Short-Term Debt and Incentives for Risk-Taking

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January 12, 2017

Abstract

We challenge the commonly accepted view that short-term debt curbs moral hazard and show that, in a world with financing frictions, short-term debt increases incentives for risk-taking. To demonstrate this result and examine its implications, we formulate a dynamic model in which firms face taxation, financing frictions, and default costs. Using this model, we show that short-term debt amplifies shocks, increases default risk, and can give rise to a rollover trap, a scenario in which firms burn cash to cover severe rollover losses. In the rollover trap, shareholders hold an option that is out-of-the-money, which provides them with risk-taking incentives.

Keywords: Short-term debt financing; rollover risk; risk-taking.
JEL Classification Numbers: G32, G35.

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*We thank Thomas Dangl, Engelbert Dockner, Sebastian Gryglewicz, Julien Hugonnier, Semyon Malamud, Kristian Miltersen, Josef Zechner, and seminar participants at Université Paris Dauphine, the University of St Gallen, and the Vienna University of Economics and Business for comments. Erwan Morellec acknowledges financial support from the Swiss Finance Institute. The views expressed are those of the authors and do not necessarily represent those of the Board of Governors of the Federal Reserve System or its staff.

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1 Introduction

In the frictionless financial markets of Modigliani and Miller (1958), capital structure is irrelevant and debt maturity has no effects on firm value. The insight that market frictions make capital structure decisions and the choice of a debt maturity relevant has spawned a large body of theoretical research, most of which suggests that short-term debt financing exposes firms to refinancing risk and reduces firm value (see e.g. Leland and Toft (1996), He and Xiong (2012a), He and Milbradt (2014), Dangl and Zechner (2016), or DeMarzo and He (2016)). That is, with short-term debt financing, firms face the risk of having to roll over maturing debt at times when debt is more expensive, either because firm fundamentals are weaker or because overall funding liquidity is reduced, leading to an increase in default risk and to a drop in value.

The fact that shorter-term debt increases rollover and bankruptcy risks and generates lower firm value poses the question of why firms issue short-term debt. One answer provided by the finance literature is that short-term debt disciplines management, reducing shareholders’ incentives for risk-taking and the agency costs of asset substitution (see e.g. Barnea, Haugen and Senbet (1980)). Under this premise, an optimal maturity of corporate debt can be derived by balancing the increased refinancing and liquidation costs of shorter-term debt against its reduced agency costs. Contributions in this literature include for example Leland and Toft (1996), Leland (1998), Cheng and Milbradt (2012), or Huberman and Repullo (2015).

1A central result in corporate finance is that, after debt is issued, equity holders hold an option on the firm’s assets and have incentives to increase the riskiness of the firm’s activities (see Jensen and Meckling (1976)). This is presumed to transfer value from debt to equity, creating the asset substitution problem. Eisdorfer (2008) and Favara, Morellec, Schroth, and Valta (2017) provide empirical support for this hypothesis.

2For example, Leland and Toft (1996, pp988) write: “[...] longer term debt also creates greater agency costs by providing incentives for equity holders to increase firm risk through asset substitution. This potential agency cost can be substantially reduced or eliminated by using shorter-term debt.” Cheng and Milbradt (2011, pp1072) write: “Although short-term debt can lead to freezes, it mitigates
In this paper, we challenge the commonly accepted view that short-term debt curbs moral hazard and show that, in a world with financing frictions, short-term debt does not decrease but instead increases incentives for risk-taking. To demonstrate this result and examine its implications for corporate policies and default risk, we formulate a dynamic model in which firms face taxation, time-varying financing frictions, and default costs. In this model, firms are financed with equity and short-term debt. They operate risky assets and have the option to invest in risk-free, liquid assets such as cash reserves. Firms maximize shareholder value by choosing their buffers of liquid assets as well as their payout, financing, risk management, and default policies. In the absence of short-term debt financing, taxation, and time variation in financing frictions, our model thus collapses to that in Décamps, Mariotti, Rochet, and Villeneuve (2011).

As in Leland (1998), He and Xiong (2012a), and much of the literature on rollover risk, we consider that when a short-term bond matures, the firm issues a new bond with the same face value, coupon rate, and maturity at market price, which can be higher or lower than the principal of the maturing bond. Short-term debt financing therefore exposes the firm to rollover risk and to rollover losses.³ To avoid default, shareholders need to absorb these rollover losses. A fundamental difference between our work and prior contributions on short-term debt and rollover risk is that we do not assume that outside equity can be issued at no cost. Rather, we consider that firms face financing frictions, which may lead to forced, inefficient liquidations. This in turn provides incentives for the firm to build up liquidity buffers that can effectively be used to absorb operating or rollover losses and reduce refinancing costs and default risk.

A first result of our paper is to show that short-term debt creates an amplification mechanism that increases the exposure of firms to cash flow shocks. Notably, we show the risk-shifting problem by imposing a punishment in the form of liquidation. [...] Our first result is thus that debt maturity should be just short enough to eliminate preemptive risk-shifting.”

³In that respect, our paper is also related to the early studies of Diamond (1991) and Flannery (1986, 1994), in which short-term debt can be repriced given interim news. A central difference with these papers however is that we show that this does not lead short-term debt to discipline shareholders.
that, as in prior models with financing frictions, negative cash flow shocks directly reduce liquid reserves, because the firm uses these reserves to absorb negative shocks. The novelty of our framework is that these negative shocks also bring along an indirect effect due to debt rollover. As the firm draws down its cash reserves to cover operating losses, default risk increases. This leads to a drop in the price of newly issued debt and to an increase in rollover losses. Rollover losses therefore compound operating losses, draining the firm’s liquid reserves and pushing the firm closer to default. Because firms issuing debt with shorter maturity need to roll over a larger fraction of their debt, this amplification mechanism is more important for firms issuing debt with shorter maturity, implying that default risk decreases with debt maturity.

A second and key result of our model is to show that, in the presence of financing frictions, short-term debt provides incentives for shareholders to increase the riskiness of assets. Notably, when firms are close to distress and debt maturity is short enough, rollover losses can be larger than expected net income, turning expected cash flows to shareholders from positive to negative. We call this scenario, in which the firm “burns” cash and cash flows to shareholders are negative because of severe rollover losses, *the rollover trap*. We show that when the firm is in *the rollover trap*, shareholders hold an option that is out-of-the-money and want to turn their expected cash flows from negative to positive, which provides them with incentives to increase risk. These risk-shifting incentives disappear as debt maturity increases and do not arise when debt maturity is infinite (as in, e.g., Bolton, Chen, and Wang (2015) or Hugonnier and Morellec (2016)) or when firms are all-equity-financed (as in e.g. Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011), or Décamps, Gryglewicz, Morellec, and Villeneuve (2016)). Indeed, when debt maturity is sufficiently long (or in the absence of debt financing), rollover losses are small (or absent) and *expected* cash flows to shareholders are always positive. In such environments, the main effect of financing frictions is to expose shareholders to the risk of a forced, inefficient liquidation, which leads them to behave in a risk-averse fashion to preserve equity value.
Our result that short-term debt increases risk-taking incentives does not arise in models of rollover risk in which shareholders have deep pockets and can optimally choose the timing of default (such as Leland and Toft (1996) or Leland (1998)). In these models, equity value is a convex function of asset value and short-term debt acts as a disciplinary device because it is less sensitive to changes in asset volatility than long-term debt. As a result, shareholders cannot shift as much value from short-term debt to equity.

Importantly, and as shown by Toft and Prucyk (1997, Figure 1), equity value can become a concave function of asset value in Leland-type models when firms cannot freely choose their default policy, either because debt contracts include protective covenants—like net-worth covenants—or because firms face leverage requirements—like in banking regulation. In such instances, shareholders have no risk-shifting incentives, even when firms are financed with infinite maturity debt. Our paper shows that shareholders in firms financed with long-term (or infinite maturity) debt also behave in a risk-averse fashion when facing financing frictions because financing frictions, like bond covenants or regulatory constraints, introduce the risk of inefficient liquidations. However, with short-term debt outstanding, shareholders in firms that are fundamentally solvent can experience a quick drop in cash flows because negative operating shocks are amplified by rollover losses. In such instances, short-term debt financing may provide shareholders with incentives to increase asset volatility when close to distress.

A third result of the paper is to show that these risk-taking or “gambling for resurrection” strategies can be value-enhancing for both debtholders and shareholders. Indeed, while making negative cash flow shocks more harmful, increasing asset volatility also makes it more likely that positive cash flow shocks will allow the firm to escape the rollover trap before it runs out of funds and is forced into inefficient liquidation. Nonethe-

4This is also the case in the Black and Scholes (1973) model, in which maximum leverage ratio or minimum interest coverage ratio requirements imply that equity is akin to a down-and-out call option on the firm’s assets (see e.g. Black and Cox (1976)). In this case, shareholders do not have incentives to shift risk when firms fundamental worsen and asset value approaches the “knock-out” barrier corresponding to the protective covenant or regulatory requirement (see Derman and Kani (1996)).
less, because shareholders capture all returns above those required to service debt and therefore benefit disproportionately from risk-taking, a conflict between debtholders and shareholders can still exist. That is, incentive compatibility is only restored at the very brink of distress, where both bondholders and shareholders want to increase risk to avoid a forced liquidation.

We also consider in the model the possibility for the firm to acquire additional financial flexibility via the use of a credit line. We show that when credit lines are senior to market debt (as is typically the case), rollover losses are larger when the firm approaches distress, which strengthens the amplification mechanism described above and, therefore, shareholders’ incentives for risk-taking. That is, we find that when short-term debt is subordinated to other claims (as is the case in banks for example where deposits are usually senior), shareholders have stronger risk-shifting incentives.

Lastly, we show that with short-term debt financing, cash holdings decisions reflect the sign and the magnitude of rollover imbalances. In turn, rollover imbalances depend on the firm’s financial strength that depends on cash savings. Through this feedback loop, a trade-off emerges in the choice of cash reserves. On the one hand, a shorter debt maturity imposes larger rollover losses when fundamentals weaken, inducing the firm to hold larger cash reserves. This is the customary precautionary motive. On the other hand, a shorter debt maturity boosts rollover gains when fundamentals improve, creating an incentive to reduce cash reserves. This is what we call the speculative motive. Because of this speculative motive, shareholders may choose to expose the firm to rollover risk in good times, when the firm faces rollover gains. A direct implication of this result is that liquidity buffers should be counter-cyclical, consistent with the available evidence (see Acharya, Shin, and Yorulmazer (2010) or Aspachs, Nier, Tiesset (2005)).

Our paper relates to the growing literature that examines the relation between short-term debt financing and credit risk in dynamic structural models with roll-over debt structure. Starting with Leland (1994b, 1998), these models show that short-term debt generally leads to an increase in default risk via rollover losses. Many of these studies
show that this effect can be magnified by other frictions; see for example Hilberink and Rogers (2002), Eom, Helwege, and Huang (2004), Ericsson and Renault (2006), Hackbarth, Miao, and Morellec (2006), He and Xiong (2012a,b), Schroth, Suarez, and Taylor (2014), He and Milbradt (2014), Chen, Xu, and Yang (2015), Chen, Cui, He, and Milbradt (2016), Dangl and Zechner (2016), or DeMarzo and He (2016).

All of these models assume that shareholders have deep pockets and can inject liquidity in the firm at no cost (i.e. there are no financing frictions), or just do not allow firms to hoard precautionary cash reserves. In our model, firms face financing frictions and optimally retain part of their earnings to build up liquid reserves that they can use to absorb rollover losses. Consistent with this modeling, Harford, Klasa, and Maxwell (2014) document that refinancing risk represents a key motivation for why non-financial firms hoard cash on their balance-sheets. Another important difference between our paper and prior work is that, in prior work, moral hazard is reduced by short-term debt financing, which seems at odds with the evidence of Graham and Harvey (2001) who find that “few executives feel that short-term debt borrowing reduces the chance that shareholders will want to take on risky projects.” Instead, short-term debt financing exacerbates incentives for risk-taking in our model.

Our work is also related to the recent papers that incorporate financing frictions into dynamic models of corporate financial decisions. These include Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011, 2013), Hartman-Glaser and Milbradt (2014), Hugonnier, Malamud and Morellec (2015), or Décamps, Gryglewicz, Morellec, and Villeneuve (2016). In this literature, it is generally assumed that firms are all-equity financed. Notable exceptions are Gryglewicz (2011), Bolton, Chen, and Wang (2014), and Hugonnier and Morellec (2016), in which firms and/or financial institutions are financed with equity and long-term (infinite maturity) debt. As discussed above, in these models firms have good fundamentals and financing frictions introduce the risk of forced liquidations, leading shareholders to behave as if they were risk-averse to preserve equity value. That is, convexity in equity value and risk-taking incentives do not arise
in these models. Our paper advances this literature by characterizing the interaction between debt maturity and corporate policies and by showing that short-term debt can encourage risk taking when firms are close to financial distress.

The paper is organized as follows. Section 2 presents the model. Section 3 derives our main results on the effects of short-term debt on risk-taking and discusses the key implications of the model. Section 4 introduces time-varying financing frictions and demonstrates the generality of our results. Section 5 concludes.

2 Model and assumptions

Throughout the paper, agents are risk-neutral and discount cash flows at a constant rate \( r > 0 \). Time is continuous and uncertainty is modeled by a probability space \((\Omega, \mathcal{F}, \mathbb{P})\), with the filtration \( \mathcal{F} = \{\mathcal{F}_t : t \geq 0\} \) satisfying the usual conditions. The subject of study is a firm held by shareholders that have limited liability. As in He and Xiong (2012a), one may interpret this firm as any firm, either financial or non-financial. However, our model is perhaps more appealing for financial firms because of their high leverage and heavy reliance on short-term debt financing.\(^5\)

Specifically, we consider a firm that owns a portfolio (or operates a set) of risky, illiquid assets as well as cash reserves and is financed with equity and short-term debt. The firm’s risky assets generate after-tax cash flows given by \( dY_t \) and governed by the process:

\[
dY_t = (1 - \theta) (\mu dt + \sigma dZ_t),
\]

where \( \mu \) and \( \sigma \) are positive constants representing respectively the mean and the volatility of pre-tax cash flows from risky investments, \((Z_t)_{t \geq 0}\) is a standard Brownian motion representing random shocks to these cash flows, and \( \theta \in (0, 1) \) is the corporate tax.

\(^5\)A number of intermediaries, such as insurance companies, hedge funds, brokers and dealers, and government-sponsored enterprises like Fannie Mae and Freddie Mac, do not take deposits directly from households, but in many ways behave like banks in debt markets (see Krishnamurthy (2010)).
rate. Equation (1) implies that over any time interval \((t, t + dt)\), the after-tax cash flows from risky assets are normally distributed with mean \((1 - \theta)\mu dt\) and volatility \((1 - \theta)\sigma \sqrt{dt}\). This in turn implies that the firm can make profits as well as losses. This cash flow specification is similar to that used for example in DeMarzo and Sannikov (2008), Bolton, Chen, and Wang (2011), DeMarzo, Fishman, He, and Wang (2012), or Hugonnier, Malamud, and Morellec (2015).

Because it pays taxes on corporate income and interest payments are tax deductible, the firm has an incentive to issue debt. As in Leland (1998), Hackbarth, Miao, and Morellec (2006), or He and Xiong (2012b), we consider finite-maturity debt structures in a stationary environment. Notably, we assume that the firm has issued debt with constant principal \(S\) and paying a constant total coupon \(C\). At each moment in time, the firm instantaneously rolls over a fraction \(m\) of its total debt. That is, the firm continuously retires outstanding debt principal at a rate \(mS\) and replaces it with new debt vintages of identical coupon, principal, and seniority. In the absence of bankruptcy, the average debt maturity equals \(M \equiv 1/m\). In the following, we assume that \(C < \mu\) to ensure that a firm issuing infinite maturity debt is solvent.

Management acts in the best interest of shareholders and chooses not only the firm’s financing policy but also its payout and default/liquidation policies. Notably, we allow management to retain earnings inside the firm and denote by \(W_t\) the firm’s cash/liquid reserves at any time \(t \geq 0\). Liquid reserves earn a rate of interest \(r - \lambda\) and can be used to cover operating or rollover losses if other sources of funds are costly or unavailable. The wedge \(\lambda > 0\) represents a carry cost of liquidity. When choosing its target level of cash reserves, the firm balances this carry cost with the benefits of liquidity.

The firm can increase its cash reserves either by retaining earnings or by raising funds

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6The cost of holding cash includes the lower rate of return on these assets because of a liquidity premium and tax disadvantages (Graham (2000) finds that cash retentions are tax-disadvantaged because corporate tax rates generally exceed tax rates on interest income). This cost of carrying cash may also be related to a free cash flow problem within the firm, as in Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011), or Hugonnier, Malamud, and Morellec (2015).
in the capital markets. As in Bolton, Chen, and Wang (2013), we consider that the
firm operates in an environment characterized by time-varying financing opportunities.
Specifically, we assume that the firm can be in one of two observable states of the world,
that we denote respectively by $i = G, B$. In the good state $G$, the firm can raise funds
at any time by incurring a fixed cost $\phi > 0$. In the bad state $B$, the firm has no access
to outside funds or, equivalently, funding costs are too high. We consider that the state
switches from $G$ to $B$ (resp. from $B$ to $G$) with probability $\pi_G dt$ (res. $\pi_B dt$) on any
time interval $(t, t + dt)$, implying that the long-run probability of being in state $G$ is
$\frac{\pi_G}{\pi_G + \pi_B}$. As we show below, these financing frictions provide incentives for the firm to
retain earnings and to build up liquid reserves.

We denote by $D_i(w; C, m, S)$ the market value of outstanding short-term debt in
state $i = G, B$ for a level of cash reserves $w$. Debt rollover implies that short-term debt
of a new vintage is issued at market price and has principal value and coupon payment
given by $mS$ and $mC$, respectively. The market value of newly issued debt – which
represents a firm inflow – may differ from the principal repayment $mS$ of debt coming
to maturity – that in turn represents an outflow to the firm. When the market value of
newly issued debt is lower than the principal, the firm bears rollover losses. Otherwise,
it enjoys rollover gains. Over any time interval $(t, t + dt)$, the rollover gains or losses are
given by $m(D_i(w; C, m, S) - S)dt$ and the dynamics of cash reserves satisfy
\begin{equation}
    dW_t = (1 - \theta)[(r - \lambda)W_t dt + (\mu - C)dt + \sigma dZ_t] \\
    + m(D_i(W_t; C, m, S) - S) dt - dP_t + dH_t - dX_t.
\end{equation}

In equation (2), $P_t$, $H_t$, and $X_t$ are non-decreasing adapted processes representing
respectively the cumulative dividends paid to shareholders, the firm’s cumulative external
financing, and the firm’s cumulative issuance costs until time $t$. This equation
shows that liquid reserves grow with earnings net of taxes, with outside financing, with
rollover gains, and with the interest earned on cash holdings. Liquid reserves decrease
with payouts to shareholders, with the coupon paid on outstanding debt, with the cost
of outside funds, and with rollover losses. In this equation, the rollover gains and losses
and the firm’s payout and financing decisions depend on the state of the world and are endogenously determined in the following.

The firm can be forced into liquidation if its cash reserves reach zero following a series of negative shocks and it is not possible/optimal to raise outside funds. We consider that the liquidation value of risky assets represents a fraction of their first best value and is given by

\[ \ell \equiv (1 - \varphi) \frac{(1 - \theta)\mu}{r}, \]

where \( \varphi \in [0,1] \) represents a haircut related to default costs. We denote by \( \tau \) the stochastic default time of the firm. If \( \tau = \infty \), the firm never chooses to liquidate.

Following Leland (1998), Hackbarth, Miao, and Morellec (2006), and He and Xiong (2012a), we consider that the firm can commit to a stationary debt structure \((C, m, S)\). Given this debt structure, management chooses the firm’s payout \((P)\), financing \((H)\), and default \((\tau)\) policies to maximize the present value of future dividends to shareholders. That is, given \((C, m, S)\), management solves:

\[
E_i(w; C, m, S) \equiv \sup_{(P,H,\tau)} \mathbb{E}_w,i \left[ \int_0^\tau e^{-rt} (dP_t - dH_t) + e^{-r\tau}(\ell + W_\tau - S)^+ \right], \quad (3)
\]

where \( x^+ = \max\{0; x\} \). The first term on the right-hand side of equation (3) represents the flow of dividends accruing to incumbent shareholders, net of the claim of new shareholders on future cash flows. The second term represents the present value of the cash flow to shareholders in default. In the following, we focus on the case in which the liquidation value of assets is lower than the face value of outstanding short-term debt, i.e. \( \ell < S \). We will show that since \( W_\tau = 0 \) in default, this implies that short-term debt is risky. Also, in most of our analysis we take the debt structure \((C, m, S)\) as given. We discuss the initial debt structure choice (maturity and leverage) in Section 3.5.

3 The rollover trap: Short-term debt and risk-taking

In the model, management chooses the firm’s dividend, financing, and savings policies to maximize shareholder value. Because creditors have rational expectations, the price
at which maturing short-term debt is rolled over reflects these policy choices and feeds back into the value of equity by determining the magnitude of rollover imbalances. The policy choices of the firm and the value of equity and short-term debt are therefore the solution to a fixed point problem.

To aid in the intuition of the model, we start by examining an environment in which the firm raises new funds only by rolling over short-term debt and does not have access to the equity market. This is the case when the bad state is absorbing or when the cost of equity financing is too high. Since there is only one regime, we omit the subscript $i$. In section 4, we will build on the results of this section to analyze a more general model in which the firm faces time-varying financing conditions.

### 3.1 Valuing corporate securities

We start our analysis by deriving the value of equity. In our model, financing frictions lead the firm to value inside equity and, therefore, to retain earnings. Keeping cash inside the firm, however, entails an opportunity cost $\lambda$ on any dollar saved. Because for sufficiently large values of cash reserves the benefit of an additional dollar saved is decreasing in the firm’s cash reserves and the marginal cost of holding cash is constant, we conjecture that there exists some target level $W^*$ for cash reserves where the marginal cost and benefit of cash reserves are equal and it is optimal to start paying dividends.

To solve for equity value, we first consider the region in $(0, \infty)$ over which it is optimal for firm shareholders to retain earnings. In this region, the firm does not deliver any cash flow to shareholders and equity value satisfies (where we omit the arguments $(C, m, S)$):

$$rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 E''(w). \quad (4)$$

The left-hand side of this equation represents the required rate of return for investing in the firm’s equity. The right-hand side is the expected change in equity value in the region where the firm retains earnings. The first term on this right-hand side captures the effects of cash savings and reflects debt rollover. That is, one important aspect of
this equation is that the value of short-term debt feeds back into the value of equity via rollover imbalances. The second term captures the effects of cash flow volatility.

Equation (4) is solved subject to the following boundary conditions. First, when cash reserves exceed $W^*$, the firm places no premium on internal funds and it is optimal to make a lump sum payment $w - W^*$ to shareholders. As a result, we have

$$E(w) = E(W^*) + w - W^*$$

for all $w \geq W^*$. Subtracting $E(W^*)$ from both sides of this equation, dividing by $w - W^*$, and taking the limit as $w$ tends to $W^*$ yields the condition:

$$E'(W^*) = 1.$$  

The equity-value-maximizing payout threshold $W^*$ is then the solution to the high-contact condition (see Dumas (1991)):

$$E''(W^*) = 0.$$  

When the firm makes losses, its cash buffer decreases. If its cash buffer decreases sufficiently, the firm may be forced to raise new equity or to liquidate. When the firm has no access to outside funding, it defaults as soon as its liquid reserves are depleted. As a result, the condition

$$E(0) = \max\{\ell - S; 0\} = 0$$

holds at zero and the liquidation proceeds are used to (partially) repay debtholders.

Consider next the value of short-term debt. Denote by $D^0 (w; C, m, S, t)$ the date–$t$ value of short-term debt issued at time 0. Since a fraction $m$ of this original debt is retired continuously, these original debtholders receive a payment rate $e^{-mt} (C + mS)$ at any time $t \geq 0$ as long as the firm is solvent. Now define the value of total outstanding debt by $D (w; C, m, S) \equiv e^{mt} D^0 (w; C, m, S, t)$. Because $D (w; C, m, S)$ receives a constant payment rate $C + mS$, it is independent of $t$. In the following, we only derive the
function $D(w; C, m, S)$, i.e. the value of total short-term debt. From this value, we can also derive the value of newly issued short-term debt, denoted by $d(w; C, m, S, 0)$. In the Appendix, we show that it satisfies: $d(w; C, m, S, 0) = mD(w; C, m, S)$.

To solve for the value of total short-term debt $D(w)$ (where we again omit the arguments $(C, m, S)$), we first consider the region in $(0, \infty)$ over which the firm retains earnings. In this region, the value of total short-term debt evolve as:

$$(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w)$$

$$+ \frac{1}{2} ((1 - \theta) \sigma)^2 D''(w) + C + mS$$

The left-hand side of equation (5) is the return required by short-term debtholders. The right-hand side represents the expected change in the value of total short-term debt. The first and second terms capture the effect of a change in cash reserves and in cash flow volatility. The third and fourth terms are the coupon and principal payments to short-term debtholders.

This equation is solved subject to the following boundary conditions. First, the firm is liquidated the first time that the cash buffer is depleted. The value of short-term debt at this point is equal to the liquidation value of assets:

$$D(0) = \min\{\ell, S\} = \ell.$$ Second, the value of short-term debt does not change when dividends are paid out, because dividend payments accrue exclusively to shareholders. We thus have:

$$D'(W^*) = 0.$$ 3.2 Amplification of shocks due to short-term debt

When a firm is financed with short-term debt (i.e. $m > 0$), it needs to roll over maturing debt, which may lead to rollover imbalances. Over each time interval of length $dt$, rollover imbalances are given by

$$R(w; C, m, S) \equiv m(D(w; C, m, S) - S),$$
and depend on the firm’s cash reserves, debt maturity, and leverage. Since default risk decreases with cash reserves \( w \), the value of debt is monotonically increasing in \( w \) in the earnings retention region (see Section 3.3). As a result, there exists at most one threshold \( \bar{W} \) at which the rollover imbalance is zero, i.e. such that:

\[
D(\bar{W}; C, m, S) = S.
\]

The firm bears rollover losses for any \( w < \bar{W} \), as the inequality \( D(w; C, m, S) < S \) holds. That is, lower cash reserves are associated with higher default risk, which reduces the value of newly-issued debt. As a result, the proceeds from newly issued debt are not sufficient to cover the principal repayment of maturing debt, and cash reserves are used to absorb the rollover losses. Conversely, for any \( w \in (\bar{W}, \bar{W}^*) \], the firm is financially strong and default risk is low. The proceeds from newly issued debt exceed the principal repayment of maturing debt and increase the firm’s cash reserves.

These rollover imbalances amplify operating shocks since total firm cash flows net of debt payouts are given by

\[
dX_t(w) \equiv (1 - \theta)[((r - \lambda)w + \mu - C)dt + \sigma dZ_t] + m(D(w) - S)dt,
\]

over each time interval of length \( dt \). In particular, rollover losses exacerbate bad times. In addition, and as shown by equation (6), this amplification effect is stronger when a firm issues debt with shorter maturity, i.e. when \( m \) is larger. To better understand this mechanism, consider the effects of a negative operating shock when cash reserves are low, in that \( w < \bar{W} \). As in prior models with financing frictions (see e.g. Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011)), this negative shock directly reduces cash reserves, because the firm uses cash to absorb it. The novelty of our framework is that this shock also brings along an indirect effect due to debt rollover. As the firm draws down its cash reserves to cover the loss, default risk increases. This leads to a drop in the price of newly issued debt and to a rollover loss. The financial loss amplifies the impact of the operating loss by further reducing cash reserves.

Insert Table 1 Here
Figure 1 plots the firm’s rollover imbalances as a function of cash reserves. The baseline values of the model parameters are reported in Table 1. We set the risk-free rate of return to $r = 3.5\%$, the corporate tax rate to $\theta = 0.3$, the mean cash flow rate to $\mu = 0.09$, and the carry cost of cash to $\lambda = 0.01$. We base the volatility of cash flows on the estimates of Sundaresan and Wang (2015) and set $\sigma = 0.08$. We base the value of liquidation costs on the estimates of Glover (2016) and set $\phi = 45\%$. Given these input parameter values, the liquidation value of assets is equal to $\ell = 0.99$. The coupon rate $C$ is set to 0.052. The face value $S = 1.27$ is uniquely determined by requiring that debt is issued at par when at the median level of cash reserves $W^*/2$ for $M = 1$. This face value implies a recovery rate of 78% in default (i.e. $\frac{\ell}{S} = 0.78$).

Figure 1 shows that the firm bears rollover losses when cash reserves are low and enjoys rollover gains when cash reserves are large. The figure shows that the effect is markedly asymmetric in that the deterioration in cash reserves caused by rollover losses is more important than the increase in rollover gains, due to the concavity of the debt value function in the rollover gains region. At the target cash level, positive operating shocks are paid out to shareholders, and debt value is insensitive to these shocks (i.e. $D'(W^*) = 0$). Lastly, the figure shows that amplification is stronger when debt maturity is shorter, because shorter debt maturity brings along larger rollover imbalances.

3.3 The “rollover trap”: Short-term debt and convexity

As we show next, an important effect of short-term debt and rollover imbalances is that they lead to convexity in equity value and to risk-taking incentives when firms face financing frictions. To understand this effect, consider a counterfactual firm financed with equity and infinite maturity debt (as in e.g. Leland (1994a), Bolton, Chen, and Wang (2015), or Hugonnier and Morellec (2016)). Since this firm does not need to roll
over debt, its equity value $E_\infty(w)$ satisfies

$$rE_\infty(w) = (1 - \theta) [(r - \lambda)w + \mu - C] E'_\infty(w) + \frac{1}{2} ((1 - \theta) \sigma)^2 E''_\infty(w),$$

in the earnings retention region. This equation is solved subject to $E_\infty(0) = E'_\infty(W^*_\infty) - 1 = E''_\infty(W^*_\infty) = 0$, where $W^*_\infty$ is the optimal payout trigger for shareholders. The value of risky, infinite-maturity debt in turn satisfies in the earnings retention region:

$$rD_\infty(w) = (1 - \theta) [(r - \lambda)w + \mu - C] D'_\infty(w) + \frac{1}{2} ((1 - \theta) \sigma)^2 D''_\infty(w) + C,$$

which is solved subject to $D_\infty(0) - \ell = D'_\infty(W^*_\infty) = 0$.

Three important features differentiate this firm from a firm financed with short-term debt. First, its expected net cash flows to shareholders, given by

$$(1 - \theta) [(r - \lambda)w + \mu - C] dt > 0,$$

are time-invariant and positive because $\mu > C$. Second, while the value of debt reflects the equity-maximizing dividend and saving policies ($W^*_\infty$ enters the debt’s boundary conditions), the value of debt does not feed back into the value of equity because it does not need to be rolled over. Third, because debt is not rolled over, there is no amplification of operating shocks due to the repricing of short-term debt. When debt maturity is infinite, we have the standard result in cash management models that firms facing financing frictions behave in a risk-averse fashion even if shareholders are risk-neutral. The reason is that shareholders want to avoid inefficient liquidation (or save on refinancing costs in the model with time-varying costs of section 4) and have no incentive to engage in risk-shifting, even when the firm is levered.

Our model shows that a risk-loving behavior can arise in dynamic models with financing frictions due to debt rollover. The reason is that as the firm approaches financial distress, the price of newly-issued debt decreases and rollover losses increase. As a result, when the firm is sufficiently close to distress, expected net cash flows to shareholders can become negative, i.e. we can have:

$$ (1 - \theta) [(r - \lambda)w + \mu - C] + m(D(w) - S) \leq 0. \tag{7} $$
Since the value of equity is non-decreasing in cash reserves, so that $E'(w) \geq 0$, it must be that the first term on the right-hand side of equation (4) is non-positive whenever (7) holds. This also implies that for the value of equity $E(w)$ to be non-negative due to limited liability, the equity value function must be convex, i.e. we must have $E''(w) \geq 0$. This leads to the following proposition:

**Proposition 1 (Short-term debt and incentives for risk-taking)** When a firm is financed with short-term debt, equity value is locally convex when rollover losses are sufficiently large for inequality (7) to hold. In such instances, short-term debt financing leads to a risk-loving behavior for shareholders.

A direct implication of Proposition 1 is that, with financing frictions and short-term debt, shareholders in a solvent firm are risk-loving if expected cash flows to equity are negative. The reason is the following. As long as inequality (7) holds, rollover losses are larger than net income and the value of an additional unit of cash to shareholders is low because it plays a minor role in helping the firm escape financial distress. Indeed, that unit of cash will be used to repay maturing debt and not to rebuild cash reserves. In expectation, the firm keeps on making rollover losses, further reducing its cash reserves and increasing default risk. In such instances, shareholders hold an option that is out of the money and want to turn their expected cash flows from negative to positive, which provides them with incentives to increase risk. Moreover, the level of cash reserves at which (7) holds as an equality is lower than the inflection point that separates the convex and concave regions. That is, the firm can be preemptively risk-loving when expected net cash flows are positive, to avoid entering the region in which expected cash flows are negative. Lastly, risk-shifting incentives decrease as debt maturity increases (because of lower rollover losses) and do not arise with infinite maturity debt.

We call this scenario, in which the firm “burns” cash and cash flows to shareholders are negative because of severe rollover losses, “the rollover trap.” When a firm is in the rollover trap, the marginal value of cash progressively increases as the firm approaches the break-even point at which (7) is binding. In this region, the value of equity is convex.
and shareholders have incentives to increase asset risk, as we show in the next section. The marginal value of cash to shareholders only starts decreasing (and equity value becomes concave) when expected cash flows become sufficiently large to guarantee that an additional unit of cash helps increase cash reserves rather than cover rollover losses.

Figure 2 plots the value of equity $E(w)$ and the marginal value of cash to shareholders $E'(w)$ as functions of the firm’s cash reserves for $w \in [0, W^*)$ for three different debt maturities. As in previous dynamic models with financing frictions, the value of equity is increasing in cash reserves. However, Figure 2 also shows that the relation between value of equity, debt maturity, and cash reserves is non-trivial and reflects the amplification mechanism generated by debt rollover. A short maturity depresses (increases) the value of equity when cash reserves are small (large) due to rollover losses (gains). Equity value is concave and shareholders are quasi risk-averse for any $w$ for long debt maturities. Equity value can be locally convex if debt maturity $M = \frac{1}{m}$ is sufficiently short.

To understand when short-term debt is more likely to induce a risk-loving behavior, Figure 2 also plots the value of equity $E(w)$ and the marginal value of equity $E'(w)$ as functions of cash reserves for varying cash flow volatilities $\sigma$ and bankruptcy costs $\varphi$. The figure shows that two dimensions of the rollover trap need to be taken into account: The extension, i.e. how large the region of convexity is, and the depth, i.e. how steep equity value is in this region. For given debt characteristics $(C, m, S)$, an increase in cash flow volatility increases the extension of the trap but decreases the depth. This property is important because, if the depth of the trap increases, negative shocks drag the firm into default more quickly. Therefore, risk-taking can effectively help the firm escape the rollover trap by trading-off the increase in the size of the trap against its depth, as shown by the second panel of Figure 2. The third panel of the figure also shows that a decrease in asset tangibility mainly increases the size of the trap. A lower recovery rate makes debt more risky and increases rollover losses, which in turn widens the rollover trap and makes it more likely that risk-taking increases equity value.
Our result that short-term debt is associated with larger risk-shifting incentives does not arise in models of rollover risk in which shareholders have deep pockets and can optimally choose the timing of default (such as Leland and Toft (1996) or Leland (1998)). In these models, equity value is a convex function of asset value and short-term debt acts as a disciplinary device because it is less sensitive to changes in asset volatility than long-term debt. Thus, shareholders are not able to shift value from debt to equity by increasing asset volatility when debt maturity is too short. As shown by Toft and Prucyk (1997, Figure 1), equity value can become concave in these models when the possibility of inefficient liquidation is introduced (e.g. via leverage requirements or protective debt covenants), thereby eliminating risk-shifting incentives. Our paper shows that shareholders in firms financed with long-term debt also behave in a risk-averse fashion when facing financing frictions, because financing frictions like bond covenants or regulatory constraints introduce the risk of inefficient liquidation. However, with short-term debt outstanding, shareholders in firms that are fundamentally solvent can experience a quick drop in cash flows because negative operating shocks are amplified by rollover losses. In such instances, short-term debt financing may provide shareholders with incentives to increase asset volatility when close to distress.

As we show next, the rollover trap can generate convexity in the value of short-term debt too. The dynamics of the value of debt in the earnings retention region are given by equation (5). Now, consider a firm for which inequality (7) holds. This inequality is necessary but not sufficient for convexity in debt value to arise. Indeed, while the value of debt increases in $w$ in that $D'(w) \geq 0$, equation (5) shows that debtholders receive the periodic payments $C + mS > 0$ when the firm is in the earnings retention region. These periodic payments imply that the level of cash reserves that separates the convexity and concavity regions is not the same for equityholders and debtholders. In particular, the region of convexity in debt value is smaller than the region of convexity in equity value (or may not exist). As a result, an incentive compatibility problem exists for the range of cash reserves for which $\frac{\partial E}{\partial \sigma} > 0$ and $\frac{\partial D}{\partial \sigma} < 0$. This leads to the following result.
Proposition 2 (Convexity in short-term debt value)  Whenever rollover losses are sufficiently large, the value of debt can be locally convex. In these instances, both debtholders and shareholders are risk-loving.

Figure 3 plots the value of debt $D(w)$ and the marginal value of cash to debtholders $D'(w)$ as functions of cash reserves for different debt maturities. $D(w)$ increases with maturity as a shortening of maturity implies an increase in rollover losses and, thus, in default risk. In addition, while debtholders suffer from the risk implied by a shorter debt maturity due to larger rollover losses, they do not capture the upside potential due to any rollover gains. Lastly, Figure 3 also shows that the convexity is less pronounced for debtholders than for equityholders, leading to an incentive compatibility problem.

3.4 Short-term-debt-induced risk-taking

We have just shown that, in a world with financing frictions, short-term debt financing generates a local convexity in the value of equity. In this section, we show that this convexity due to short-term debt financing may lead to a risk-shifting behavior, in which the firm engages in zero NPV investments with random returns in an attempt to improve equity value. That is, we show that financing frictions imply a behavior that is in sharp contrast with the long-standing idea that short-term debt has a disciplinary role and reduces the agency costs of asset substitution (see e.g. Barnea, Haugen, and Senbet (1980), Leland and Toft (1996), Leland (1998), or Cheng and Milbradt (2012)).

To analyze the firm’s incentives to increase asset risk, we follow Bolton, Chen, and Wang (2011), Hugonnier, Malamud, and Morellec (2015), and Décamps, Gryglewicz, Morellec, and Villeneuve (2016) and assume that the firm has access to a futures contract whose price is a Brownian motion $B_t$, uncorrelated with the Brownian motion $Z_t$ driving the firm cash flows. A position $\gamma_t$ in the futures contract thus implies that the firm cash flows change from $dY_t$ to $dY_t + (1 - \theta)\gamma_t dB_t$. Hedging positions are generally constrained
by margin requirements. To capture these requirements, we consider that the firm’s futures position \( \gamma_t \) cannot exceed some fixed size (or collateral constraint) \( \Gamma \) and study the effects of varying \( \Gamma \) on optimal policies and equity value.

Assuming frictionless trading in the futures contract, standard arguments show that in the region over which the firm retains earnings, equity value satisfies:

\[
rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w)
+ \max_{0 \leq \gamma \leq \Gamma} \left\{ \frac{1}{2}(1 - \theta)^2 \left( \sigma^2 + \gamma^2 \right) E''(w) \right\},
\]

where the last term on the right hand side captures the effects of risk-taking on equity value. By differentiating with respect to \( \gamma \), we can determine the optimal risk-taking strategy. This leads to the following Proposition.

**Proposition 3 (Optimal risk-taking)** For all \( w \) such that \( E''(w) > 0 \), shareholders are risk-loving and find it optimal to increase the volatility of assets by taking the maximum position in future contracts (\( \gamma = \Gamma \)). For all \( w \) such that \( E''(w) < 0 \), shareholders are risk-averse and take no positions in future contracts (\( \gamma = 0 \)).

Proposition 3 reveals that the optimal risk-taking policy is of a bang-bang type: If risk-taking is optimal, it happens at the maximal rate. When risk-taking is allowed, the value of equity is defined over three intervals: \([0, W_{\Gamma})\), \([W_{\Gamma}, W^*(\Gamma))\), and \([W^*(\Gamma), \infty)\), where \( W_{\Gamma} \) represents the threshold marking off the convex and concave regions and \( W^*(\Gamma) \) is the optimal payout threshold. Equity value solves equation (8) subject to boundary conditions at zero and at the target cash level \( W^*(\Gamma) \), as well as continuity and smoothness conditions at \( W_{\Gamma} \). The details are reported in the Appendix.

Figure 4 shows that risk-taking substantially increases the value of equity when it is convex, that is when the firm is close to financial distress. In particular, the main effect of risk-taking is to reduce the depth of the trap. If the trap is less deep, negative operating shocks drag the firm into default at a slower pace. The top panel of Figure 4 plots equity value under different risk-taking strategies and shows that the increase in
equity value due to risk-taking is stronger when cash flows are less volatile.

Recall from Section 3.3 that the value of debt can also be locally convex when rollover imbalances are large and the firm is sufficiently close to distress. In such instances, risk-taking strategies are value-enhancing for debtholders too. Thus, both shareholders and debtholders can benefit from an increase in asset risk. This is the case in the right panel of Figure 4, in which we consider a firm characterized by a low cash flow volatility. For such a firm, an increase in cash flow volatility increases the likelihood of escaping the rollover trap and, thus, decreases the probability of default and yield spreads. In addition, and as shown by the bottom panel of Figure 5, the effect of risk-taking on yield spreads is more important for larger values of $\Gamma$.

The left panel of Figure 4 shows that a conflict of interest can arise between shareholders and debtholders when cash flow volatility is sufficiently high. Indeed, because shareholders capture all returns above those required to service debt and benefit disproportionately from risk-taking, they may have incentives to gamble when this is suboptimal for debtholders. In this case, increasing asset volatility leads to a modest decrease in yield spreads at the very brink of distress (when cash reserves are close to zero), but to an increase in yield spreads for higher levels of cash reserves (see the top panel of Figure 5 which shows that change in yields due to risk-taking).

3.5 Optimal capital structure

We now investigate the effects of rollover imbalances on the firm’s optimal capital structure. To do so, we allow the debt principal to be a function of the coupon $C$ and impose that debt is issued at par at a given level of cash reserves, where we use different cash levels to account for varying initial setup costs or financial constraints. The optimal
coupon is such that firm value, i.e., the sum of equity and debt values as calculated in Section 3, is maximized when debt is issued (at par) for the first time. That is, at the time of debt issuance, management selects the capital structure to maximize

\[ V(w; C, m, S) \equiv \sup_{C \in \mathbb{R}_+} \left[ E(w; C, m, S) + D(w; C, m, S) \right], \]

under the constraint

\[ w = W_0 - S - I, \]

where \( I \) is the initial investment cost and \( W_0 \) is the initial cash endowment before financing at time 0 and the constraint that short-term debt is initially issued at par:

\[
E_w \left[ \int_0^\tau e^{-(r+m)t} (C + mS) dt + e^{-(r+m)\tau} f \right] = S.
\]

Table 2 shows the value-maximizing capital structure as a function of debt maturity. When debt maturity is infinite, there are no rollover imbalances. In this case, the optimal debt level balances the tax benefits of debt with bankruptcy costs. When debt maturity is finite, two additional factors shape capital structure choices. First, a short debt maturity imposes larger rollover losses when cash reserves are low, which increases the cost of debt and, thus, decreases the firm’s incentives to issue debt. Second, a short debt maturity increases the proceeds from debt rollover when cash reserves are large (and default risk is low), which decreases the cost of debt and, thus, creates an incentive to increase leverage ratios. Table 2 shows that when debt maturity is relatively short, the first effect dominates and the threat of large rollover losses lead the firm to decrease the optimal coupon rate compared to the infinite maturity case. That is, by generating substantial rollover losses, a shorter maturity decreases the firm’s debt capacity and optimal leverage. Conversely, the second effect can dominate when debt maturity is finite and relatively long; i.e., the lower cost of debt leads the firm to increase the optimal coupon rate compared to the infinite maturity case.
Table 2 also shows that our model can deliver a finite optimal debt maturity. In our model, a decrease in average debt maturity always decreases the value of risky debt by increasing rollover losses and default risk. As mentioned earlier, debtholders suffer from the downside risk and do not capture any upside from issuing short-term. Therefore, the value of debt is the largest when maturity is infinite. This is what we call the debt effect. For shareholders, however, a shortening of average debt maturity has contrasting effects depending on the firm’s cash reserves (see Figure 2). When cash reserves are low, a shorter debt maturity implies larger rollover losses, which decreases the value of equity. When cash reserves are large, a shorter maturity leads to larger net proceeds from rolling over maturing debt, which increases the value of equity. This is what we call the equity effect. The underlying motive for choosing short-term debt maturities in our model is thus very different from previous contributions, in which short-term debt maturity allows firms to reduce the agency costs of risk-shifting (Leland and Toft (1996) or Cheng and Milbradt (2012)) or to reduce the cost of bond illiquidity (Ericsson and Renault (2006), He and Xiong (2012a) or He and Milbradt (2014)). In our model, short-term debt maturity decreases the cost of debt financing for solvent firms, but this benefit needs to be weighted against severe rollover losses when cash flows deteriorate.

3.6 Rollover risk and credit lines

In our benchmark analysis, the firm is forced into liquidation when cash reserves are depleted and access to the equity market is prohibitively expensive. We now assess the robustness of our main results by allowing the firm to take on additional debt via a credit line. In practice, credit lines provide firms with immediate liquidity that can be used in times of need (see Sufi, 2009). In our model, they allow the firm to acquire flexibility in their debt policy, with a total amount of (net) debt varying between $S - W^*$ and $S + L$, where $L$ is the pre-established limit on the credit line.

Specifically, assume the firm has access to a credit line with pre-determined limit $L \geq 0$. For the amount of credit that the firm uses, the interest spread over the risk-free
rate is $\beta > 0$. As in Bolton, Chen, and Wang (2014), the spread $\beta$ can be interpreted as an intermediation cost. Because of this spread, the firm will optimally avoid using its credit line before exhausting internal funds. That is, the firm uses cash as the marginal source of financing if $w \in [0, W^*(L)]$ (the cash region), where $W^*(L)$ denotes the target cash level when the firm has access to a credit line. Conversely, the firm draws funds from the credit line when $w \in [-L, 0]$ (the credit line region). In the following, we assume that the credit line has priority over short-term debt and that $L < \ell$, implying that the credit line is fully collateralized. We report the system of equations satisfied by equity and debt values when the firm has access to a credit line in Appendix A.3.

FIGURE 6

Figure 6 describes the effects of credit lines on the values of corporate securities and rollover imbalances. The figure shows that credit lines reduce the need for large cash balances in that the target cash level is smaller when $L > 0$ (see also Bolton, Chen, and Wang (2011) or Décamps, Gryglewicz, Morelec, and Villeneuve (2016)). By reducing the expected cost of financing frictions when cash reserves are depleted, credit lines increase the values of debt and equity in the cash region. Nonetheless, credit lines reduce the value of short-term debt in the credit line region. The reason is that the credit line has to be paid in full before debtholders can collect any liquidation proceeds. The resulting lower payoff to short-term debt in liquidation leads to larger rollover losses when the firm is close to exhausting the credit line (see the bottom right panel).

While this analysis confirms our qualitative results obtained in the benchmark environment, it shows that the quantitative importance of rollover losses can be different in the presence of a *senior* credit line. When the firm has access to a credit line, rollover losses are smaller in the cash region, but larger in the credit line region. This implies that *senior* credit lines strengthen the amplification mechanism highlighted in Section 3.2 and, therefore, shareholders’ incentives for risk-taking.
4 Time-varying financing conditions

Having explained the effects of short-term debt on corporate policies and incentives for risk-taking in a simplified framework, we now analyze a more general environment in which funding conditions are time-varying. In such an environment, firms also find it optimal to hold cash reserves but the target cash level is state-dependent, denoted by $W^*_i$. Notably, because financial frictions are more severe in state $B$, we expect the level of target cash reserves to be larger in that state. That is, we expect $W^*_B > W^*_G$.

To solve for equity value, we first consider the region in $(0, \infty)$ over which it is optimal for firm shareholders to retain earnings. In this region, the firm does not deliver any cash flow to shareholders and equity value satisfies for $i = G, B$, $i \neq j$:

\[
re_i(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D_i(w) - S)]E'_i(w)
+ \frac{1}{2}((1 - \theta)^2 \sigma^2)E''_i(w) + \pi_i [E_j(w) - E_i(w)].
\]

Equation (9) is similar to equation (4) except for the third term on the right hand side, which reflects the effect of time-varying financing conditions on equity value. This term is the product of the instantaneous probability of a change in financing conditions and the variation in equity value triggered by such a change.

Equation (9) is solved subject to the following boundary conditions. First, when cash reserves exceed $W^*_i$, the firm places no premium on internal funds and it is optimal to make a lump sum payment $w - W^*_i$ to shareholders. As a result, we have

\[
E_i(w) = E_i(W^*_i) + w - W^*_i
\]

for all $w \geq W^*_i$. Subtracting $E_i(W^*_i)$ from both sides of this equation, dividing by $w - W^*_i$, and taking the limit as $w$ tends to $W^*_i$ yields the condition:

\[
E'_i(W^*_i) = 1.
\]

The equity-value-maximizing payout threshold $W^*_i$ is then the solution to:

\[
E''_i(W^*_i) = 0.
\]
When the firm makes losses, its cash buffer decreases. If its cash buffer decreases sufficiently, the firm may be forced to raise new equity or to liquidate. Consider first state \( G \) in which refinancing is possible. In this state, the firm may raise funds before its cash buffer gets completely depleted to avoid that financing conditions worsen when cash reserves are close to zero (as in Bolton, Chen, Wang (2013)). We denote the issuance boundary in state \( G \) by \( W \in [0, W^*_G) \). For any \( w \leq W \) in state \( G \), the firm raises new equity and resets its cash buffer to \( W^*_G \) if optimal to do so. This implies that

\[
E_G(w) = E_G(W^*_G) - (W^*_G - w) - \phi, \quad \forall w \leq W.
\]

If \( W \) is strictly greater than zero, the firm effectively taps the equity markets before its cash reserves are depleted. In this case, it must be that the condition

\[
E'_G(W) = 1
\]

holds. Indeed, management delays equity issues until the marginal value of cash to shareholders equals the marginal cost of refinancing, that is equal to one.

Consider next state \( B \). In that state, the firm has no access to outside funding and defaults as soon as its liquid reserves are depleted. As a result, the condition

\[
E_B(0) = \max\{\ell - S; 0\} = 0
\]

holds at zero and the liquidation proceeds are used to repay debtholders.

It is important to note that the boundaries of the earnings retention region are different in the two states. Notably, cash reserves evolve in \([0, W^*_B]\) in the bad state and in \([W, W^*_G]\) in the good state. This implies that if the financing state switches from bad to good while the firm’s cash reserves are in \((0, W]\), the firm immediately taps the equity market to raise its liquid reserves to their optimal level \( W^*_G \). In these instances,

\[
7\text{There exists a critical issue cost } \Phi \text{ that makes current shareholders indifferent between default and continuation through a costly issue. This critical value binds the following condition}
\]

\[
E_G(W^*_G) - W^*_G = \Phi.
\]
the value of equity jumps from $E_B(w)$ to $E_G(W_G^*) - (W_G^* - w) - \phi$ for any $w \in [0, W]$. If instead the financing state switches from bad to good when $w \in [W_G^*, W_B^*]$, the firm makes a lump sum payment to shareholders and cash reserves go down to $W_G^*$.

To solve for the value of total short-term debt $D_i(w)$ (where we again omit the arguments $(C, m, S)$), we also first consider the region in $(0, \infty)$ over which the firm retains earnings. In this region, $D_i(w)$ satisfies for $i = G, B$, $i \neq j$:

\[
(r + m)D_i(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D_i(w) - S)]D_i'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 D_i''(w) + C + mS + \pi_i[D_j(w) - D_i(w)].
\]

This system of equations is solved subject to the following boundary conditions. First, the firm is liquidated the first time that the cash buffer is depleted in the bad state. The value of short-term debt at this point is equal to the liquidation value of assets:

\[
D_B(0) = \min\{\ell, S\} = \ell.
\]

In the good state, management raises new equity up to the target level $W_G^*$ whenever cash reserves are below $W$. Since the net proceeds from the issue are stored in the cash reserve, the value of short-term debt satisfies:

\[
D_G(w) = D_G(W_G^*), \quad \text{for } w \leq W.
\]

Lastly, the value of short-term debt does not change when dividends are paid out, because dividend payments accrue to shareholders. We thus have:

\[
D_i'(W_i^*) = 0, \quad \text{for } i = G, B.
\]

To fully characterize the value of short-term debt, note that if the state switches from bad to good when $w \in (0, W]$, shareholders raise new funds to reset cash reserves to $W_G^*$ and the value of short-term debt jumps from $D_B(w)$ to $D_G(W_G^*)$. In addition, for any $\phi < \Phi$, shareholders strictly prefer continuation to default. Conversely, when $\phi \geq \Phi$, shareholders prefer to default. Unless otherwise stated, we assume in the following that $\phi < \Phi$. 

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if the state switches from bad to good when \( w \in (W_G^*, W_B^*), \) the firm makes a payment \( w - W_G^* \) to shareholders, leading to a jump in the value of debt from \( D_B(w) \) to \( D_G(W_G^*) \). Therefore, in the region \((0, W] \cup [W_G^*, W_B^*], \) \( D_B(w) \) satisfies

\[
(r + m)D_B(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D_B(w) - S)]D_B'(w) \\
+ \frac{1}{2}((1 - \theta)\sigma)^2 D_B''(w) + C + mS + \pi_B[D_G(W_G^*) - D_B(w)].
\]

We now turn to the analysis of the model. We first analyze how time-varying financing conditions affect the price at which short-term debt is rolled over and the magnitude and sign of rollover imbalances. Consider first the bad state. In that state, the firm may be forced into liquidation after a series of negative shocks because it is unable to raise new equity if it runs out of funds. Thus, the bad state displays a pattern that is analogous to the case analyzed in Section 3. Specifically, there exists a level of cash reserves \( \bar{W}_B \) such that \( D_B(\bar{W}_B; C, m, S) = S \), i.e. such that new debt is issued at par. Rollover imbalances are positive (respectively, negative) above (below) \( \bar{W}_B \), and the size of the imbalance decreases with debt maturity.

Consider next the good state. In that state, default never occurs because the firm can always raise new capital by paying the fixed cost \( \phi \). The value of currently-issued debt increases compared to the bad state, and even more so if debt maturity is shorter. As noted by Acharya, Krishnamurthy, and Perotti (2011): “Creating exposure to liquidity risk is profitable in good times, but creates vulnerability to massive losses when the risk perception changes.” In line with this intuition, Figure 7 (top panel) shows that short-term debt financing may be attractive to shareholders in the good state, because the price of debt is large and so are the proceeds from debt rollover, which increase the value of equity (second panel). However, short-term debt leads to rollover losses in the bad state, which increases default risk.

The analysis in Section 3 has shown that the value of equity can be locally convex when rollover losses are large. When financing conditions are time-varying, this pattern is preserved in the bad state. The value of equity can also be locally convex in the
good state, but for a different reason (Figure 7, third panel). In the good state, this convexity is related to the possibility to time the market by issuing securities when the cost of external finance is low, as in Bolton, Chen, and Wang (2013). In our model, the value of the market timing option is magnified by the threat of being caught in the rollover trap—that is, the possibility of a switch from the good to the bad state when cash reserves are low. The timing option is then more valuable when maturity is shorter.

The last panel of Figure 7 shows that the value of short-term debt is quite insensitive to cash reserves in good times due to the low default risk. The value of debt is the highest at $W^*_G$—because the firm holds its target cash level—and at $W$—because the firm raises new equity to restore the cash reserve at the target $W^*_G$. In the bad state, the value of short-term debt displays a pattern analogous to the case analyzed in Section 3, i.e. it is increasing in $w$ as financial distress risk is lower when $w$ is larger. Overall Figure 7 demonstrates that the patterns identified in section 3 remain and that, here again, short-term debt encourages shareholders to engage in risk-shifting strategies.

In our model, cash reserves and rollover imbalances both depend on debt maturity and are jointly determined by the following recursion. On the one hand, the decision to retain cash within the firm reflects the sign and magnitude of rollover imbalances. On the other hand, rollover imbalances depend on the firm’s financial resilience, that in turn depends on its cash reserves. Through this feedback effect, two opposite forces drive the target level of cash reserves in relation to debt maturity. First, a shorter maturity imposes larger rollover losses in bad times, pushing the firm to increase its target cash reserves. This is what we call the precautionary motive. Second, a shorter maturity increases the net proceeds from debt rollover. Rollover gains replenish the cash reserves, creating an incentive to reduce target cash reserves to save on the carry cost of cash. This is what we call the speculative motive.
Table 3 reports the target level of cash reserves in the bad and good states for different debt maturities and reveals that the precautionary motive dominates, in line with Harford, Klasa, and Maxwell (2014). Lastly, the bad state commands a larger target cash level ($W^*_B > W^*_G$), consistent with the evidence in Acharya, Shin, Yorulmazer (2010) and Aspachs, Nier, Tiesset (2005) that bank liquidity buffers are counter-cyclical.

5 Conclusion

We examine the relation between debt maturity and risk-taking in a model in which firms face financing frictions. To do so, we develop a dynamic structural model in which firms face corporate taxation, time-varying issuance costs of securities, and default costs. In this model, firms are financed with equity and short-term risky debt. Firms hold risky assets and have the option to invest in risk-free, liquid assets such as cash reserves or safe government bonds. Firms maximize shareholder value by choosing their buffers of liquid assets as well as their financing, risk management, and default policies.

With this model, we show that when a firm has short-term debt outstanding, negative operating shocks lead to a drop in liquid reserves and cause the firm to suffer losses when rolling over short term debt, due to weaker fundamentals. This amplification mechanism leads to an increase in default risk, that gets more pronounced as debt maturity decreases and rollover losses increase. When firms are close to distress and debt maturity is short enough, rollover losses can be larger than expected operating profits, dragging the bank closer to default. We call this scenario, in which the firm “burns” cash because of severe rollover losses, “the rollover trap.” In contrast with extant models with long-term debt financing and financing frictions or with short-term debt but without financing frictions, the existence of the rollover trap can make both debtholders and equity holders risk-loving when close to distress. This in turn may lead firms to engage in risk-taking strategies in an attempt to improve equity value. That is, we show that financing frictions imply behavior that is in sharp contrast with the long-standing idea that short-term debt has a disciplinary role and reduces the agency costs of asset substitution.
Appendix

A.1. The value of short-term debt

We start by deriving the value of total short-term debt, denoted by $D(w)$. Since the firm keeps a stationary debt structure, $D(w)$ receives a constant payment rate $C + mS$ that is independent of $t$. Following standard arguments, the function $D(w)$ satisfies the following ordinary differential equation (ODE):

$$(r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + d(w) - mS]D'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 D''(w) + C + mS$$

where $d(w)$ is the value of currently-issued short-term debt. For any given time $t$, we denote by $d(w, \tau)$ the value of the outstanding debt of generation $\tau \leq t$, with $\tau \in [-\infty, 0]$. Therefore, $d(w, 0) = d(w)$ represents the value of currently-issued short-term debt (i.e., $\tau = 0$ at the current time), and we have the following relation

$$d(w, \tau) = e^{mt}d(w).$$

All remaining units of short-term debt from prior issues have the same value per unit, as units of all vintages pay the same coupon, and the remaining units of all vintages will be retired at the same fractional rate. However, there are fewer outstanding units of debt of older generations due to accumulated debt retirement. Integrating $d(w, \tau)$ over $\tau \in [-\infty, 0]$ gives the total value of short-term debt outstanding $D_i(w)$, and then the following important relation

$$D(w) = d(w) \int_{-\infty}^{0} e^{mt}d\tau = \frac{d(w)}{m}$$

holds. Using this relation, together with the ODE describing the dynamics of $D(w)$, we finally get the ODE for currently issued short-term debt, given by

$$rd(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + d(w) - mS]d'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 d''(w) + mC + m[mS - d(w)].$$
The third term on the right-hand side implies that the short-term debt issued today promises a coupon payment $mC$ on any time interval. Recall that exponential repayment of debt with average maturity $1/m$ implies that debt matures randomly at the jump times of a Poisson process with intensity $m$. The fourth term on the right-hand side then represents the payoff obtained by the debtholders when the debt randomly matures times the probability of this occurrence.

A.2. Risk-taking

We derive the optimal risk-taking policy and the value of the firm’s securities under the assumptions in Section 3.4. Assuming frictionless trading in futures contracts, standard arguments imply that, in the earnings retention region, the value of equity satisfies:

$$rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w)$$

$$+ \max_{0 \leq \gamma \leq \Gamma} \left\{ \frac{1}{2} (1 - \theta)^2 (\sigma^2 + \gamma^2) E''(w) \right\}.$$ 

By simply differentiating this equation with respect to the control, it follows that management takes on the maximum position $\Gamma$ in the future contract if $E''(w) > 0$, i.e. if the value of equity is convex. Conversely, management takes no position in the contract if $E''(w) < 0$, i.e. if the value of equity is concave. We denote by $W_\Gamma$ the cash level that separates the convex and the concave region, i.e. such that

$$E''(W_\Gamma) = 0.$$ 

The optimal risk-taking policy is thus of a bang-bang type:

$$\gamma = \begin{cases} 
\Gamma & \text{if } 0 \leq w < W_\Gamma, \\
0 & \text{if } W_\Gamma \leq w < W^*(\Gamma).
\end{cases}$$ 

That is, if risk-taking is optimal, it happens at the maximal rate. Note that the target level of cash holdings is denoted by $W^*(\Gamma)$ in this environment.

In analogy to Section 3, management finds it optimal to pay out dividends to shareholders when the cash reserves exceed $W^*(\Gamma)$, and the value of equity is linear
above this target level. Differently, the optimal risk-taking policy means that, when \( W_T \in (0, W^*(\Gamma)) \), the cash retention region \([0, W^*(\Gamma))\) is characterized by a risk-taking region, \([0, W_T)\), and a no-risk-taking region, \([W_T, W^*(\Gamma))\). In the risk-taking region \([0, W_T)\), the value of equity satisfies the following differential equation

\[
 rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2}(1 - \theta)^2 (\sigma^2 + \Gamma^2) E''(w).
\]

In the no-risk-taking region \([W_T, W^*(\Gamma))\), the value of equity satisfies

\[
 rE(w) = (1 - \theta)((r - \lambda)w + \mu - C + m(D(w) - S)]E'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 E''(w).
\]

The system of ODEs for the value of equity is solved subject to the following boundary condition at the default/liquidation threshold, \( E(0) = 0 \), and the boundary conditions at the target cash level, \( \lim_{w \uparrow W^*(\Gamma)} E'(w) = 1 \) and \( \lim_{w \downarrow W^*(\Gamma)} E''(w) = 0 \). These boundary conditions are similar to those derived in Section 3 and admit an analogous interpretation. In addition, we now need to impose continuity and smoothness at \( W_T \),

\[
 \lim_{w \uparrow W_T} E(w) = \lim_{w \downarrow W_T} E(w) \quad \text{and} \quad \lim_{w \uparrow W_T} E'(w) = \lim_{w \downarrow W_T} E'(w),
\]

to ensure that the risk-taking region and the no-risk-taking regions are smoothly pasted.

Since debtholders have rational expectations, the value of debt reflects this risk-taking policy. As this policy is chosen by management to maximize shareholders’ value, this means that risk-taking may occur even when the value of debt is concave — then decreasing the value of debt. In the risk-taking region \([0, W_T)\), the value of short-term debt \( D(w) \) satisfies

\[
 (r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w) + \frac{1}{2}(1 - \theta)^2 (\sigma^2 + \Gamma^2) D''(w) + C + mS.
\]

In the no-risk-taking region \([W_T, W^*(\Gamma))\), \( D(w) \) satisfies

\[
 (r + m)D(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]D'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 D''(w) + C + mS.
\]
On top of the boundary conditions at 0 and $W^*(\Gamma)$ as in Section 3, respectively $D(0) = (\ell - \Delta)^+$ and $D'(W^*(\Gamma)) = 0$, we impose continuity and smoothness at $W_T$, i.e.

$$\lim_{w \uparrow W_T} D(w) = \lim_{w \downarrow W_T} D(w) \quad \text{and} \quad \lim_{w \uparrow W_T} D'(w) = \lim_{w \downarrow W_T} D'(w).$$

### A.3. Credit lines

We derive the system of equations for the values of equity and short-term debt in the presence of a credit line, as analyzed in Section 3.6. The firm uses cash as the marginal source of financing if $w \in [0, W^*(L)]$ (the cash region), where $W^*(L)$ denotes the target cash level as a function of $L$. Conversely, the firm draws funds from the credit line when $w \in [-L, 0]$ (the credit line region). In the cash region, the value of equity satisfies the following ODE

$$rE(w) = [(1 - \theta)((r - \lambda)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 E''(w).$$

In the credit line region, the firm needs to pay interests on borrowed funds, and the value of equity satisfies

$$rE(w) = [(1 - \theta)((r + \beta)w + \mu - C) + m(D(w) - S)]E'(w) + \frac{1}{2}((1 - \theta)\sigma)^2 E''(w).$$

Similarly, the value of short-term debt satisfies the following ODE

$$(r+m)D(w) = [(1-\theta)((r-\lambda)w+\mu-C)+m(D(w)-S)]D'(w)+\frac{1}{2}((1-\theta)\sigma)^2 D''(w)+C+mS$$

in the cash region, whereas it satisfies the following ODE

$$(r+m)D(w) = [(1-\theta)((r+\beta)w+\mu-C)+m(D(w)-S)]D'(w)+\frac{1}{2}((1-\theta)\sigma)^2 D''(w)+C+mS$$

in the credit line region.

The system of equations is solved subject to the following boundary conditions at the liquidation boundary $(-L)$ and at the payout boundary $(W^*(L))$,

$$E(-L) = E'(W^*(L)) - 1 = E''(W^*(L)) = 0,$$

$$35$$
\[ D(-L) - (\ell - L) = D'(W^*(L)) = 0, \]

and the continuity and smoothness conditions where the credit line and cash regions are pieced together

\[
\begin{align*}
\lim_{w \uparrow 0} E(w) &= \lim_{w \downarrow 0} E(w) \\
\lim_{w \uparrow 0} E'(w) &= \lim_{w \downarrow 0} E'(w) \\
\lim_{w \uparrow 0} D(w) &= \lim_{w \downarrow 0} D(w) \\
\lim_{w \uparrow 0} D'(w) &= \lim_{w \downarrow 0} D'(w).
\end{align*}
\]
References


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### Table 1: Baseline parametrization.

#### A. Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean cash flow rate</td>
<td>$\mu$</td>
<td>0.09</td>
</tr>
<tr>
<td>Cash flow volatility</td>
<td>$\sigma$</td>
<td>0.08</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$r$</td>
<td>0.035</td>
</tr>
<tr>
<td>Carry cost of cash</td>
<td>$\lambda$</td>
<td>0.025</td>
</tr>
<tr>
<td>Liquidation cost</td>
<td>$\varphi$</td>
<td>0.45</td>
</tr>
<tr>
<td>Tax rate</td>
<td>$\theta$</td>
<td>0.30</td>
</tr>
<tr>
<td>Coupon on short-term debt</td>
<td>$C$</td>
<td>0.052</td>
</tr>
<tr>
<td>Principal on short-term debt</td>
<td>$S$</td>
<td>1.27</td>
</tr>
<tr>
<td>Average maturity on short-term debt</td>
<td>$M$</td>
<td>1</td>
</tr>
<tr>
<td>Fixed financing cost</td>
<td>$\phi$</td>
<td>0.012</td>
</tr>
<tr>
<td>Switching intensity (good to bad)</td>
<td>$\pi_G$</td>
<td>0.20</td>
</tr>
<tr>
<td>Switching intensity (bad to good)</td>
<td>$\pi_B$</td>
<td>0.60</td>
</tr>
</tbody>
</table>

#### B. Implied variables in one-state model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity value</td>
<td>$E(W^*)$</td>
<td>1.196</td>
</tr>
<tr>
<td>Target level of liquid reserves</td>
<td>$W^*$</td>
<td>0.456</td>
</tr>
<tr>
<td>Yield spreads</td>
<td></td>
<td>0.02 – 3004</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td></td>
<td>51.6% – 100%</td>
</tr>
</tbody>
</table>
Table 2: **Optimal capital structure.**

The table reports the value-maximizing capital structure (coupon, principal, leverage ratio) as well as firm value at debt issuance, under the baseline parametrization and varying the average maturity of corporate debt.

<table>
<thead>
<tr>
<th>Maturity ( (M) )</th>
<th>Coupon ( (C) )</th>
<th>Principal ( (S) )</th>
<th>Leverage ratio</th>
<th>Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>At par at ( W^*/3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.036</td>
<td>1.012</td>
<td>51.2%</td>
<td>1.976</td>
</tr>
<tr>
<td>3</td>
<td>0.044</td>
<td>1.138</td>
<td>57.0%</td>
<td>1.996</td>
</tr>
<tr>
<td>5</td>
<td>0.050</td>
<td>1.239</td>
<td>61.6%</td>
<td>2.012</td>
</tr>
<tr>
<td>10</td>
<td>0.056</td>
<td>1.371</td>
<td>67.4%</td>
<td>2.036</td>
</tr>
<tr>
<td>Inf</td>
<td>0.056</td>
<td>1.488</td>
<td>72.6%</td>
<td>2.049</td>
</tr>
<tr>
<td>At par at ( W^*/2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.040</td>
<td>1.123</td>
<td>53.5%</td>
<td>2.100</td>
</tr>
<tr>
<td>3</td>
<td>0.048</td>
<td>1.292</td>
<td>60.3%</td>
<td>2.142</td>
</tr>
<tr>
<td>5</td>
<td>0.054</td>
<td>1.418</td>
<td>65.3%</td>
<td>2.172</td>
</tr>
<tr>
<td>10</td>
<td>0.063</td>
<td>1.619</td>
<td>73.0%</td>
<td>2.219</td>
</tr>
<tr>
<td>Inf</td>
<td>0.062</td>
<td>1.673</td>
<td>75.6%</td>
<td>2.215</td>
</tr>
</tbody>
</table>

Table 3: **Financing decisions.**

The table reports the target level of cash reserves in good \( (W^*_G) \) and in bad times \( (W^*_B) \), the issue threshold \( \bar{W} \), and the issue size \( (W^*_G - \bar{W}) \) for average debt maturities \( M \) of 1 year, 5 years, 10 years, and infinite maturity.

<table>
<thead>
<tr>
<th>( M )</th>
<th>( W^*_G )</th>
<th>( W^*_B )</th>
<th>( \bar{W} )</th>
<th>( W^*_G - \bar{W} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.320</td>
<td>0.344</td>
<td>0.150</td>
<td>0.170</td>
</tr>
<tr>
<td>5</td>
<td>0.245</td>
<td>0.269</td>
<td>0.071</td>
<td>0.174</td>
</tr>
<tr>
<td>10</td>
<td>0.237</td>
<td>0.261</td>
<td>0.063</td>
<td>0.174</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.234</td>
<td>0.258</td>
<td>0.054</td>
<td>0.180</td>
</tr>
</tbody>
</table>
Figure 1: Rollover imbalances, Debt maturity, and Cash Reserves.

The figure plots the rollover imbalance \( R(w) \equiv m(D(w) - S) \) as a function of cash reserves \( w \in [0, W^*] \) for average debt maturities \( M \) of 1 year (solid line), 5 years (dashed line), and infinite maturity (dotted line).
Figure 2: Value of equity and the Rollover trap.

The figure plots the value of equity (left panel) and the marginal value of cash for the shareholders (right panel) as a function of cash reserves $w \in [0, W^*]$, for different debt maturities $M$ (top panel), cash flow volatilities $\sigma$ (middle panel), and liquidation costs $\varphi$ (bottom panel).
Figure 3: Value of debt.

The figure plots the aggregate value of debt $D(w)$ and the marginal value of cash for the debtholders $D'(w)$ as a function of cash reserves $w \in [0, W^*]$ and for average debt maturities $M$ of 1 year (solid line), 5 years (dashed line), and infinite maturity (dotted line).
Figure 4: Risk-taking.

The figure plots the value of equity $E(w)$ (top panel) and the aggregate value of short-term debt $D(w)$ (bottom panel) as a function of cash reserves $w \in [0, W^*]$ under different risk-taking strategies and for cash flow volatility equal to 0.08 (left panel) and 0.06 (right panel).
Figure 5: Risk-taking and yield spreads.

The figure plots the difference in yield spreads when shareholders do and do not engage in risk-taking strategies as a function of cash reserves $w \in [0, W^*]$, for cash flow volatility equal to 0.08 (top panel) and 0.06 (bottom panel).
Figure 6: CREDIT LINE.

The figure plots the value of equity, the marginal value of equity, the aggregate value of short-term debt, and the rollover imbalance in the absence (solid line) and in the presence of credit line availability (dashed line for $L = 0.06$ and dotted line for $L = 0.12$).
Figure 7: Time-varying funding liquidity.

The figure plots the rollover imbalance $R_i(w)$, the value of equity $E_i(w)$, the marginal value of cash for shareholders $E'_i(w)$, and the value of aggregate short-term debt $D_i(w)$, as a function of cash reserves $w \in [0, W^*_i]$ in the good state (left panel) and in the bad state (right panel) for average debt maturities $M$ of 1 year (solid line), 5 years (dashed line), and infinite maturity (dotted line).