optimal leverage. Conversely, the second effect can dominate when debt maturity is finite and relatively long; i.e., the lower cost of debt increases the firm debt capacity and the optimal coupon rate compared to the infinite maturity case.

4 Robustness to alternative model specifications

4.1 Increasing debt exposure via credit line

In our benchmark analysis, the firm is forced into liquidation when cash reserves are depleted and access to the equity market is prohibitively expensive. We now assess the robustness of our main results by allowing the firm to take on additional debt via credit line drawdowns. In practice, credit lines provide firms with immediate liquidity that can be used in times of need (see Sufi (2009)). In our model, they allow the firm to acquire flexibility in their debt and liquidity policies, with a total amount of (net) debt varying between $S - W^*$ and $S + L$, where $L$ is the pre-established limit on the credit line.

Specifically, assume that the firm has access to a credit line with pre-determined limit $L \geq 0$. For the amount of credit that the firm uses, the interest spread over the risk-free rate is $\beta > 0$. As in Bolton, Chen, and Wang (2011), the spread $\beta$ can be interpreted as an intermediation cost. Because of this spread, the firm will optimally avoid using its credit line before exhausting internal funds. That is, the firm uses cash as the marginal source of financing if $w \in [0, W^*(L)]$ (the cash region), where $W^*(L)$ denotes the target cash level when the firm has access to a credit line. Conversely, the firm draws funds
from the credit line when \( w \in [-L, 0] \) (the credit line region). In the following, we assume that the credit line has priority over short-term debt and that \( L < \ell \), implying that the credit line is fully collateralized. We report the system of equations satisfied by equity and debt values when the firm has access to a credit line in Appendix A.5.

Figure 6 describes the effects of credit lines on the values of corporate securities and rollover imbalances. Consistent with economic intuition, the figure shows that credit lines reduce the need for large cash balances in that the target cash level is smaller when \( L > 0 \) (see also Bolton, Chen, and Wang (2011) or Décamps, Gryglewicz, Morellec, and Villeneuve (2017)). By reducing the expected cost of financing frictions, credit lines increase the values of debt and equity in the cash region. Nonetheless, credit lines reduce the value of short-term debt in the credit line region. The reason is that the credit line has to be paid in full before debtholders can collect any liquidation proceeds. The resulting lower payoff to short-term debt in liquidation leads to larger rollover losses when the firm is close to exhausting the credit line (see the bottom right panel). This implies that senior credit lines strengthen the amplification mechanism highlighted in Section 3.3 and, therefore, shareholders’ incentives for risk-taking. This analysis therefore not only confirms our results on the effects of short-term debt on risk-taking incentives, but also shows that these effects can magnified by the presence of a senior credit line.

4.2 Time-varying financing conditions

Having explained the effects of short-term debt on corporate policies and incentives for risk-taking in a model in which firms do not have access to outside equity, we now analyze a more general environment in which funding conditions are time-varying, as described in Section 2.

In such an environment, the firm still finds it optimal to hold cash reserves, but the target level of cash reserves is state-dependent, denoted by \( W^*_i \). Notably, because
financial frictions are more severe in state $B$ than in state $G$, we expect the target level of cash reserves to be larger in state $B$. That is, we expect $W^*_B > W^*_G$. Another key difference with the model presented in Section 3 is that the firm can raise equity at a cost when in state $G$. As in Bolton, Chen, Wang (2013), the firm may choose to raise funds before its cash buffer gets completely depleted to avoid that financing conditions worsen when cash reserves are close to zero. We denote the issuance boundary in state $G$ by $W \in [0, W^*_G)$. We report in Appendix A.6 the system of equations satisfied by the values of equity $E_i(w; C, m, S)$ and short-term debt $D_i(w; C, m, S)$ in each state $i$.

We first analyze how time-varying financing conditions affect the price at which short-term debt is rolled over and the magnitude and sign of rollover imbalances. Consider first the bad state. In that state, the firm may be forced into default after a series of negative shocks because it is unable to raise new equity if it runs out of funds. Thus, the bad state displays a pattern that is analogous to the case analyzed in Section 3. Specifically, there exists a level of cash reserves $W_B$ such that $D_B(W_B; C, m, S) = S$, i.e. such that new debt is issued at par. Rollover imbalances are positive (respectively negative) above (respectively below) $W_B$, and their size decreases with debt maturity.

Consider next the good state. In this state, default never occurs because the firm can always raise capital by paying the cost $\phi$. The value of newly-issued debt increases compared to the bad state, and even more so if debt maturity is shorter. As noted by Acharya, Krishnamurthy, and Perotti (2011): “Creating exposure to liquidity risk is profitable in good times, but creates vulnerability to massive losses when the risk perception changes.” In line with this intuition, Figure 7 (top panel) shows that short-term debt financing may be attractive to shareholders in the good state, because the market value of debt is relatively larger and so are the proceeds from debt rollover, which increases equity value (second panel). However, short-term debt leads to rollover losses in the bad state, which increases default risk and decreases the value of equity.

The analysis in Section 3 has shown that the value of equity can be locally convex
when rollover losses are large. When financing conditions are time-varying, this pattern is preserved in the bad state. The value of equity can also be locally convex in the good state, but for a different reason (Figure 7, third panel). In the good state, this convexity is related to the possibility to time the market by issuing securities when the cost of external finance is low, as in Bolton, Chen, and Wang (2013). Consistent with the fact that the timing option in the good state is more valuable when the firm has issued shorter-term debt, Table 4 shows that the refinancing threshold \( W \) (i.e. the exercise threshold for the market timing option) decreases as debt maturity increases. Overall, Figure 7 demonstrates that the patterns identified in Section 3 still hold (i.e., they are not specific to the way financing frictions are modeled) and that short-term debt encourages shareholders to engage in risk-taking strategies.

To better understand the relation between debt maturity, time-varying financing frictions, and cash holdings policies, Table 4 reports the target level of cash reserves in the bad and good states for different debt maturities.

The table reveals that by imposing larger rollover losses in bad times, a shorter maturity pushes the firm to increase its target cash reserves, in line with Harford, Klasa, and Maxwell (2014). Table 4 also shows that the bad state commands a larger target cash level (\( W_B^* > W_G^* \)), consistent with the evidence in Acharya, Shin, Yorulmazer (2010) that bank liquidity buffers are counter-cyclical. Lastly, consistent with the fact that the timing option in the good state is more valuable when the firm has issued shorter-term debt, Table 4 shows that the refinancing threshold \( W \) (i.e. the exercise threshold for the timing option) decreases as debt maturity increases.

4.3 Introducing financing frictions in a Leland-type setup

Our result that short-term debt generates risk-taking incentives goes against the long-standing idea that short-term debt reduces the agency cost of asset substitution, as
discussed for example in Leland and Toft (1996) or Leland (1994b, 1998). In this section, we show that this result is not driven by the assumption about the stochastic process governing the firm’s cash flows but rather by the fact that financing frictions constrain shareholders’ default decision.\(^\text{12}\) To do so, we consider in this section a setup à la Leland (1994b, 1998) in which we relax the assumption that shareholders have deep pockets and can choose the timing of default that maximizes equity value.\(^\text{13}\)

Consider a firm whose unlevered asset value \(V = (V_t)_{t\geq 0}\) follows a geometric Brownian motion:

\[ dV_t = (\mu - \delta)V_t dt + \sigma V_t dZ_t, \tag{9} \]

where \(\mu\) is the total expected rate of return, \(\delta\) is the constant payout rate, and \(dZ_t\) is the increment of a standard Brownian motion. The firm is financed with equity and short-term debt, as described in Section 2. In Leland (1994b, 1998) or Leland and Toft (1996), the process in (9) continues without time limit unless \(V\) falls to a default-triggering value \(V_B\), which is endogenously determined to maximize equity value. In this setup, shareholders can inject cash in the firm instantaneously and at no cost, and equity value is a convex function of asset value when debt maturity is infinite or sufficiently long. Shareholders thus have incentives to increase asset risk, which is detrimental to debtholders. Conversely, if maturity is sufficiently short, increasing risk does not benefit shareholders, except when default is imminent. In this setting, short-term debt acts as a disciplining device by decreasing shareholders’ risk-taking incentives.

Suppose now that shareholders can no longer optimize the timing of default because of financing frictions, debt covenants, or regulatory constraints, and denote by \(V_b\) the exogenous threshold (asset value) triggering default. To consider relevant cases, we assume

\(^{12}\)In line with previous dynamic models with financing frictions such as Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011, 2013), or Hugonnier, Malamud and Morellec (2015), cash flows are governed by an arithmetic Brownian motion in our model. In Leland-type models, total cash flows and asset values are governed by a geometric Brownian motion.

\(^{13}\)The results derived in this section also hold in a Leland and Toft (1996) setup, in which bond expirations are uniformly spread out over time.
that $V_b$ is greater than $V_B$, so that shareholders are forced to liquidate the firm’s assets early, when suboptimal for them. Default can be interpreted in this context as being triggered by the breach of a net-worth covenant. Alternatively, it can be interpreted as a liquidity default caused by financing frictions. As shown by Leland (1994a) and Toft and Prucyk (1997), when the default boundary is exogenous and sufficiently large, equity value becomes a concave function of asset value. In this case, shareholders are effectively risk-averse and have no risk-taking incentives.\footnote{A similar result obtains in the models of cash management with financing frictions and infinite maturity debt developed by Bolton, Chen, and Wang (2015) or Hugonnier and Morellec (2017).}

We now show that short-term debt can restore the convexity of equity in asset value when the default boundary is exogenous. To understand why this is the case, consider the expected net cash flow to equityholders when varying average debt maturity $M$. When $M = \infty$, the expected net cash flow to equityholders on any interval of length $dt$ is given by

$$[\delta V_t - (1 - \theta)C]dt,$$

which is the total firm payout minus the after-tax coupon payment. When $M$ is finite, the net cash flow to equityholders on any interval of length $dt$ is given by

$$[\delta V_t - (1 - \theta)C + m(D(V_t; m) - S)]dt$$

where the last term in the square bracket represents the firm’s rollover imbalance. When asset value is low and $D(V_t; m) < S$, the firm faces rollover losses. When average debt maturity is shorter, the fraction $m$ of debt that needs to be rolled over on each time interval is larger, which magnifies rollover losses as fundamentals deteriorate. For sufficiently short debt maturity, the expression in (11) can be negative even for values of $V_t$ that make (10) positive. That is, if rollover losses are sufficiently large, expected net cash flows to shareholders flip from positive to negative. In this case, shareholders hold an out-of-the-money option and have incentives to increase asset risk.

\footnote{A similar result obtains in the models of cash management with financing frictions and infinite maturity debt developed by Bolton, Chen, and Wang (2015) or Hugonnier and Morellec (2017).}
Figure 8 provides an illustration of this result by plotting the value of equity and the marginal value of equity as functions of the value of the firm’s assets, for different debt maturities. In this figure, we base our parametrization on Leland (1994b) and set the risk-free rate to 7.5%, the cash flow volatility to 0.20, bankruptcy costs to 0.5, the tax rate to 0.35, and the payout rate to 0.07. Additionally, we set the value of debt principal and coupon to 65 and 6, respectively. In the top panel, we impose an exogenous default boundary equal to $V_b = 90$, which is larger than the endogenous default boundaries. The top panels of the figure highlight that incentives for risk taking are less pronounced for long-term debt than for short-term debt when shareholders are constrained in their default decisions. If debt maturity is sufficiently long, equity value is concave for all asset values, and equityholders have no risk-taking incentives. If maturity is short, equity value becomes convex when asset value is sufficiently close to the default threshold. As shown in the bottom panels, this is not the case when shareholders are unconstrained in their default decisions. In this case, long-term debt is associated with risk-taking incentives for any $V$.

It is worth noting that risk-taking incentives in this context arise in fundamentally solvent firms, i.e. firms for which $(10)$ is positive. They are driven by liquidity problems rather than by solvency problems. Because financing frictions are key to this mechanism and because they lead shareholders to value inside equity, our baseline model is one in which we allow firms to keep cash reserves, as in Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011).

5 Conclusion

A commonly accepted view in banking and corporate finance is that short-term debt can improve firm value by disciplining management and curbing moral hazard. This view does not seem to be supported however by the available evidence. This paper shows that, for firms facing financing frictions or regulatory constraints, short-term debt does not decrease but, instead, increases incentives for risk-taking. To demonstrate this result
and examine its implications, we develop a model in which firms are financed with equity and risky short-term debt and face taxation, time-varying financing frictions, and default costs. In this model, firms own a portfolio/operate a set of risky assets and have the option to invest in risk-free, liquid assets such as cash reserves or safe government bonds. Firms maximize shareholder value by choosing their buffers of liquid assets as well as their financing, risk management, and default policies. A key difference with prior work is that financing frictions and/or regulatory constraints affect the firm’s default decision and can make it inefficient from shareholders’ perspective.

With this model, we show that when a firm has short-term debt outstanding and debt is fairly priced, negative operating shocks lead to a drop in liquid reserves and cause the firm to suffer losses when rolling over short term debt, due to weaker fundamentals. This amplification mechanism leads to an increase in default risk, that gets more pronounced as debt maturity decreases and rollover losses increase. When firms are close to distress and debt maturity is short enough, rollover losses can be larger than expected operating profits, dragging the firm closer to default. We call this scenario, in which the firm “burns” cash because of severe rollover losses, “the rollover trap.” In contrast with extant models with long-term debt financing and financing frictions or with short-term debt but without financing frictions or regulatory constraints, the existence of the rollover trap can make equity holders risk-loving when close to distress. This in turn may lead firms to engage in risk-taking strategies in an attempt to improve fundamentals and debt repricing and avoid inefficient closure. That is, we show that financing frictions or regulatory constraints combined with fair debt pricing imply behavior that is in sharp contrast with the long-standing idea that short-term debt has a disciplinary role and reduces the agency costs of asset substitution.
Appendix

A.1 Deriving the value of short-term debt

We start by deriving the value of total short-term debt, denoted by \( D(w) \). Since the firm keeps a stationary debt structure, \( D(w) \) receives a constant payment rate \( C + mS \) that is independent of \( t \). Following standard arguments, the function \( D(w) \) satisfies the following ordinary differential equation (ODE):

\[
(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + d(w) - mS]D'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 D''(w) + C + mS
\]

where \( d(w) \) is the value of currently-issued short-term debt. For any given time \( t \), we denote by \( d(w, \tau) \) the value of the outstanding debt of generation \( \tau \leq t \), with \( \tau \in [-\infty, 0] \). Therefore, \( d(w, 0) = d(w) \) represents the value of currently-issued short-term debt (i.e., \( \tau = 0 \) at the current time), and we have the following relation

\[
d(w, \tau) = e^{m\tau}d(w).
\]

All remaining units of short-term debt from prior issues have the same value per unit, as units of all vintages pay the same coupon, and the remaining units of all vintages will be retired at the same fractional rate. However, there are fewer outstanding units of debt of older generations due to accumulated debt retirement. Integrating \( d(w, \tau) \) over \( \tau \in [-\infty, 0] \) gives the total value of short-term debt outstanding \( D_i(w) \), and then the following important relation

\[
D(w) = d(w) \int_{-\infty}^{0} e^{\tau m} d\tau = \frac{d(w)}{m}
\]

holds. Using this relation, together with the ODE describing the dynamics of \( D(w) \), we finally get the ODE for currently issued short-term debt, given by

\[
rd(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + d(w) - mS]d'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 d''(w) + mC + m[mS - d(w)].
\]
The third term on the right-hand side implies that the short-term debt issued today promises a coupon payment $mC$ on any time interval. Recall that exponential repayment of debt with average maturity $1/m$ implies that debt matures randomly at the jump times of a Poisson process with intensity $m$. The fourth term on the right-hand side then represents the payoff obtained by the debtholders when the debt randomly matures times the probability of this occurrence.

A.2 Proof of Proposition 1

Condition (7) in Proposition 1 can be rewritten as

$$(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S) \leq 0,$$

where $m$ is the fraction of total debt that is rolled over. When shareholders have limited liability, equity value satisfies $E(w) \geq 0$. In addition, equity value is increasing in cash reserves, in that $E'(w) > 0$. As a result, when the above condition is satisfied we have

$$rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2} ((1 - \theta)\sigma^2) E''(w),$$

which implies that $E''(w) \geq 0$. That is, equity value is locally convex.

A.3 Proof of Proposition 2

Debt value is non-negative ($D(w) \geq 0$) and non-decreasing in cash reserves ($D'(w) \geq 0$). Moreover, the periodic payment to debtholders $C + mS \geq 0$ is non-negative. When condition (7) in Proposition 1 is satisfied, we have

$$(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w)$$

$$+ \frac{1}{2} ((1 - \theta)\sigma^2) D''(w) + C + mS \geq 0,$$

which implies that $D''(w)$ can be positive (and, thus, the value of debt is convex) if condition (7) is sufficiently negative.
As in the main text, we denote by $W_D$ the level of cash reserves that separates the region of concavity and of convexity in debt value (i.e., such that $D''(W_D) = 0$). We also denote by $W_E$ the level of cash reserves that separates the region of concavity and of convexity in equity value (i.e., such that $E''(W_E) = 0$). Because debtholders receive the periodic payment $C + mS$ (which increases the right-hand side of the above ODE), the inequality $W_D \leq W_E$ holds. As a result, the region of convexity in debt value is smaller than the region of convexity in equity value.

### A.4 Proof of Proposition 3

We derive the optimal risk-taking policy and the value of the firm’s securities under the assumptions in Section 3.5. Assuming frictionless trading in futures contracts, standard arguments imply that, in the earnings retention region, the value of equity satisfies the Hamilton-Jacobi-Bellman equation reported in Section 3.5, equation (8). By simply differentiating this equation with respect to the control, it follows that management takes on the maximum position $\Gamma$ in the future contract if $E''(w) > 0$, i.e. if the value of equity is convex. Conversely, management takes no position in the contract if $E''(w) < 0$, i.e. if the value of equity is concave. We denote by $W_{\Gamma}$ the cash level that separates the convex and the concave region, i.e. such that

$$E''(W_{\Gamma}) = 0.$$  

The optimal risk-taking policy is thus of a bang-bang type:

$$\gamma = \begin{cases} 
\Gamma & \text{if } 0 \leq w < W_{\Gamma}, \\
0 & \text{if } W_{\Gamma} \leq w < W^*(\Gamma). 
\end{cases}$$  

That is, if risk-taking is optimal, it happens at the maximal rate. Note that the target level of cash holdings is denoted by $W^*(\Gamma)$ in this environment.

In analogy to Section 3, management finds it optimal to pay out dividends to shareholders when the cash reserves exceed $W^*(\Gamma)$, and the value of equity is linear above this target level. Differently, the optimal risk-taking policy means that, when
$W_{\Gamma} \in (0, W^*(\Gamma))$, the cash retention region $[0, W^*(\Gamma))$ is characterized by a risk-taking region, $[0, W_{\Gamma})$, and a no-risk-taking region, $[W_{\Gamma}, W^*(\Gamma))$. In the risk-taking region $[0, W_{\Gamma})$, the value of equity satisfies the following differential equation

$$rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2}(1 - \theta)^2 (\sigma^2 + \Gamma^2) E''(w).$$

In the no-risk-taking region $[W_{\Gamma}, W^*(\Gamma))$, the value of equity satisfies

$$rE(w) = (1 - \theta)((r - \lambda)w + \mu - C + m(D(w) - S)]E'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 E''(w).$$

The system of ODEs for the value of equity is solved subject to the following boundary condition at the default/liquidation threshold, $E(0) = 0$, and the boundary conditions at the target cash level, $\lim_{w \uparrow W^*(\Gamma)} E'(w) = 1$ and $\lim_{w \downarrow W^*(\Gamma)} E''(w) = 0$. These boundary conditions are similar to those derived in Section 3 and admit an analogous interpretation. In addition, we now need to impose continuity and smoothness at $W_{\Gamma}$,

$$\lim_{w \uparrow W_{\Gamma}} E(w) = \lim_{w \downarrow W_{\Gamma}} E(w) \quad \text{and} \quad \lim_{w \uparrow W_{\Gamma}} E'(w) = \lim_{w \downarrow W_{\Gamma}} E'(w),$$

to ensure that the risk-taking region and the no-risk-taking regions are smoothly pasted.

Since debtholders have rational expectations, the value of debt reflects this risk-taking policy. As this policy is chosen by management to maximize shareholders’ value, this means that risk-taking may occur even when the value of debt is concave — then decreasing the value of debt. In the risk-taking region $[0, W_{\Gamma})$, the value of short-term debt $D(w)$ satisfies

$$(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w) + \frac{1}{2}(1 - \theta)^2 (\sigma^2 + \Gamma^2) D''(w) + C + mS.$$ 

In the no-risk-taking region $[W_{\Gamma}, W^*(\Gamma))$, $D(w)$ satisfies

$$(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 D''(w) + C + mS.$$
On top of the boundary conditions at 0 and \( W^*(\Gamma) \) as in Section 3, respectively \( D(0) = (\ell - \Delta)^+ \) and \( D'(W^*(\Gamma)) = 0 \), we impose continuity and smoothness at \( W^\Gamma \), i.e.

\[
\lim_{w \uparrow W^\Gamma} D(w) = \lim_{w \downarrow W^\Gamma} D(w) \quad \text{and} \quad \lim_{w \uparrow W^\Gamma} D'(w) = \lim_{w \downarrow W^\Gamma} D'(w).
\]

### A.5 Credit lines

We derive the system of equations for the values of equity and short-term debt in the presence of a credit line, as analyzed in Section 4.1. The firm uses cash as the marginal source of financing if \( w \in [0, W^*(L)] \) (the cash region), where \( W^*(L) \) denotes the target cash level as a function of \( L \). Conversely, the firm draws funds from the credit line when \( w \in [-L, 0] \) (the credit line region). In the cash region, the value of equity satisfies the following ODE

\[
rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2} ((1 - \theta)\sigma)^2 E''(w).
\]

In the credit line region, the firm needs to pay interests on borrowed funds, and the value of equity satisfies

\[
rE(w) = [(1 - \theta)((r + \beta)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2} ((1 - \theta)\sigma)^2 E''(w).
\]

Similarly, the value of short-term debt satisfies the following ODE

\[
(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w) + \frac{1}{2} ((1 - \theta)\sigma)^2 D''(w) + C + mS
\]

in the cash region, whereas it satisfies the following ODE

\[
(r + m)D(w) = [(1 - \theta)((r + \beta)w + \mu - C) + m(D(w) - S)]D'(w) + \frac{1}{2} ((1 - \theta)\sigma)^2 D''(w) + C + mS
\]

in the credit line region.

The system of equations is solved subject to the following boundary conditions at the liquidation boundary \((-L)\) and at the payout boundary \((W^*(L))\):

\[
E(-L) = E'(W^*(L)) - 1 = E''(W^*(L)) = 0,
\]

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and
\[ D(-L) - (\ell - L) = D'(W^*(L)) = 0, \]
as well as the continuity and smoothness conditions where the credit line and cash regions are pieced together
\[
\lim_{w \uparrow 0} E(w) = \lim_{w \downarrow 0} E(w), \quad \text{and} \quad \lim_{w \uparrow 0} E'(w) = \lim_{w \downarrow 0} E'(w)
\]
\[
\lim_{w \uparrow 0} D(w) = \lim_{w \downarrow 0} D(w), \quad \text{and} \quad \lim_{w \uparrow 0} D'(w) = \lim_{w \downarrow 0} D'(w).
\]

A.6 Time-varying financing conditions

To solve for equity value, we first consider the region in \((0, \infty)\) over which it is optimal for firm shareholders to retain earnings. In this region, the firm does not deliver any cash flow to shareholders and equity value satisfies for \(i = G, B, i \neq j:\)
\[
rE_i(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D_i(w) - S)]E'_i(w)
\]
\[
+ \frac{1}{2}((1 - \theta)\sigma)^2 E''_i(w) + \pi_i [E_j(w) - E_i(w)]. \tag{A1}
\]
Equation (A1) is solved subject to the following boundary conditions. First, when cash reserves exceed \(W^*_{i}\), the firm places no premium on internal funds and it is optimal to make a lump sum payment \(w - W^*_i\) to shareholders. As a result, we have
\[
E_i(w) = E_i(W^*_i) + w - W^*_i
\]
for all \(w \geq W^*_i\). Subtracting \(E_i(W^*_i)\) from both sides of this equation, dividing by \(w - W^*_i\), and taking the limit as \(w\) tends to \(W^*_i\) yields the condition:
\[
E'_i(W^*_i) = 1.
\]
The equity-value-maximizing payout threshold \(W^*_i\) is then the solution to:
\[
E''_i(W^*_i) = 0.
\]
When the firm makes losses, its cash buffer decreases. If its cash buffer decreases sufficiently, the firm may be forced to raise new equity or to liquidate. Consider first state $G$ in which refinancing is possible. In this state, the firm may raise funds before its cash buffer gets completely depleted to avoid that financing conditions worsen when cash reserves are close to zero (as in Bolton, Chen, Wang (2013)). We denote the issuance boundary in state $G$ by $W \in [0, W_G^*)$. For any $w \leq W$ in state $G$, the firm raises new equity and resets its cash buffer to $W_G^*$ if optimal to do so. This implies that

$$E_G(w) = E_G(W_G^*) - (W_G^* - w) - \phi, \quad \forall w \leq W.$$

If $W$ is strictly greater than zero, the firm effectively taps the equity markets before its cash reserves are depleted. In this case, it must be that the condition

$$E'_G(W) = 1$$

holds. Indeed, management delays equity issues until the marginal value of cash to shareholders equals the marginal cost of refinancing, that is equal to one.

Consider next state $B$. In that state, the firm has no access to outside funding and defaults as soon as its liquid reserves are depleted. As a result, the condition

$$E_B(0) = \max\{\ell - S; 0\} = 0$$

holds at zero and the liquidation proceeds are used to repay debtholders.

Note that the cash reserves process evolves in $[0, W_B^*)$ in the bad state and in $[W, W_G^*)$ in the good state. This implies that if the financing state switches from bad to good while the firm’s cash reserves are in $(0, W]$, the firm immediately taps the equity market to raise its liquid reserves to their optimal level $W_G^*$. In these instances, the value of equity jumps from $E_B(w)$ to $E_G(W_G^*) - (W_G^* - w) - \phi$ for any $w \in [0, W]$. If, instead, the financing state switches from bad to good when $w \in [W_G^*, W_B^*]$, the firm makes a lump sum payment to shareholders and cash reserves go down to $W_G^*$.

To solve for the value of total short-term debt $D_i(w)$ (where we again omit the arguments $(C, m, S)$), we also first consider the region in $(0, \infty)$ over which the firm
retains earnings. In this region, $D_i(w)$ satisfies for $i = G, B$, $i \neq j$:

$$(r + m)D_i(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D_i(w) - S)]D_i'(w)$$

$$+ \frac{1}{2}((1 - \theta)\sigma)^2 D_i''(w) + C + mS + \pi_i [D_j(w) - D_i(w)].$$

This system of equations is solved subject to the following boundary conditions. First, the firm is liquidated the first time that the cash buffer is depleted in the bad state. The value of short-term debt at this point is equal to the liquidation value of assets:

$$D_B(0) = \min\{\ell, S\} = \ell.$$  

In the good state, management raises new equity up to the target level $W^*_G$ whenever cash reserves are below $W$. Since the net proceeds from the issue are stored in the cash reserve, the value of short-term debt satisfies:

$$D_G(w) = D_G(W^*_G), \quad \text{for } w \leq W.$$  

Lastly, the value of short-term debt does not change when dividends are paid out, because dividend payments accrue to shareholders. We thus have:

$$D'_i(W^*_i) = 0, \quad \text{for } i = G, B.$$  

To fully characterize the value of short-term debt, note that if the state switches from bad to good when $w \in (0, W]$, shareholders raise new funds to reset cash reserves to $W^*_G$ and the value of short-term debt jumps from $D_B(w)$ to $D_G(W^*_G)$. In addition, if the state switches from bad to good when $w \in (W^*_G, W^*_B]$, the firm makes a payment $w - W^*_G$ to shareholders, leading to a jump in the value of debt from $D_B(w)$ to $D_G(W^*_G)$. Therefore, in the region $(0, W] \cup [W^*_G, W^*_B]$, $D_B(w)$ satisfies

$$(r + m)D_B(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D_B(w) - S)]D_B'(w)$$

$$+ \frac{1}{2}((1 - \theta)\sigma)^2 D_B''(w) + C + mS + \pi_B [D_G(W^*_G) - D_B(w)].$$
References


## Table 1: Baseline Parametrization.

### A. Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean cash flow rate</td>
<td>( \mu )</td>
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</tr>
<tr>
<td>Cash flow volatility</td>
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<tr>
<td>Risk-free rate</td>
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</tr>
<tr>
<td>Carry cost of cash</td>
<td>( \lambda )</td>
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</tr>
<tr>
<td>Liquidation cost</td>
<td>( \varphi )</td>
<td>0.45</td>
</tr>
<tr>
<td>Tax rate</td>
<td>( \theta )</td>
<td>0.30</td>
</tr>
<tr>
<td>Coupon on short-term debt</td>
<td>( C )</td>
<td>0.052</td>
</tr>
<tr>
<td>Principal on short-term debt</td>
<td>( S )</td>
<td>1.27</td>
</tr>
<tr>
<td>Average maturity on short-term debt</td>
<td>( M )</td>
<td>1</td>
</tr>
<tr>
<td>Fixed financing cost</td>
<td>( \phi )</td>
<td>0.012</td>
</tr>
<tr>
<td>Switching intensity (good to bad)</td>
<td>( \pi_G )</td>
<td>0.20</td>
</tr>
<tr>
<td>Switching intensity (bad to good)</td>
<td>( \pi_B )</td>
<td>0.60</td>
</tr>
</tbody>
</table>

### B. Implied variables in one-state model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity value</td>
<td>( E(W^*) )</td>
<td>1.196</td>
</tr>
<tr>
<td>Target level of liquid reserves</td>
<td>( W^* )</td>
<td>0.456</td>
</tr>
<tr>
<td>Yield spreads</td>
<td></td>
<td>0.02 – 3004</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td></td>
<td>51.6% – 100%</td>
</tr>
</tbody>
</table>
Table 2: Risk-taking thresholds.

The table reports the inflection points for debt ($W_D$) and equity ($W_E$), the size of the region over which equity value is convex and debt value is concave (agency region, $AR$), and the target cash level ($W^*$) for different debt maturities ($M$), cash flow drift ($\mu$), cash flow volatility ($\sigma$), and liquidation costs ($\varphi$).

<table>
<thead>
<tr>
<th></th>
<th>$W_D$</th>
<th>$W_E$</th>
<th>$AR$</th>
<th>$W^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 1$</td>
<td>0.144</td>
<td>0.183</td>
<td>0.039</td>
<td>0.456</td>
</tr>
<tr>
<td>$M = 3$</td>
<td>0.052</td>
<td>0.085</td>
<td>0.033</td>
<td>0.368</td>
</tr>
<tr>
<td>$M = 5$</td>
<td>0.017</td>
<td>0.046</td>
<td>0.029</td>
<td>0.341</td>
</tr>
<tr>
<td>$M = 10$</td>
<td>n.a.</td>
<td>0.003</td>
<td>0.003</td>
<td>0.322</td>
</tr>
<tr>
<td>$M = \infty$</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.000</td>
<td>0.312</td>
</tr>
<tr>
<td>$\mu = 0.07$</td>
<td>0.358</td>
<td>0.414</td>
<td>0.056</td>
<td>0.709</td>
</tr>
<tr>
<td>$\mu = 0.08$</td>
<td>0.248</td>
<td>0.295</td>
<td>0.047</td>
<td>0.579</td>
</tr>
<tr>
<td>$\mu = 0.09$</td>
<td>0.144</td>
<td>0.183</td>
<td>0.039</td>
<td>0.456</td>
</tr>
<tr>
<td>$\mu = 0.10$</td>
<td>0.050</td>
<td>0.083</td>
<td>0.033</td>
<td>0.343</td>
</tr>
<tr>
<td>$\mu = 0.11$</td>
<td>n.a.</td>
<td>0.011</td>
<td>0.011</td>
<td>0.270</td>
</tr>
<tr>
<td>$\sigma = 0.06$</td>
<td>0.154</td>
<td>0.173</td>
<td>0.019</td>
<td>0.357</td>
</tr>
<tr>
<td>$\sigma = 0.08$</td>
<td>0.144</td>
<td>0.183</td>
<td>0.039</td>
<td>0.456</td>
</tr>
<tr>
<td>$\sigma = 0.10$</td>
<td>0.130</td>
<td>0.194</td>
<td>0.064</td>
<td>0.556</td>
</tr>
<tr>
<td>$\sigma = 0.12$</td>
<td>0.115</td>
<td>0.208</td>
<td>0.093</td>
<td>0.655</td>
</tr>
<tr>
<td>$\sigma = 0.14$</td>
<td>0.097</td>
<td>0.222</td>
<td>0.125</td>
<td>0.752</td>
</tr>
<tr>
<td>$\varphi = 0.30$</td>
<td>n.a.</td>
<td>n.a.</td>
<td>0.000</td>
<td>0.296</td>
</tr>
<tr>
<td>$\varphi = 0.35$</td>
<td>0.008</td>
<td>0.051</td>
<td>0.043</td>
<td>0.331</td>
</tr>
<tr>
<td>$\varphi = 0.40$</td>
<td>0.071</td>
<td>0.112</td>
<td>0.041</td>
<td>0.388</td>
</tr>
<tr>
<td>$\varphi = 0.45$</td>
<td>0.144</td>
<td>0.183</td>
<td>0.039</td>
<td>0.456</td>
</tr>
<tr>
<td>$\varphi = 0.50$</td>
<td>0.220</td>
<td>0.257</td>
<td>0.037</td>
<td>0.526</td>
</tr>
</tbody>
</table>
Table 3: Optimal capital structure.
The table reports the value-maximizing capital structure (coupon, principal, leverage ratio) as well as firm value at debt issuance, under the baseline parametrization and varying the average maturity of corporate debt.

<table>
<thead>
<tr>
<th>Maturity ($M$)</th>
<th>Coupon ($C$)</th>
<th>Principal ($S$)</th>
<th>Leverage ratio</th>
<th>Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At par at $W^*/3$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.036</td>
<td>1.012</td>
<td>51.2%</td>
<td>1.976</td>
</tr>
<tr>
<td>3</td>
<td>0.044</td>
<td>1.138</td>
<td>57.0%</td>
<td>1.996</td>
</tr>
<tr>
<td>5</td>
<td>0.050</td>
<td>1.239</td>
<td>61.6%</td>
<td>2.012</td>
</tr>
<tr>
<td>10</td>
<td>0.056</td>
<td>1.371</td>
<td>67.4%</td>
<td>2.036</td>
</tr>
<tr>
<td>Inf</td>
<td>0.056</td>
<td>1.488</td>
<td>72.6%</td>
<td>2.049</td>
</tr>
<tr>
<td><strong>At par at $W^*/2$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.040</td>
<td>1.123</td>
<td>53.5%</td>
<td>2.100</td>
</tr>
<tr>
<td>3</td>
<td>0.048</td>
<td>1.292</td>
<td>60.3%</td>
<td>2.142</td>
</tr>
<tr>
<td>5</td>
<td>0.054</td>
<td>1.418</td>
<td>65.3%</td>
<td>2.172</td>
</tr>
<tr>
<td>10</td>
<td>0.063</td>
<td>1.619</td>
<td>73.0%</td>
<td>2.219</td>
</tr>
<tr>
<td>Inf</td>
<td>0.062</td>
<td>1.673</td>
<td>75.6%</td>
<td>2.215</td>
</tr>
</tbody>
</table>

Table 4: Financing decisions.
The table reports the target level of cash reserves in good ($W^*_G$) and in bad times ($W^*_B$), the issue threshold ($W$), and the issue size ($W^*_G - W$) for average debt maturities $M$ of 1 year, 5 years, 10 years, and infinite maturity.

<table>
<thead>
<tr>
<th>$M = 1$</th>
<th>$M = 5$</th>
<th>$M = 10$</th>
<th>$M = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^*_G$</td>
<td>0.320</td>
<td>0.245</td>
<td>0.237</td>
</tr>
<tr>
<td>$W^*_B$</td>
<td>0.344</td>
<td>0.269</td>
<td>0.261</td>
</tr>
<tr>
<td>$W$</td>
<td>0.150</td>
<td>0.071</td>
<td>0.063</td>
</tr>
<tr>
<td>$W^*_G - W$</td>
<td>0.170</td>
<td>0.174</td>
<td>0.174</td>
</tr>
</tbody>
</table>
Figure 1: Rollover imbalances.

The figure plots the rollover imbalance $R(w) \equiv m(D(w) - S)$ as a function of cash reserves $w \in [0, W^*]$ for different values of average debt maturity $M$ and for different values of asset profitability $\mu$. 
Figure 2: Value of equity and the Rollover trap.
The figure plots the value of equity (left panel) and the marginal value of cash for shareholders (right panel) as functions of cash reserves $w \in [0, W^*]$, for different debt maturities $M$ (top panel), asset profitability $\mu$ (middle panel), and liquidation costs $\varphi$ (bottom panel).
Figure 3: **Short-term debt and agency conflicts.**

The figure illustrates shareholders’ and debtholders’ risk-taking incentives as a function of cash holdings \([0, W^*]\).
Figure 4: Value of debt.

The figure plots the aggregate value of debt $D(w)$ and the marginal value of cash for the debtholders $D'(w)$ as a function of cash reserves $w \in [0, W^*)$ and for average debt maturities $M$ of 1 year (solid line), 5 years (dashed line), and infinite (dotted line).
Figure 5: Risk-taking.

The figure plots the value of equity $E(w)$ (top panel) and the difference in yield spreads when shareholders do and do not engage in risk-taking strategies (bottom panel) as a function of cash reserves $w \in [0,W^*]$ under different risk-taking strategies and for maturity $M = 1$ (left panel) and $M = 3$ (right panel).
Figure 6: CREDIT LINE.

The figure plots the value of equity, the marginal value of equity, the aggregate value of short-term debt, and the rollover imbalance in the absence (solid line) and in the presence of credit line availability (dashed line for $L = 0.06$ and dotted line for $L = 0.12$).
The figure plots the rollover imbalance $R_i(w)$, the value of equity $E_i(w)$, and the marginal value of cash for shareholders $E'_i(w)$ as a function of cash reserves $w \in [0, W^*_i]$ in the good state (left panel) and in the bad state (right panel) for average debt maturities $M$ of 1 year (solid line), 5 years (dashed line), and infinite (dotted line).
Figure 8: Leland Setup.

The figure plots the value of equity $E(V)$ and its sensitivity to asset value $E'(V)$ when the default threshold is exogenous (top panel) or endogenously chosen to maximize equity value (bottom panel), for average debt maturities $M$ of 1 year (solid line), 5 years (dashed line), and infinite (dotted line).