The term structure of interbank risk

Damir Filipović, Anders B. Trolle*

Ecole Polytechnique Fédérale de Lausanne and Swiss Finance Institute, Switzerland

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ABSTRACT

We infer a term structure of interbank risk from spreads between rates on interest rate swaps indexed to the London Interbank Offered Rate (LIBOR) and overnight indexed swaps. We develop a tractable model of interbank risk to decompose the term structure into default and non-default (liquidity) components. From August 2007 to January 2011, the fraction of total interbank risk due to default risk, on average, increases with maturity. At short maturities, the non-default component is important in the first half of the sample period and is correlated with measures of funding and market liquidity. The model also provides a framework for pricing, hedging, and risk management of interest rate swaps in the presence of significant basis risk.

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1. Introduction

Interbank risk, as defined in this paper, is the risk of direct or indirect loss resulting from lending in the interbank money market. The recent financial crisis has highlighted the implications of such risk for financial markets and economic growth. While existing studies provide important insights on the determinants of short-term interbank risk, very little is known about the term structure of interbank risk. We provide a comprehensive analysis of this topic. First, we develop a tractable model of the term structure of interbank risk, connecting the interbank money market to the interest rate swap (IRS) and credit default swap (CDS) markets. Second, we apply the model to analyze interbank risk since the onset of the financial crisis, decomposing the term structure of interbank risk into default and non-default (liquidity) components, and study their associated risk premiums.

We follow most existing studies by measuring interbank risk as the spread between a London Interbank Offered Rate (LIBOR) and the rate on a maturity-matched overnight indexed swap (OIS). The former is a reference...
rate for unsecured interbank borrowing and lending, and the latter is a common risk-free rate proxy. We show that the spread between the fixed rate on an IRS with floating-leg payments indexed to, say, three-month LIBOR and an OIS of the same maturity reflects risk-neutral expectations about future three-month LIBOR-OIS spreads and, therefore, future three-month interbank risk. This allows us to infer a term structure of interbank risk from IRS-OIS spreads of different maturities.

Studying the term structure of interbank risk has a number of advantages. First, the term structure contains important information about interbank risk that is not contained in observed LIBOR-OIS spreads. This is clearly illustrated by Fig. 1, where the solid line shows the three-month LIBOR-OIS spread and the dotted line shows the five-year IRS-OIS spread indexed to three-month LIBOR. Prior to the onset of the credit crisis, the term structure of interbank risk was essentially flat, with IRS-OIS spreads only a few basis points (bps) higher than LIBOR-OIS spreads. Then, at the onset of the crisis in August 2007, LIBOR-OIS spreads increased much more than IRS-OIS spreads. This resulted in a strongly downward-sloping term structure of interbank risk, indicating that market participants expected the extremely high levels of interbank risk to be a relatively short-lived phenomenon. Finally, from fall 2009 to the end of our sample period in early 2011, LIBOR-OIS spreads were more or less back to pre-crisis levels (except for a transitory increase related to the escalation of the European sovereign debt crisis), while IRS-OIS spreads remained well above pre-crisis levels and significantly higher than LIBOR-OIS spreads. The result was an upward-sloping term structure of interbank risk, indicating that market participants expected interbank risk to increase in the future (or required a large risk premium for bearing future interbank risk, or both).

A second advantage to studying the term structure is that it allows the risk-neutral dynamics of interbank risk to be estimated with high precision. With a parsimonious risk premium specification, which tightens the relation between actual and risk-neutral dynamics, this also increases the precision with which the actual dynamics of interbank risk is estimated. That is, taking term structure information into account helps provide a more accurate characterization of interbank risk than can be obtained from a pure time series model.

Several dimensions of interbank risk can give rise to a LIBOR-OIS spread. An obvious candidate is default risk. LIBOR is a benchmark indicating the average rate at which large, creditworthy banks belonging to the LIBOR panel can obtain unsecured funding for longer terms (typically three or six months) in the interbank money market. An OIS is a swap with floating payments based on a reference rate for unsecured overnight funding, which we assume equals the average cost of unsecured overnight funding for LIBOR panel banks. An important feature of the LIBOR panel is that its composition is updated over time to include only creditworthy banks. A bank that experiences a significant deterioration in its credit quality is dropped from the panel and is replaced by a bank with superior credit quality. Therefore, the OIS rate reflects the average credit quality of a periodically refreshed pool of creditworthy banks, while LIBOR incorporates the risk that the average credit quality of an initial set of creditworthy banks would deteriorate over the term of the loan.

Consequently, LIBOR exceeds the OIS rate.

To formalize this, we set up an intensity-based model in which, at a given time, we distinguish between the average credit quality (i.e., default intensity) of the periodically refreshed panel and the credit quality of an average bank within an initial panel. Deterioration in the credit quality of this bank relative to the average credit quality of the periodically refreshed panel occurs according to a jump process. The first jump time is interpreted as the time when the bank is dropped from the panel. The risk of credit quality deterioration (i.e., the intensity and size of the jump process) varies stochastically over time.

In this setting, the default component of the LIBOR-OIS

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1 While this assumption is true in the euro market, it is only approximately true in the US dollar market, because the reference rate (the effective federal funds rate) reflects the average funding cost for a broader set of banks than the LIBOR panel. Nevertheless, Afonso, Kovner, and Schoar (2009) show that credit risk in the federal funds market is managed via credit rationing instead of interest rates, so that in practice only creditworthy banks participate in the federal funds market. Such credit rationing was particularly prevalent in the aftermath of the Lehman Brothers default.

2 We stress that there is negligible default risk in the OIS contract due to collateralization. However, the OIS rate does reflect default risk due to the indexation of the floating leg to an unsecured overnight rate.
spread is driven by the expected rate of credit quality deterioration of an average bank within the initial panel.

A LIBOR-OIS spread could also arise due to factors not directly related to default risk, primarily liquidity. There are several reasons that liquidity in the market for interbank funding beyond the ultra-short term can deteriorate. For instance, banks could refrain from lending longer term for precautionary reasons, if they fear adverse shocks to their own funding situation, or for speculative reasons, if they anticipate possible fire sales of assets by other financial institutions. Instead of modeling these mechanisms directly, we posit a residual factor that captures the component of the LIBOR-OIS spread that is not due to default risk. To the extent that liquidity effects are correlated with default risk, the residual factor captures the part of liquidity that is unspanned by default risk.

The default and non-default components in an IRS-OIS spread reflect risk-neutral expectations about the default and non-default components in future LIBOR-OIS spreads. To separate the two components, we use information from the credit default swap market. At each observation date, we construct a CDS spread term structure for an average panel bank as a composite of the CDS spread term structures for the individual panel banks. The composite CDS spread term structure allows us to estimate the risk-neutral process for credit quality deterioration of an average panel bank and then to infer the default component in LIBOR-OIS spreads. Importantly, the potential for refreshment of the LIBOR panel combined with the risk of credit quality deterioration makes the default component in an IRS-OIS spread lower than the default risk reflected by a maturity-matched CDS spread.

Our model is set within a general affine framework. Depending on the specification, two factors drive OIS rates, one or two factors drive the default component of LIBOR-OIS spreads (i.e., the risk of credit quality deterioration), and one or two factors drive the non-default component of LIBOR-OIS spreads. The model is highly tractable with analytical expressions for LIBOR, OIS, IRS, and CDS. In valuing swap contracts, we match as closely as possible current market practice regarding collateralization.

We apply the model to study interbank risk from the outset of the financial crisis in August 2007 until January 2011. We utilize a panel data set consisting of term structures of OIS rates, IRS-OIS spreads indexed to three-month and six-month LIBOR, and CDS spreads—all with maturities up to ten years. The model is estimated by maximum likelihood in conjunction with the Kalman filter.

We conduct a specification analysis, which shows that a specification with two factors driving OIS rates, two factors driving the default component of LIBOR-OIS spreads, and one factor driving the non-default component of LIBOR-OIS spreads has a satisfactory fit to the data, while being fairly parsimonious. We then use this specification to decompose the term structure of interbank risk into default and non-default components. We find that, on average, the fraction of total interbank risk due to default risk increases with maturity. At the short end of the term structure, the non-default component is important in the first half of the sample period. At longer maturities, the default component is the dominant driver of interbank risk throughout the sample period.

To what extent are our results affected by possible strategic behavior by certain LIBOR contributors during parts of the sample period? First, the procedure for computing LIBOR should limit the impact of strategic behavior. Second, even to the extent that LIBOR were affected, this is unlikely to impact our results, because interbank risk is primarily inferred from the cross section of swap rates, which are determined in highly competitive markets. Instead, idiosyncratic variation in LIBOR rates show up as a pricing error in our Kalman filter setting. Nonetheless, to verify the robustness of our results, we reestimate the model using only swap rates and no LIBOR rates but find no significant changes to the results. This robustness check can be found in the online Appendix.

To understand the determinants of the non-default component of interbank risk, we relate the residual factor to a number of proxies for funding liquidity and market liquidity, which tend to be highly intertwined. Specifically, we regress the residual factor on the components of the liquidity proxies, which are unspanned by interbank default risk. The R² reaches 64% in a multivariate regression specification, strongly suggesting that the non-default component of interbank risk largely captures liquidity effects not spanned by default risk.

We also provide tentative evidence on the pricing of interbank risk in the interest rate swap market. We find that swap market participants require compensation for

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4 The main component of a composite CDS spread is the expected credit quality deterioration, over the maturity of the CDS contract, of an average bank within the current panel. In contrast, the default component of an IRS-OIS spread reflects a series of expected short-term credit quality deteriorations, from each LIBOR fixing date to the next, of average banks within future refreshed panels.

6 A recent study by Kuo, Skeie, and Vickery (2012) compares LIBOR with rates on actual interbank borrowing by LIBOR panel banks at the one-month, three-month, and six-month maturities during the crisis period. They find that LIBOR is a reasonably accurate reflection of the average unsecured funding costs of LIBOR panel banks in the interbank money market. The main exception is the two-week period following the Lehman Brothers default, when LIBOR underestimates funding costs by about 20 bps. The difference between quoted and transacted rates, however, is only marginally statistically significant and should be put in relation to the extremely elevated LIBOR-OIS spreads during this period, as shown in Fig. 1. Going forward, the proposals for greater regulatory oversight and increased transparency set out in the Wheatley (2012) report should diminish concerns about the integrity of LIBOR.

7 For each liquidity measure, the component that is unspanned by interbank default risk is given by the residual from a regression of the liquidity measure on the first two principal components of the composite CDS term structure.
exposure to variation in interbank default risk, while we are not able to reliably estimate the compensation required for exposure to the residual factor. This implies that in the first part of the sample period, when the non-default component dominates, the overall compensation for variation in interbank risk is low. In contrast, in the second part of the sample period, when the default component dominates, the overall compensation for variation in interbank risk is significant. For instance, the instantaneous Sharpe ratio on a strategy of being long the five-year IRS-OIS spread indexed to three-month LIBOR is estimated to have averaged 0.35 from early 2009 to the end of the sample period.

Throughout, we also report results for the euro (EUR) market. Not only does this serve as a robustness check, but this market is also interesting in its own right. First, by several measures, the market is even larger than the US dollar (USD) market. Second, the structure of the EUR market is such that the reference overnight rate in an OIS exactly matches the average cost of unsecured overnight funding of EURIBOR (European Interbank Offered Rate, the EUR equivalent of LIBOR) panel banks, providing a check of this assumption. And, third, the main shocks to the interbank money market in the second half of the sample period emanated from the Eurozone with its sovereign debt crisis. Indeed, we find that interbank risk in the EUR market is generally higher than interbank risk in the USD market during the second half of the sample period, and that the opposite is true during the first half. Nevertheless, results on the decomposition of interbank risk, the drivers of the residual factor, and the risk compensation in the swap market are similar to the USD market.

As a plausibility check of the model-based decomposition, we consider a simple regression-based decomposition in which each interest rate spread (LIBOR-OIS or IRS-OIS) is regressed on a maturity-matched CDS spread. This regression-based decomposition gives results that are qualitatively similar to the model-based decomposition.

In addition to providing insights into the dynamics and determinants of interbank risk, the model should be useful for pricing, hedging, and risk management in the interest rate swap market. Since the onset of the credit crisis, market participants have been exposed to significant basis risk: Swap cash flows are indexed to LIBOR but, because of collateral agreements, are discounted using rates inferred from the OIS market. Furthermore, swap portfolios at most financial institutions are composed of swap contracts indexed to LIBOR rates of various maturities, creating another layer of basis risk. Our model provides a framework for managing overall interest rate risk and these basis risks in an integrated way. From a regulatory standpoint, the model could be useful for determining the right discount curve for the valuation of long-term insurance liabilities, where discount factors are typically allowed to include a liquidity component but not a default risk component.

Our paper is most closely related to Collin-Dufresne and Solnik (2001, henceforth CS), who study the term structure of spreads between yields on corporate bonds issued by LIBOR banks and IRS rates. In their model, a spread arises because bond yields reflect the possibility of deterioration in the credit quality of current LIBOR banks relative to that of future LIBOR banks. A similar mechanism is present in our model. However, several important differences exist between CS and our paper. First, in the CS model, LIBOR reflects only default risk; in our model, LIBOR reflects both default and non-default (liquidity) risk, which allows a decomposition of interbank risk. Second, CS assume a constant intensity of credit quality deterioration, causing the implicit IRS-OIS spread term structures to be virtually flat and constant across time. In contrast, we allow the intensity of credit quality deterioration to vary stochastically, which produces rich dynamics of the default component of spreads (with additional spread dynamics coming from the non-default component). Third, in the CS model, shocks to credit quality are permanent, while we allow for gradual improvement in credit quality following a shock, further improving the fit of the model. Fourth, in the empirical analysis we use CDSs instead of corporate bonds and OISs instead of Treasuries. Corporate bonds and Treasuries were heavily affected by liquidity and flight-to-quality issues during the crisis. By considering only swap contracts, we expect liquidity to be less of an issue and to be more uniform across instruments leading to a clean decomposition of the term structure of interbank risk.

A number of papers have analyzed the three-month LIBOR-OIS spread and attempted to decompose it into default and liquidity components. These papers include Schwartz (2010), Taylor and Williams (2009), McAndrews, Sarkar, and Wang (2008), Michaud and Upper (2008), and Eisenschmidt and Tapking (2009). They all study the early phase of the financial crisis before the collapse of Lehman Brothers and find, with the exception of Taylor and Williams (2009), that liquidity was a key driver of interbank risk during this period. We find a similar result for the short end of the term structure of interbank risk. However, at longer maturities, default risk appears to have been the dominant driver even during the early phase of the financial crisis, underscoring the importance of taking the entire term structure into account when analyzing interbank risk.

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9 CS use the Treasury curve instead of the OIS curve as the reference curve. In their model, IRS-Treasury spreads do vary over time, because the default intensity of the periodically refreshed panel is stochastic.

10 Our paper is also related to Liu, Longstaff, and Mandell (2006), Johannes and Sundareshan (2007), and Feldhutter and Lando (2008), who study the term structure of IRS-Treasury spreads. Feldhutter and Lando (2008) allow for a non-default component in the spread between LIBOR and the (unobservable) risk-free rate, which they argue is related to hedging flows in the IRS market during their pre-crisis sample period.

11 Smith (2010) studies LIBOR-OIS spreads of maturities up to 12 months within a dynamic term structure model and attributes most of the variation in spreads to variation in risk premiums. A somewhat problematic aspect of her analysis is that the default component of LIBOR-OIS spreads is identified by the spread between LIBOR and repo.
Several papers including Bianchetti (2009), Fujii, Shimada, and Takahashi (2009), Henrard (2009), and Mercurio (2009, 2010) have developed pricing models for interest rate derivatives that take the stochastic IRS-OIS spread into account. These are highly reduced-form models in that swap spreads indexed to different LIBOR rates are modeled independently of each other and are not decomposed into different components. In contrast, we provide a unified model of all such spreads, making it possible to aggregate the risk of large swap portfolios and analyze their underlying determinants.

The rest of the paper is organized as follows: Section 2 describes the market instruments. Section 3 describes the model of the term structure of interbank risk. Section 4 discusses the data and the estimation approach. Section 5 presents the results. Section 6 concludes, and several appendices contain pricing formulas, proofs, and details on the estimation. An online Appendix contains additional material, including a large number of robustness checks showing that the results hold true for alternative model parameterizations and measures of interbank default risk.

2. Market instruments

We describe the market instruments that we use in the paper. We first consider the basic reference rates and then a variety of swap contracts that are indexed to these reference rates.

2.1. Reference rates

A large number of fixed income contracts are tied to an interbank offered rate. The main reference rate in the USD-denominated fixed income market is USD LIBOR. In the EUR-denominated fixed income market it is EURIBOR (there also exists a EUR LIBOR, although this rate has not received the same benchmark status as EURIBOR). Both LIBOR and EURIBOR are trimmed averages of rates submitted by sets of banks. In the case of LIBOR, each contributor bank bases its submission on the question: “At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size?”. In the case of EURIBOR, the wording is slightly different and each contributor bank submits “the rates at which euro interbank term deposits are being offered within the Eurozone by one prime bank to another”. Therefore, LIBOR is an average of the rates at which banks believe they can obtain unsecured funding, and EURIBOR is an average of the rates at which banks believe a prime bank can obtain unsecured funding. This subtle difference becomes important when quantifying the degree of default risk inherent in the two rates. Both rates are quoted for a range of terms, with three and six months being the most important and most widely followed. In the following, we let \( L(t,T) \) denote the \((T-t)\)-maturity LIBOR or EURIBOR rate that fixes at time \( t \).

For both LIBOR and EURIBOR, contributor banks are selected based on their credit quality and the scale of their market activities. During our sample period, the LIBOR panel consisted of 16 banks and the EURIBOR panel was significantly larger, consisting of 42 banks.\(^\text{12}\) An important feature of both panels is that they are reviewed and revised periodically. A bank that experiences a significant deterioration in its credit quality (or its market share, or both) is dropped from the panel and is replaced by a bank with superior credit quality.

An increasing number of fixed income contracts are tied to an index of overnight rates. In the USD market, the benchmark is the effective federal funds (FF) rate, which is a transaction-weighted average of the rates on overnight unsecured loans of reserve balances held at the Federal Reserve that banks make to one another. In the EUR market, the benchmark is the Euro Overnight Index Average (EONIA) rate, computed as a transaction-weighted average of the rates on all overnight unsecured loans in the interbank money market initiated by EURIBOR panel banks. Therefore, in the EUR market, the benchmark overnight rate reflects the average cost of unsecured overnight funding of panel banks. We assume that the same holds for the USD market, although the set of banks from which the effective federal funds rate is computed does not exactly match the LIBOR panel.\(^\text{13}\)

For the sake of convenience, we use “LIBOR” as a generic term for an interbank offered rate, comprising both LIBOR and EURIBOR, whenever there is no ambiguity.

2.2. Pricing collateralized contracts

Swap contracts between major financial institutions are virtually always collateralized to the extent that counterparty risk is negligible.\(^\text{14}\) In this subsection, we provide the generic pricing formula of collateralized cash flows that we use to price swap contracts. Similar formulas have been derived in various contexts by Johannes and Sundaresan (2007), Fuji, Shimada, and Takahashi (2009), and Piterbarg (2010). Consider a contract with a contractual nominal cash flow \( X \) at maturity \( T \). Its present value at \( t < T \) is denoted by \( V(t) \). We assume that the two parties in the contract agree on posting cash-collateral on a continuous marking-to-market basis. We also assume that, at any time \( t < T \), the posted amount of collateral equals 100% of the contract’s present value \( V(t) \). The receiver of the collateral can invest it at the risk-free rate \( r(t) \) and has to pay an agreed rate \( r_c(t) \) to the poster of collateral. The present

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\(^{12}\) After the end of our sample period, the USD LIBOR panel was expanded to 20 banks and the EURIBOR panel was expanded to 44 banks.

\(^{13}\) Participants in the federal funds market are those with accounts at Federal Reserve banks, which include US depository institutions, US branches of foreign banks, and government-sponsored enterprises.

\(^{14}\) Even in the absence of collateralization, counterparty risk usually has only a very small effect on the valuation of swap contracts; see, e.g., Duffie and Huang (1996). This led to the approach to interest rate swap pricing in Duffie and Singleton (1997).
value thus satisfies the integral equation

\[ V(t) = E^Q_t \left[ e^{-\int_t^T r_s \, ds} X + \int_t^T e^{-\int_t^s r_u \, ds} (r_u - r_c(u)) V(u) \, du \right], \] (1)

where, throughout, we assume a filtered probability space \((\Omega, \mathcal{F}, \mathcal{F}_t, Q)\), where \(Q\) is a risk-neutral pricing measure, and \(E^Q_t = E^Q[\cdot | \mathcal{F}_t]\) denotes conditional expectation under \(Q\). Appendix A shows that this implies the pricing formula

\[ V(t) = E^Q_0 \left[ e^{-\int_t^T r_s \, ds} X \right]. \] (2)

For \(X = 1\), we obtain the price of a collateralized zero-coupon bond

\[ P_c(t, T) = E^Q_0 \left[ e^{-\int_t^T r_s \, ds} \right]. \] (3)

In the sequel, we assume that the collateral rate \(r_c(t)\) is equal to an instantaneous proxy \(L(t, t)\) of the overnight rate, which we define as

\[ r_c(t) = L(t, t) = \lim_{\Delta t \rightarrow 0} \frac{L(t, t + \Delta t) - L(t, t)}{\Delta t}. \] (4)

In reality, best practice among major financial institutions is daily marking-to-market and adjustment of collateral. Furthermore, cash collateral is the most popular form of collateral, because it is free from the issues associated with rehypothecation and allows for faster settlement times. Finally, FF and EONIA are typically the contractual interest rates earned by cash collateral in the USD and EUR markets, respectively. The assumptions we make, therefore, closely approximate current market reality.15

### 2.3. Interest rate swaps

In a regular interest rate swap, counterparties exchange a stream of fixed-rate payments for a stream of floating-rate payments indexed to LIBOR of a particular maturity. More specifically, consider two discrete tenor structures:

\[ t = t_0 < t_1 < \cdots < t_N = T \] (5)

and

\[ t = T_0 < T_1 < \cdots < T_n = T, \] (6)

and let \(\delta = t_i - t_{i-1}\) and \(\Delta = T_i - T_{i-1}\) denote the lengths between tenor dates, with \(\delta < \Delta\). At every time \(t_i\), \(i = 1, \ldots, N\), one party pays \(\Delta L(t_{i-1}, t_i)\), and at every time \(t_i\), \(i = 1, \ldots, n\), the other party pays \(\Delta K \), where \(K\) denotes the fixed rate on the swap. The swap rate, \(IRS_{\delta,\Delta}(t, T)\), is the value of \(K\) that makes the IRS value equal to zero at inception and is given by

\[ IRS_{\delta,\Delta}(t, T) = \frac{\sum_{i=1}^N E^Q_t \left[ e^{-\int_t^{t_i} r_s \, ds} \delta L(t_{i-1}, t_i) \right]}{\sum_{i=1}^N \Delta P_c(t, T_i)}. \] (7)

In the USD market, the benchmark IRS pays three-month LIBOR floating versus six-month fixed. In the EUR market, the benchmark IRS pays six-month EURIBOR floating versus one-year fixed. Rates on IRS indexed to LIBOR of other maturities are obtained via basis swaps (BS).

### 2.4. Basis swaps

In a basis swap, counterparties exchange two streams of floating-rate payments indexed to LIBOR of different maturities, plus a stream of fixed payments. The quotation convention for basis swaps differs across brokers and across markets, and it could also have changed over time. However, as demonstrated in the online Appendix, the differences between the conventions are negligible. Consider a basis swap in which one party pays the \(\delta_1\)-maturity LIBOR and the other party pays the \(\delta_2\)-maturity LIBOR with \(\delta_1 < \delta_2\). We use the quotation convention in which the basis swap rate, \(BS_{\delta_1,\delta_2}(t, T)\), is given as the difference between the fixed rates on two IRS indexed to \(\delta_2\)- and \(\delta_1\)-maturity LIBOR, respectively. That is,

\[ BS_{\delta_1,\delta_2}(t, T) = IRS_{\delta_2}(t, T) - IRS_{\delta_1}(t, T). \] (8)

This convention has the advantage that rates on non-benchmark IRS are very easily obtained via basis swaps.

### 2.5. Overnight indexed swaps

In an overnight indexed swap, counterparties exchange a stream of fixed-rate payments for a stream of floating-rate payments indexed to a compounded overnight rate (FF or EONIA). In contrast to an IRS, an OIS typically has fixed-rate payments and floating-rate payments occurring at the same frequency. Consider the tenor structure given in (6) with \(\Delta = T_i - T_{i-1}\). At every time \(T_i\), \(i = 1, \ldots, n\), one party pays \(\Delta K\) and the other party pays \(\Delta L(T_{i-1}, T_i)\), where \(L(T_{i-1}, T_i)\) is the compounded overnight rate for the period \([T_{i-1}, T_i]\). This rate is given by

\[ L(T_{i-1}, T_i) = \frac{1}{\Delta} \prod_{j=1}^{K_i} (1 + (t_{j} - t_{j-1})L(t_{j-1}, t_j)) - 1, \] (9)

where \(T_k = t_0 < t_1 < \cdots < t_K = T_i\) denotes the partition of the period \([T_{i-1}, T_i]\) into \(K_i\) business days, and \(L(t_{j-1}, t_j)\) denotes the respective overnight rate. As in Andersen and Piterbarg (2010, Subsection 5.5), we approximate simple by continuous compounding and the overnight rate by the instantaneous rate \(L(t, t)\) given in Eq. (4), in which case \(L(T_{i-1}, T_i)\) becomes

\[ L(T_{i-1}, T_i) = \frac{1}{\Delta} \left( e^{\int_{T_{i-1}}^{T_i} r_s \, ds} - 1 \right). \] (10)

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15 International Swaps and Derivatives Association (2010) is a detailed survey of current market practice. Further evidence for the pricing formula given in this subsection is provided by Whittall (2010), who reports that the main clearhouse of swap contracts now uses discount factors extracted from the OIS term structure to value collateralized swap cash flows.

16 In practice, the length between dates varies slightly depending on the day-count convention. To simplify notation, we suppress this dependence.
The OIS rate is the value of $K$ that makes the OIS value equal to zero at inception and is given by

$$OIS(t, T) = \frac{\sum_{i=1}^{n} e^{-\int_{t}^{T} r_s(s) ds} \Delta P_c(t_i, T_i)}{\sum_{i=1}^{n} \Delta P_c(t, T_i)} = \frac{1 - P_c(t, T_n)}{\sum_{i=1}^{n} \Delta P_c(t, T_i)}.$$  

(11)

In both the USD and EUR markets, OIS payments occur at a one-year frequency, i.e., $\Delta = 1$. For OISs with maturities less than one year, there is only one payment at maturity.

2.6. The IRS-OIS spread

Combining Eqs. (7) and (11), a few calculations yield

$$IRS(t, T) - OIS(t, T) = \sum_{i=1}^{n} e^{-\int_{t}^{T} r_s(s) ds} (\Delta P_t(t_i, T_i) - OIS(t_i, T_i)) \Delta P_c(t, T_i).$$  

(12)

This equation shows that the spread between the rates on, say, a five-year IRS indexed to $\delta$-maturity LIBOR and a five-year OIS reflects (risk-neutral) expectations about future $\delta$-maturity LIBOR-OIS spreads during the next five years.17

To the extent that the LIBOR-OIS spread measures short-term interbank risk, the IRS-OIS spread reflects expectations about future short-term interbank risk—more specifically, about short-term interbank risk among the banks that constitute the LIBOR panel at future tenor dates, which could vary due to the periodic updating of the LIBOR panel. Consequently, we refer to the term structure of IRS-OIS spreads as the term structure of interbank risk.

2.7. Credit default swaps

In a credit default swap, counterparties exchange a stream of coupon payments for a single default protection payment in the event of default by a reference entity. As such, the swap has a premium leg (the coupon stream) and a protection leg (the contingent default protection payment). More specifically, consider the tenor structure given in (5) and let $\tau$ denote the default time of the reference entity.18 The present value of the premium leg with coupon rate $C$ is given by

$$V_{\text{prem}}(t, T) = C I_1(t, T) + C I_2(t, T),$$  

(13)

where $CI_{1}(t, T)$ with

$$I_1(t, T) = E^T_0\left[\sum_{i=1}^{N} e^{-\int_{t_i}^{T} r_s(s) ds} (t_i-t_{i-1}) 1_{(t_i < \tau)}\right]$$  

is the value of the coupon payments prior to default time $\tau$, and $CI_{2}(t, T)$ with

$$I_2(t, T) = E^T_0\left[\sum_{i=1}^{N} e^{-\int_{t_i}^{T} r_s(s) ds} (t - t_{i-1}) 1_{(t_i < \tau \wedge \tau < t)}\right]$$  

is the accrued coupon payment at default time $\tau$. The present value of the protection leg is

$$V_{\text{prot}}(t, T) = E^T_0\left[e^{-\int_{t}^{\tau} r_s(s) ds} (1 - R(\tau)) 1_{(\tau \leq T)}\right],$$  

(16)

where $R(\tau)$ denotes the recovery rate at default time $\tau$. The CDS spread, $CDS(t, T)$, is the value of $C$ that makes the premium and protection leg equal in value at inception and is given by

$$CDS(t, T) = \frac{V_{\text{prot}}(t, T)}{I_1(t, T) + I_2(t, T)}.$$  

(17)

While these par spreads are quoted in the market, CDS contracts have been executed since 2009 with a standardized coupon and an upfront payment to compensate for the difference between the par spread and the coupon. However, our CDS database consists of par spreads throughout the sample period.

3. Modeling the term structure of interbank risk

We describe our model of the term structure of interbank risk. We first consider the general framework and then specialize to a tractable model with analytical pricing formulas.

3.1. The general framework

Instead of modeling the funding costs of individual panel banks, we consider an average bank that represents the panel at a given time. More specifically, we assume the extended doubly stochastic framework provided in Appendix B, where for any $t_o \geq 0$, the default time of an average bank within the $t_o$-panel is modeled by some random time $r(t_o) > t_o$. This default time admits a non-negative intensity process $\lambda(t_o)$, for $t > t_o$, with initial value $\lambda(t_o, t_o) = \lambda(t_o)$. In other words, at a given time $t > t_o$, $\lambda(t)$ is the average default intensity (i.e., credit quality) of the current $t$-panel, and $\lambda(t_o, t)$ is the default intensity of an average bank within the initial $t_o$-panel.

In view of the doubly stochastic property given in Eq. (53), the time $t_o$-value of an unsecured loan with notional 1 to an average bank within the $t_o$-panel over period $[t_o, T]$ equals

$$B(t_o, T) = E^T_0\left[e^{-\int_{t_o}^{T} r_s(s) ds} 1_{(r(t_o)) > T}\right] = E^T_0\left[e^{-\int_{t_o}^{T} r_s(s) ds} 1_{(r(t_o)) > T}\right].$$  

(18)

Here we assume zero recovery of interbank loans, which is necessary to keep the subsequent affine transform analysis...
tractable.\textsuperscript{19} Absent market frictions, the \((T-t_0)\)-maturity LIBOR rate \(L(t_0, T)\) satisfies \(1 + (T-t_0)L(t_0, T) = B(t_0, T)\).

In practice, LIBOR could be affected by factors not directly related to default risk. For instance, banks could refrain from lending beyond the ultra-short term for precautionary reasons as in the models of Allen, Carletti, and Gale (2009) and Acharya and Skeie (2011) or for speculative reasons as in the models of Acharya, Gromb, and Yorulmaz (2012), Acharya, Shin, and Yorulmaz (2011), and Diamond and Rajan (2011). Either way, the volume of longer term interbank loans decreases and the rates on such loans increase beyond the levels justified by default risk. We allow for a non-default component in LIBOR by setting

\[
L(t_0, T) = \frac{1}{T-t_0} \left( \frac{1}{B(t_0, T)} - 1 \right) \Xi(t_0, T),
\]

where \(\Xi(t_0, T)\) is a multiplicative residual term that satisfies

\[
\lim_{T \to t_0} \Xi(t_0, T) = 1.
\]

It follows from Eq. (4) that the collateral rate \(r_c(t_0)\) becomes

\[
r_c(t_0) = \lim_{T \to t_0} \frac{1}{T-t_0} \left( \frac{1}{B(t_0, T)} - 1 \right) \Xi(t_0, T)
= \frac{d}{dt} B(t_0, T)_{|_{T=t_0}} = r(t_0) + \Lambda(t_0).
\]

Combining Eq. (11) (in the case of a single payment) and Eq. (19), we get the following expression for the LIBOR-OIS spread:

\[
L(t_0, T) - OIS(t_0, T) = \frac{1}{T-t_0} \left[ \left( \frac{1}{B(t_0, T)} - P_c(t_0, T) \right)
- \left( \frac{1}{B(t_0, T)} - 1 \right) \Xi(t_0, T) - 1 \right].
\]

The first bracketed term in Eq. (22) is the default component. The periodic updating of the LIBOR panel implies that \(\lambda(t_0, T) \geq \Lambda(t)\), for \(t > t_0\). From Eqs. (18) and (3) in conjunction with Eq. (21) it follows that \(B(t_0, T) < P_c(t_0, T)\), which implies that the default component is positive. The second bracketed term in Eq. (22) is the non-default component, which is positive provided that \(\Xi(t_0, T) > 1\).

For the analysis, we also need expressions for the CDS spreads of an average bank within the \(t_0\)-panel. The factors \(I_1(t_0, T)\) and \(I_2(t_0, T)\) in the present value of the premium leg given in Eqs. (14) and (15) become

\[
I_1(t_0, T) = \sum_{i=1}^{N} (t_1-t_{i-1}) E_{t_0}^T \left[ e^{- \int_{t_0}^{t_1} \lambda(t) dt} 1_{\{t < t_0\}} \right]
= \sum_{i=1}^{N} (t_1-t_{i-1}) E_{t_0}^T \left[ e^{- \int_{t_0}^{t_{i-1}} \lambda(t) dt + \int_{t_{i-1}}^{t_1} \lambda(t) dt} \right]
= \sum_{i=1}^{N} (t_1-t_{i-1}) E_{t_0}^T \left[ e^{- \int_{t_0}^{t_{i-1}} \lambda(t) dt + \int_{t_{i-1}}^{t_1} \lambda(t) dt} \right],
\]

and

\[
I_2(t_0, T) = \sum_{i=1}^{N} \left[ e^{- \int_{t_0}^{t_2} \lambda(t) dt} \int_{t_{i-1}}^{t_1} \lambda(t) dt \right] 1_{\{t_1 < t_0\}} 1_{\{t < t_0\}}.
\]

\textsuperscript{19} Alternatively, we could follow Duffie and Singleton (1999) and let \(\hat{\lambda}(t_0, s) = B(t_0, s)\), which is the product of a default intensity process, \(\lambda(t_0, s)\), and a fractional default loss process, \(l(t_0, s)\). That is, \(l(t_0, s)\in[0, 1]\) defines the fraction of market value of the loan that is lost upon default.

\[
\Xi(t_0, T) = 1 + \left( \frac{1}{B(t_0, T)} - 1 \right) \Xi(t_0, T) = \frac{d}{dt} B(t_0, T)_{|_{T=t_0}} = r(t_0) + \Lambda(t_0).
\]

where we use the fact that, employing the terminology of Appendix B, \(\exp \left[ - \int_{t_0}^{T} \lambda(t, s) dt \right] \lambda(t, u)\) is the \(\mathcal{F}_t\)-conditional density function of \(\tau(t)\); see, e.g., Filipović (2009, Subsection 12.3). In line with the assumption of zero recovery of interbank loans in the derivation of Eq. (18), we shall assume zero recovery for the CDS protection leg. Its present value given in Eq. (16) thus becomes

\[
V_{prot}(t_0, T) = E_{t_0}^0 \left[ e^{- \int_{t_0}^{T} \lambda(t) dt} 1_{\{t_0 < T\}} \right]
= \int_{t_0}^{T} E_{t_0}^0 \left[ e^{- \int_{t_0}^{t} \lambda(t) dt} \right] \lambda(t) dt.
\]

3.2. An affine factor model

We now introduce an affine factor model of \(\tau(t)\), the intensities \(\Lambda(t)\) and \(\lambda(t_0, t)\), and the residual \(\Xi(t_0, T)\). We assume that the risk-free short rate, \(r(t)\), is driven by a two-factor Gaussian process

\[
dr(t) = \kappa_r (\theta_r - r(t)) dt + \sigma_r dW_r(t)
\]

\[
d\gamma(t) = \kappa_r (\theta_r - \gamma(t)) dt + \sigma_r (\rho dW_r(t) + \sqrt{1-\rho^2} dW_f(t)),
\]

where \(\gamma(t)\) is the stochastic mean-reversion level of \(r(t)\) and \(\rho\) is the correlation between innovations to \(r(t)\) and \(\gamma(t)\). The model is equally tractable with \(\tau(t)\) being driven by a two-factor square-root process. While this could seem more appropriate given the low interest rate environment during much of the sample period, we find that the fit to the OIS term structure is slightly worse with this specification. The fit to the IRS-OIS and CDS spread term structures is virtually the same for the two specifications.

We have investigated several specifications for the average default intensity of the periodically refreshed panel, \(\Lambda(t)\). In the interest of parsimony, we assume that \(\Lambda(t)\) is constant

\[
\Lambda(t) = \Lambda,
\]

In the online Appendix, we analyze a setting, in which \(\Lambda(t)\) is stochastic. This adds complexity to the model without materially affecting the results.

The default intensity of an average bank within the \(t_0\)-panel, \(\hat{\lambda}(t_0, t)\), is modeled by

\[
\hat{\lambda}(t_0, t) = \Lambda + \int_{t_0}^{t} \kappa (\Lambda - \hat{\lambda}(t_0, s)) ds + \sum_{j=N(t)+1}^{\infty} Z_{t, j},
\]

where \(N(t)\) is a simple counting process with jump intensity \(\kappa\) and \(Z_{1, 1}, Z_{2, 2}, \ldots\) are identically and independently distributed (i.i.d.) exponential jump sizes with mean \(1/\xi\). That is, we assume that deterioration in the credit quality of an average bank within the \(t_0\)-panel relative to the average credit quality of the periodically refreshed panel occurs according to a jump process. The first jump time of \(\hat{\lambda}(t_0, t)\) is interpreted as the time when
the bank is dropped from the panel. Between jumps, we allow for \( \lambda(t_0, t) \) to revert toward \( \Lambda \) (in the online Appendix, we explore an alternative specification in which deterioration in credit quality is permanent).

The intensity of credit quality deterioration, \( \nu(t) \), is stochastic and evolves according to either a one-factor square-root process

\[
d\nu(t) = \kappa_\nu(t_0, t) dt + \sigma_\nu \sqrt{\nu(t)} dW_\nu(t)
\]

or a two-factor square-root process

\[
d\nu(t) = \kappa_\nu(t_0, t) dt + \sigma_\nu \sqrt{\nu(t)} dW_\nu(t)
\]

where \( \mu(t) \) is the stochastic mean-reversion level of \( \nu(t) \).

Our doubly stochastic framework for modeling default of \( d \) deteriorates in credit quality is permanent).

\[
dX(t) = \kappa_{\lambda, \nu}(t) dt + \sigma_\lambda \sqrt{\lambda(t)} dW_\lambda(t)
\]

\[
d\lambda(t) = \kappa_\lambda(t_0, t) dt + \sigma_\lambda \sqrt{\lambda(t)} dW_\lambda(t)
\]

Finally, the multiplicative residual term, \( \Xi(t_0, T) \), is modeled by

\[
\Xi(t_0, T) = E^Q_0 \left[ \exp\left( \frac{-1}{T} \int_0^T \xi(s) ds \right) \right]
\]

where \( \xi(t) \) evolves according to either a one-factor square-root process

\[
d\xi(t) = \kappa_\xi(t_0, t) dt + \sigma_\xi \sqrt{\xi(t)} dW_\xi(t)
\]

or a two-factor square-root process

\[
d\xi(t) = \kappa_\xi(t_0, t) dt + \sigma_\xi \sqrt{\xi(t)} dW_\xi(t)
\]

where \( \epsilon(t) \) is the stochastic mean-reversion level of \( \xi(t) \).

We estimate the model on a panel data set that covers the period starting with the onset of the credit crisis on August 9, 2007 and ending on January 12, 2011. We do not include the pre-crisis period, given that a regime switch in the perception of interbank risk appears to have occurred at the onset of the crisis (see Fig. 1).

4. Data and estimation

We estimate the model on a panel data set that covers the period starting with the onset of the credit crisis on August 9, 2007 and ending on January 12, 2011. We do not include the pre-crisis period, given that a regime switch in the perception of interbank risk appears to have occurred at the onset of the crisis (see Fig. 1).

4.1. Interest rate data

The interest rate data are from Bloomberg. We collect daily OIS rates with maturities of three and six months and one, two, three, four, five, seven, and ten years (in Bloomberg, data on the USD seven-year OIS rate is missing and the time series for the USD ten-year OIS rate starts on July 28, 2008). We also collect daily IRS and BS rates with maturities of one, two, three, four, five, seven, and ten years as well as three-month and six-month LIBOR and EURIBOR rates. The rates on OIS, IRS, and BS are composite quotes computed from quotes that Bloomberg collects from major banks and inter-dealer brokers.

In the USD market, the benchmark IRS is indexed to three-month LIBOR (with fixed-rate payments occurring at a six-month frequency), and the rate on an IRS indexed to six-month LIBOR is obtained via a BS as

\[
IRS_{6M,6M}(t, T) = IRS_{3M,6M}(t, T) + BS_{3M,6M}(t, T).
\]

Conversely, in the EUR market, the benchmark IRS is indexed to six-month EURIBOR (with fixed-rate payments occurring at a one-year frequency), and the rate on an IRS indexed to three-month EURIBOR is obtained via a BS as

\[
IRS_{3M,1Y}(t, T) = IRS_{6M,1Y}(t, T) - BS_{3M,6M}(t, T).
\]

We focus on the spreads between rates on IRS and OIS with the same maturities. Therefore, for each currency and on each day in the sample, we have two spread term structures given by

\[
SPREAD_d(t, T) = IRS_{d,6M}(t, T) - OIS(t, T)
\]

for \( \delta = 3M \) or \( \delta = 6M \) and \( \Delta = 6M(1Y) \) in the USD (EUR) market.

Table 1 shows summary statistics of the data. For a given maturity, interest rate spreads are always increasing in the tenor (the maturity of the LIBOR rate to which an IRS is indexed). This is consistent with a six-month LIBOR loan containing more default and liquidity risk than two consecutive three-month LIBOR loans. For a given tenor, the mean and volatility of spreads decrease with maturity. While the mean spreads are similar across the two markets, spread volatility tends to be higher in the USD market.
Table 1
Summary statistics of data.

The table shows means and, in parentheses, standard deviations. $SPREAD_{1m}$ denotes the difference between the fixed rates on an interest rate swap (IRS) indexed to the three-month London Interbank Offered Rate (LIBOR) or European Interbank Offered Rate (EURIBOR) and an overnight indexed swap (OIS) with the same maturity. $SPREAD_{10}$ denotes the difference between the fixed rates on an IRS indexed to six-month LIBOR or EURIBOR and an OIS with the same maturity. $CDS_{\text{Median}}$ and $CDS_{\text{Mean}}$ are the credit default swap (CDS) spread term structures for an average bank within the LIBOR and EURIBOR panels, respectively. OIS rates are measured in percentages, and interest rate spreads and CDS spreads are measured in basis points. Each time series consists of 895 daily observations from August 9, 2007 to January 12, 2011, except those marked with †, which consist of 643 daily observations from July 28, 2008 to January 12, 2011.

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<th>6M</th>
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<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
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<td>(1.46)</td>
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<td>(1.21)</td>
<td>(1.12)</td>
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<td>(0.96)</td>
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<td>(0.55)</td>
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<td>39.0</td>
<td>35.4</td>
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</table>

4.2. CDS spread data

The CDS data are from Markit, which is the leading provider of CDS quotes. Markit collects quotes from major market participants and constructs daily composite quotes. Because data supplied by Markit are widely used for marking-to-market CDS contracts, its quotes are closely watched by market participants. For each bank in the LIBOR and EURIBOR panels, we collect daily spread term structures for CDS contracts written on senior obligations. On each date in the sample, we construct a CDS spread term structure for an average bank within the panel as a composite of the CDS spread term structures for the individual panel banks. As discussed in Section 2.1, LIBOR is a trimmed mean of the rates at which banks estimate they can obtain unsecured funding for a given term. Because we expect a positive relation between the submitted rates and banks’ own default risk, LIBOR presumably reflects a trimmed mean of the default risk of the panel banks. Therefore, we measure the default risk of an average bank within the LIBOR panel by aggregating the CDS spreads of the individual LIBOR panel banks in the same way that LIBOR is computed from the submitted rates, namely, by removing the top and bottom 25% of spreads and computing a simple average of the remaining spreads. The resulting default risk measure is denoted $CDS_{\text{Median}}$.

In contrast, EURIBOR is a trimmed mean of the rates at which banks estimate a prime bank (not necessarily themselves) can obtain unsecured funding for a given term. While the notion of a prime bank is ambiguous, we interpret it as a representative bank among the panel. Because the median, not the mean, seems to be the appropriate statistics in this case, it is plausible that the submitted rates reflect what each bank perceives is the median default risk in the panel. Being a trimmed mean of the submitted rates, EURIBOR itself then also reflects the median default risk in the panel. Therefore, we measure the default risk of an average bank within the EURIBOR panel by taking the median of the CDS spreads of the individual EURIBOR panel banks. We denote this default risk measure $CDS_{\text{Median}}$.

Table 1 shows summary statistics of the composite CDS spreads. On average, the level of CDS spreads increases with maturity and CDS spread volatility decreases with maturity. The magnitudes of $CDS_{\text{Median}}$ in the USD market and $CDS_{\text{Median}}$ in the EUR market are similar despite the EURIBOR panel being composed of significantly more banks than the LIBOR panel.

4.3. Maximum likelihood estimation

We estimate the model specifications using maximum likelihood in conjunction with Kalman filtering. For this
purpose, we cast the model in state space form with a measurement equation describing the relation between the state variables and the observable interest rates and spreads, as well as a transition equation describing the discrete-time dynamics of the state variables.\footnote{In the online Appendix, we consider an alternative two-stage maximum likelihood procedure inspired by Duffee (1999) and Duffie, Pedersen, and Singleton (2003), which breaks the large estimation problem into two smaller and more manageable estimation problems. The two estimation approaches give very similar results, and we, therefore, report results based on the single-stage maximum likelihood procedure outlined here.}

Let $X_t$ denote the vector of state variables and let $Z_t$ denote the vector consisting of the term structure of OIS rates, the two term structures of IRS-OIS spreads, and the term structure of CDS spreads observed at time $t$. The measurement equation is given by

$$Z_t = h(X_t; \theta) + u_t, \quad u_t \sim N(0, \Sigma),$$

where $h$ is the pricing function, $\theta$ is the vector of model parameters, and $u_t$ is a vector of i.i.d. Gaussian pricing errors with covariance matrix $\Sigma$. To reduce the number of parameters in $\Sigma$, we follow usual practice in the empirical term structure literature in assuming that the pricing errors are cross-sectionally uncorrelated (that is, $\Sigma$ is diagonal) and that the same variance, $\sigma^2$, applies to all pricing errors. The observed instruments (OIS rates, IRS-OIS spreads, and CDS spreads) are linked to the state variables as follows: OIS rates are related to the $r(t)$-process and $\Lambda$ via Eqs. (11) and (3). CDS spreads are related to $\lambda(t)$ and, hence, the $\nu(t)$-process and $\Lambda$ via Eqs. (17) and (23)–(25). Finally, IRS-OIS spreads are related to $\lambda(t)$ and $\Xi(t)$ and, hence, the $\xi(t)$-process (the effect of $\Lambda$ more or less cancels) via Eqs. (12) and (22) in conjunction with Eqs. (3) and (18).\footnote{CDS and IRS-OIS spreads also depend on the $r(t)$-process and $\Lambda$ due to the discounting by $r(t)$, but this provides only weak identification.}

While the transition density of $X_t$ is unknown, its conditional mean and variance is known in closed form, because $X_t$ follows an affine diffusion process under the objective probability measure. We approximate the transition density with a Gaussian density with identical first and second moments, in which case the transition equation is of the form

$$X_t = \Phi_0 + \Phi_1 X_{t-1} + w_t, \quad w_t \sim N(0, Q_t),$$

where $Q_t$ is an affine function of $X_{t-1}$.

Due to the nonlinearities in the relation between observations and state variables, we apply the nonlinear unscented Kalman filter, which is found by Christoffersen, Jacobs, Karoui, and Mimouni (2009) to have very good finite-sample properties in the context of estimating dynamic term structure models with swap rates. Details are provided in Appendix D. The Kalman filter produces one-step-ahead forecasts for $Z_t$, $\tilde{Z}_{t-1}$, and the corresponding error covariance matrices, $F_{t-1}$, from which we construct the log-likelihood function

$$L(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left[ n_t \log 2\pi + \log F_{z,t-1} + (Z_t - \tilde{Z}_{t-1})^T F_{z,t-1}^{-1} (Z_t - \tilde{Z}_{t-1}) \right].$$

where $T=895$ is the number of observation dates and $n_t$ is the (time-varying) number of observations in $Z_t$. The maximum likelihood estimator, $\hat{\theta}$, is then

$$\hat{\theta} = \arg \max_{\theta} L(\theta).$$

Approximating the true transition density with a Gaussian makes this a quasi-maximum likelihood (QML) procedure. While QML estimation has been shown to be consistent in many settings, it is in fact not consistent in a Kalman filter setting, because the conditional covariance matrix $Q_t$ in the recursions depends on the Kalman filter estimates of the state variables instead of the true, but unobservable, values; see, e.g., Duan and Simonato (1999). However, simulation results in several papers have shown this issue to be negligible in practice.

In terms of identification, we face several issues. First, we show in the online Appendix that it is very difficult to separately identify $\xi$ with $1/\gamma$ being the mean jump size in the default intensity) and the process for $\nu(t)$ (the intensity of credit quality deterioration). Instead, it is the mean rate of credit quality deterioration of an average panel bank, $(1/\gamma)\nu(t)$, that matters for valuation. In the estimation, we fix $\xi$ at 10, but the implied process for the mean rate of credit quality deterioration is invariant to the choice of $\xi$.

Second, in a preliminary analysis, we find that it is difficult to reliably estimate the default intensity of the periodically refreshed panel, $\Lambda$. Its value is not identified from the OIS term structure and, in the absence of very short-term CDS spreads, is also hard to pin down from the CDS term structure.\footnote{The OIS term structure depends on the model for the collateral rate, $r(t) = r(t) + \Lambda$, under the risk-neutral measure, with $r(t)$ given by Eq. (26). An equivalent model in which $c(t)$ is a state variable is

$$dr(t) = \kappa (\mu - r(t)) \, dt + \sigma \, dW_t(t)$$

$$d\gamma(t) = \kappa (\tilde{\gamma} - \gamma(t)) \, dt + \sigma \rho \, dW_t(t) + \sqrt{1-\rho^2} \, dW_t(t),$$

where $\mu(t) = r(t) + \Lambda$ and $\tilde{\gamma} = \gamma - \Lambda$. The latter specification is maximally flexible with six identifiable parameters, see Dai and Singleton (2000). Therefore, $\nu$, and $\Lambda$ are not separately identified from the OIS term structure.} From Eqs. (4) and (21), we have that $\Lambda$ is the difference between the instantaneous proxy of the overnight unsecured interbank rate, $I(t)$, and the truly risk-free rate, $r(t)$. Therefore, one can get an idea about the magnitude of $\Lambda$ by examining the spreads between short-term OIS rates and repo rates, which are virtually risk-free due to the practice of overcollateralization of repo loans; see, e.g., Longstaff (2000). The sample averages of the spreads between one-week OIS rates and one-week general collateral (GC) repo rates for Treasuries, agency securities, agency-issued mortgage-backed securities (MBSs), and European government bonds are 13 bps, 3 bps, 1 bp, and 0 bp,
respectively.\textsuperscript{25} Plots of these spreads can be found in the online Appendix. In the case of Treasury collateral, the spread spikes at the beginning of the crisis and around the Bear Stearns and Lehman Brothers episodes. However, movements in the spread likely reflect periodic scarcity of Treasury collateral, rather than variation in default risk, because the spikes are mostly due to downward spikes in the Treasury repo rate, not upward spikes in the OIS rate. Also, the correlation between the OIS-Treasury repo spread and the short-maturity (six-month) LIBOR panel CDS spread is virtually zero.\textsuperscript{26} In the cases of agency securities and agency-issued MBSs, the spreads are volatile in the first half of the sample period, but without systematic patterns around crisis events. In the case of the EUR market, the spread is very stable throughout the sample period. Taken together, these results suggest that very little default risk exists in the market for overnight interbank deposits. We fix \( \lambda \) at 5 bps, but reasonable variations in the value of \( \lambda \) do not change our results. In the online Appendix, we show that our results are robust to extending the model with a stochastic \( \Lambda(t) \) identified via OIS-Treasury repo spreads.

Third, given the relatively short sample period, many of the market price of risk parameters are imprecisely estimated (in contrast to the risk-neutral parameters, most of which are strongly identified with low standard errors). For each model specification, we obtain a more parsimonious risk premium structure by reestimating the model after setting to zero those market price of risk parameters for which the absolute \( t \)-statistics did not exceed one.\textsuperscript{27} The likelihood functions were virtually unaffected by this, so we henceforth study these constrained model specifications.

5. Results

We discuss estimates of parameters and state variables, compare model specifications, decompose the term structure of interbank risk, and quantify risk premiums.

5.1. Maximum likelihood estimates

Table 2 displays parameter estimates and their asymptotic standard errors. It is straightforward to verify that, for all the specifications, the parameter values satisfy the sufficient admissibility conditions in Lemma C.4 in Appendix C. The estimates are strikingly similar across the two markets and, therefore, we focus on the USD estimates.

In the \( \mathcal{A}(2, 2, 1) \) and \( \mathcal{A}(2, 2, 2) \) specifications, \( \nu(t) \) is relatively volatile and displays fast mean-reversion toward \( \mu(t) \), which in turn is less volatile and has much slower mean-reversion. Hence, \( \nu(t) \) captures transitory shocks to the intensity of credit quality deterioration, while \( \mu(t) \) captures more persistent shocks. In the \( \mathcal{A}(2, 1, 1) \) specification, the speed of mean-reversion and the volatility of \( \nu(t) \) lie between those of \( \nu(t) \) and \( \mu(t) \) in the more general specifications. Also, between jumps, the reversion of the default intensity toward \( \lambda \) occurs relatively fast. Although estimated with some uncertainty, the market price of risk parameters \( \Gamma_\nu \) and \( \Gamma_\nu \) are negative in all specifications. This implies that the expected rate of credit quality deterioration is lower under the physical measure than under the risk-neutral measure, indicating that market participants require a premium for bearing exposure to variation in default risk. This is consistent with several papers finding that variation in default risk carries a risk premium; see, e.g., Duffie (1999) in the case of corporate default risk and Pan and Singleton (2008) in the case of sovereign default risk. We return to this issue in Section 5.6.

In all specifications, the residual factor, \( \xi(t) \), is very volatile, exhibits very fast mean-reversion, and has a long-run mean of essentially zero. In the \( \mathcal{A}(2, 2, 2) \) specifications, \( \xi(t) \) is mean-reverting toward \( \epsilon(t) \), which is less volatile and has slower mean-reversion. Hence, \( \xi(t) \) captures transitory shocks to the non-default component and \( \epsilon(t) \) captures moderately persistent shocks. In none of the specifications were we able to reliably estimate \( \Gamma_\xi \) and \( \Gamma_\xi \). Consequently, these parameters were constrained to zero (see end of Section 4.3).

5.2. State variables

Fig. 2 displays the state variables for the three specifications estimated on USD data. The corresponding figure for the EUR market is similar and available in the online Appendix. It is instructive to see the reaction of the state variables to the three most important shocks to the interbank money market during the sample period: the Bear Stearns near-bankruptcy on March 16, 2008, the Lehman Brothers bankruptcy on September 15, 2008, and the escalation of the European sovereign debt crisis often marked by the downgrade of Greece’s debt to non-investment-grade status by Standard and Poor’s on April 27, 2010. The figure shows that \( \nu(t) \) increases leading up to the Bear Stearns near-default but quickly decreases after the takeover by J.P. Morgan. If anything, the opposite is true of \( \xi(t) \). Immediately following the Lehman default, \( \xi(t) \) spikes while \( \nu(t) \) increases more gradually and does not reach its maximum until March 2009. Finally, with the escalation of the European sovereign debt crisis, \( \nu(t) \)
increases while ξ(t) does not react. These dynamics hold true regardless of the model specification and suggest that an increase in the risk of credit quality deterioration was the main factor driving interbank risk around the first and third episode, while an increase in risk factors not directly related to default risk was the main driver in the aftermath of the Lehman default.

To better interpret the model’s implications for current and future interbank default risk, we compute risk-neutral three-month and six-month expected default probabilities (EDPs) for an average bank within the current panel as well as for an average bank within the refreshed panel in five year’s time.28 We focus on risk-neutral EDPs, since these are more precisely estimated than EDPs under the physical measure (EDPs are lower under the physical measure, since the expected rate of credit quality deterioration is lower). The risk-neutral EDPs are displayed in Fig. 3 for the USD market, with the corresponding figure for the EUR market available in the online Appendix. Taking the Λ(2, 2, 1) specification as an example, over our sample period, the spot three-month EDP averaged 0.09% but peaked at 0.32% in March 2009 (Panel A2). Because of mean-reversion in the intensity of credit quality deterioration, the forward three-month EDP is much less volatile, averaging 0.07% and peaking at 0.11%.29 Until the Lehman Brothers default, the term structure of EDPs was mostly upward-sloping, implying that risk-neutral expectations were for interbank default risk to increase in the future. This contrasts with the downward-sloping spread term structure during this period (see Fig. 1), indicating an important role for non-default risk factors in determining shorter-term spreads. From the Lehman Brothers default until fall 2009, the term structure of EDPs was

28 Specifically, we compute \( E^p \left[ 1_{\tau < \tau_{2,3,6}^p} \right] \) and \( E^p \left[ 1_{\tau - \tau_{2,3,6}^p} \right] \), where \( \delta \) equals three or six months and \( \tau \) equals \( t \) plus five years. Both expressions have analytical solutions in our affine framework.

29 The expected default probabilities depend on our assumption of zero recovery (or 100% loss rate). As a rule of thumb, halving the loss rate is nearly the same as doubling the expected default probabilities.
downward-sloping, while it is again mostly upward-sloping during the last part of the sample period.

Similar dynamics are observed for the six-month EDPs (Panel B2). The potential for refreshment of the LIBOR panel combined with the risk of credit quality deterioration implies that a strategy of lending for six months to a LIBOR counterparty involves more default risk than lending for two consecutive three-month periods, as the latter strategy includes the option of switching to a more creditworthy counterparty after three months. Indeed, the spot six-month EDP is consistently larger than the sum of the spot three-month EDP and the three-month forward three-month EDP for a refreshed panel. The sample mean of the former is 0.27%, and the sample mean of the latter sum is only 0.17%.

5.3. Specification analysis

For each of the model specifications, we compute the fitted OIS rates, interest rate spreads, and CDS spreads based on the filtered state variables. For each day in the sample and within each category (OIS, SPREAD$_{3M}$, SPREAD$_{6M}$, and CDS) we then compute the root mean squared pricing errors (RMSEs) of the available rates or spreads, thereby constructing time series of RMSEs.

The first three rows of Panel A in Table 3 display the means of the RMSE time series in the USD market. The next two rows report the mean difference in RMSEs between two model specifications along with the associated $t$-statistics. Given that all specifications have two factors driving the OIS term structure, they produce almost the same fit to OIS rates. However, they differ significantly in their fit to interest rate spreads and CDS spreads. $\mathcal{A}(2,2,1)$ has a significantly better fit than $\mathcal{A}(2,1,1)$ to the CDS term structure, with the mean RMSE decreasing from 11.6 bps to 6.6 bps. It also appears to trade off a statistically significant better fit to the term structure of swap spreads indexed to six-month LIBOR, for a statistically insignificant worse fit to the term structure of swap spreads indexed to three-month LIBOR. $\mathcal{A}(2,2,2)$ improves upon $\mathcal{A}(2,2,1)$ with a statistically significant better fit to the term structures of CDS spreads and swap spreads.

Fig. 2. State variables, US dollar market. The figure shows the state variables for the three model specifications estimated on US dollar data. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece’s debt to non-investment-grade status by Standard and Poor’s on April 27, 2010. Each time series consists of 895 daily observations from August 9, 2007 to January 12, 2011.
Table 3
Comparison of model specifications.

The table reports means of the root mean squared pricing error (RMSE) time series of overnight indexed swap (OIS) rates, interest rate spreads, and credit default swap (CDS) spreads. SPREAD\textsuperscript{3M} denotes the difference between the fixed rates on an interest rate swap (IRS) indexed to the three-month London Interbank Offered Rate (LIBOR) or European Interbank Offered Rate (EURIBOR) and an OIS with the same maturity. SPREAD\textsuperscript{6M} denotes the difference between the fixed rates on an IRS indexed to six-month LIBOR or EURIBOR and an OIS with the same maturity. Units are basis points. t-Statistics, corrected for heteroscedasticity and serial correlation up to 50 lags using the method of Newey and West (1987), are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively. Each time series consists of 895 daily observations from August 9, 2007 to January 12, 2011.

<table>
<thead>
<tr>
<th></th>
<th>OIS</th>
<th>SPREAD\textsuperscript{3M}</th>
<th>SPREAD\textsuperscript{6M}</th>
<th>CDS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: US dollar market</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A(2, 1, 1)</td>
<td>7.14</td>
<td>7.65</td>
<td>7.63</td>
<td>11.55</td>
</tr>
<tr>
<td>A(2, 2, 1)</td>
<td>7.06</td>
<td>8.12</td>
<td>6.99</td>
<td>6.62</td>
</tr>
<tr>
<td>A(2, 2, 2)</td>
<td>7.02</td>
<td>7.65</td>
<td>6.37</td>
<td>6.19</td>
</tr>
<tr>
<td>Δ(2, 2, 1)−Δ(2, 1, 1)</td>
<td>-0.09*</td>
<td>-0.63*</td>
<td>-4.93***</td>
<td></td>
</tr>
<tr>
<td>Δ(2, 2, 2)−Δ(2, 2, 1)</td>
<td>-0.04*</td>
<td>-0.62***</td>
<td>-0.43***</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Euro market</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A(2, 1, 1)</td>
<td>6.16</td>
<td>7.66</td>
<td>8.23</td>
<td>11.77</td>
</tr>
<tr>
<td>A(2, 2, 1)</td>
<td>5.93</td>
<td>7.83</td>
<td>7.16</td>
<td>7.07</td>
</tr>
<tr>
<td>A(2, 2, 2)</td>
<td>5.39</td>
<td>7.13</td>
<td>6.22</td>
<td>6.34</td>
</tr>
<tr>
<td>Δ(2, 2, 1)−Δ(2, 1, 1)</td>
<td>-0.23***</td>
<td>-1.06***</td>
<td>-4.70***</td>
<td></td>
</tr>
<tr>
<td>Δ(2, 2, 2)−Δ(2, 2, 1)</td>
<td>-0.34***</td>
<td>-0.95***</td>
<td>-0.73***</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3. Default probabilities, US dollar market. Panels A1–A3 display the risk-neutral three-month (3M) expected default probability (EDP) for an average bank within the current panel as well as for an average bank within the refreshed panel in five year’s time. Panels B1–B3 display the corresponding six-month (6M) EDPs. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece’s debt to non-investment-grade status by Standard and Poor’s on April 27, 2010. Each time series consists of 895 daily observations from August 9, 2007 to January 12, 2011.

Table 3 indexed to six-month LIBOR, and a marginally statistically significant better fit to the term structure of swap spreads indexed to three-month LIBOR. Economically, however, the improvement of A(2, 2, 2) over A(2, 2, 1) is modest (about 0.5 bp in terms of average RMSEs), and we do not expect more elaborate models to perform much better.

Panel B in Table 3 displays the results for the EUR market, which are similar to those obtained for the USD market. Overall, the model has a slightly better fit to the EUR data than the USD data. This is also apparent from Table 2, where, for each specification, the estimated variance on the pricing errors is smaller for the EUR market.

More information about pricing errors are provided in the online Appendix, where we display RMSEs for each point on the term structures of OIS rates, interest rate spreads, and CDS spreads. For the A(2, 2, 1) and A(2, 2, 2) specifications, the fit is generally rather uniform along the term structures, although in some cases we observe a deterioration in the fit at very short or very long maturities—an issue that is often encountered in term structure modeling.\(^{30}\)

\(^{30}\) In our model, we assume that the OIS reference rate equals the average cost of unsecured overnight funding for LIBOR panel banks, which implies that the LIBOR-OIS spread goes to zero as maturity goes to zero. In principle, an interesting out-of-sample test of the model is the extent to which very short term LIBOR-OIS spreads implied by the model correspond to those observed in the data. In practice, however, very short-term LIBOR-OIS spreads are extremely noisy and display little correlation with longer-term spreads. For instance, in the USD market, the shortest LIBOR maturity is overnight and the correlation between changes in the overnight LIBOR-FF spread and changes in the three-month and six-month LIBOR-OIS spreads are 0.08 and −0.01, respectively. One would need to include an additional factor to the model to capture the largely idiosyncratic behavior at the very short end of the spread curve.
Because we value parsimony, in the following we use the \( A(2, 2, 1) \) specification to analyze the term structure of interbank risk in more detail.

5.4. Decomposing the term structure of interbank risk

We measure the default component as the hypothetical swap spread that would materialize if default risk were the only risk factor in the interbank money market. This is computed by setting the residual term to one, \( \Delta(t, T) = 1 \). The non-default component is then given by the difference between the fitted swap spread and the default-induced swap spread.

Table 4 displays, for each maturity, summary statistics of the two components. Focus first on the USD market. Panel A1 shows the decomposition of swap spreads indexed to three-month LIBOR. At the short end of the term structure, the default component is, on average, slightly smaller than the non-default component. As maturity increases, the default component, on average, decreases rapidly with maturity. The upshot is that, as maturity increases, default increasingly becomes the dominant component. Panel A2 shows the decomposition of swap spreads indexed to six-month LIBOR. At the short end of the term structure, the default component is, on average, larger than the non-default component. Otherwise, the pattern is the same, with default increasingly becoming the dominant component as maturity increases. Another observation from Table 4 is that both components are very volatile, particularly at the short end of the term structure.

Fig. 4 displays the time series of the default and non-default components of the three-month and six-month LIBOR-OIS spreads (Panels A and B) and the five-year swap spreads indexed to three-month or six-month LIBOR (Panels C and D) in the USD market. Consider first the LIBOR-OIS spreads. Prior to the Lehman default, the default component constitutes a relatively small part of spreads, except for a brief period around the Bear Stearns near-default. In the aftermath of the Lehman default, the non-default component increases rapidly but then declines, while the default component increases gradually. The result is that by March 2009 and for the rest of the sample period, including the European sovereign debt crisis, spreads are almost exclusively driven by the default component. Consider next the five-year swap spreads. Clearly, default is the overall more important component. Even prior to the Lehman default, the default component is the dominant driver of spreads. Immediately after the Lehman default, both the default and non-default components increase after which the default component gradually becomes the sole driver of spreads.

Turn next to the EUR market. The summary statistics of the default and non-default components in Panels B1 and B2 in Table 4 are similar to those of the USD market. However, comparing Fig. 5 with Fig. 4 shows that interbank risk in the EUR market is generally lower than interbank risk in the USD market during the first half of the sample period, while the opposite is true during the second half. This is consistent with the observation that banks’ exposures to US structured credit products was an important source of interbank risk in the first half of the sample period, and their exposures to European sovereign debt was a major source of interbank risk in the second half.

As a plausibility check of our model-based decomposition, we also perform a simple regression-based decomposition in which each interest rate spread (LIBOR-OIS

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**Table 4**

Decomposition of the term structure of interbank risk.

The table shows the decomposition of the spread term structures using the \( A(2, 2, 1) \) specification and the \( \text{CSD}\text{SPREAD}_{\text{default}} \) and \( \text{CSD}\text{SPREAD}_{\text{non-default}} \) measures of interbank default risk in the US dollar and euro markets, respectively. Each spread is decomposed into a default and a non-default component, and the table displays means and, in parentheses, standard deviations of the two components. \( \text{SPREAD}_{\text{default}} \) and \( \text{SPREAD}_{\text{non-default}} \) denote the spread term structures indexed to the three-month and six-month London Interbank Offered Rate (LIBOR) or European Interbank Offered Rate (EURIBOR), respectively. Units are basis points. Each time series consists of 895 daily observations from August 9, 2007 to January 12, 2011, except those marked with †, which consist of 643 daily observations from July 28, 2008 to January 12, 2011.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>3M</th>
<th>6M</th>
<th>1Y</th>
<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
</tr>
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<tbody>
<tr>
<td><strong>Panel A1: SPREAD\text{_}^{\text{3M, US dollar market}}</strong></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Default</td>
<td>28.1</td>
<td>25.2</td>
<td>24.0</td>
<td>23.8</td>
<td>23.9</td>
<td>21.4</td>
<td>19.2</td>
<td></td>
<td>28.6†</td>
</tr>
<tr>
<td>Non-default</td>
<td>33.4</td>
<td>20.4</td>
<td>10.6</td>
<td>7.2</td>
<td>5.5</td>
<td>4.5</td>
<td></td>
<td></td>
<td>1.9†</td>
</tr>
<tr>
<td><strong>Panel A2: SPREAD\text{_}^{\text{3M, US dollar market}}</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Default</td>
<td>45.9</td>
<td>43.1</td>
<td>40.9</td>
<td>40.5</td>
<td>40.7</td>
<td>41.0</td>
<td></td>
<td></td>
<td>48.6†</td>
</tr>
<tr>
<td>Non-default</td>
<td>38.3</td>
<td>29.6</td>
<td>15.6</td>
<td>10.6</td>
<td>8.1</td>
<td>6.7</td>
<td></td>
<td></td>
<td>2.9†</td>
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<tr>
<td><strong>Panel B1: SPREAD\text{_}^{\text{3M, Euro market}}</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td>28.6</td>
<td>24.2</td>
<td>22.5</td>
<td>22.1</td>
<td>22.1</td>
<td>22.2</td>
<td>22.7</td>
<td>23.4</td>
<td></td>
</tr>
<tr>
<td>Non-default</td>
<td>30.5</td>
<td>21.9</td>
<td>11.7</td>
<td>8.0</td>
<td>6.2</td>
<td>5.1</td>
<td>3.9</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B2: SPREAD\text{_}^{\text{3M, Euro market}}</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td>46.7</td>
<td>42.4</td>
<td>39.3</td>
<td>38.5</td>
<td>38.5</td>
<td>38.7</td>
<td>39.4</td>
<td>40.8</td>
<td></td>
</tr>
<tr>
<td>Non-default</td>
<td>34.1</td>
<td>31.0</td>
<td>17.3</td>
<td>11.9</td>
<td>9.2</td>
<td>7.6</td>
<td>5.8</td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>

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† Indicate observations during the first half of the sample period.
or IRS-OIS) is regressed on a constant and the CDS spread with the same maturity (except in case of the three-month LIBOR-OIS spread, when we regress on the six-month CDS spread due to the absence of a three-month CDS spread).

That is, we run the regression

$$\text{SPREAD}_δ = β_0 + β\text{CDS}_t + ε_t,$$

for each spread maturity and for δ equal to three and six months. The default component is taken to be $β\text{CDS}_t$, ensuring that the default component is zero when the CDS spread is zero. However, in view of Eqs. (12), (17), and (22), no linear relation exists between the default component in an IRS-OIS spread and the maturity-matched CDS spread. Therefore, the regression can serve only as an approximate check of the model-based decomposition.

In case of the USD market, Table 5 displays the means and, in parentheses, standard deviations of the default components obtained via the two approaches. The regression-based default component tends to be lower, on average, and less volatile than the model-based default component, particularly at longer maturities.31

The table also displays the $β$ estimates, which are all highly significant. For both tenors, $β$ decreases with swap maturity. This is consistent with the fact that the potential for refreshment of the LIBOR panel combined with the risk of credit quality deterioration makes the difference between the default component in an IRS-OIS spread and the maturity-matched CDS spread increase with swap maturity. For a given swap maturity, $β$ is lower for the three-month tenor than the six-month tenor, because a series of three-month refreshed default risk carries less risk than a series of six-month refreshed default risk.

Fig. 6 shows the time series of the model-based and regression-based default components of the three-month and six-month LIBOR-OIS spreads (Panels A and B) and the five-year swap spreads indexed to three-month and six-month LIBOR (Panels C and D) in the USD market. Qualitatively, the two approaches give similar results, confirming the plausibility of the model-based decomposition.

5.5. Understanding the non-default component

To understand the determinants of the non-default component, we investigate its relation with funding liquidity and market liquidity, which tend to be highly
interconnected; see, e.g., Brunnermeier and Pedersen (2009).

Funding liquidity. Given its over-the-counter structure, we do not have liquidity proxies that are specific to the market for unsecured interbank term funding. However, liquidity in this market is likely correlated with liquidity in the market for secured term funding, which is another vital source of financing for banks. We consider two liquidity proxies for term repos. The first proxy is the spread between three-month GC repo rates for agency-issued MBSs and Treasuries. This measure reflects funding cost differentials between securities that differ in their market liquidity. In addition to wider spreads, larger initial margins (or haircuts) would indicate lower repo market liquidity. However, in contrasts to haircuts on structured product collateral, haircuts on Treasuries and agency-issued MBSs were fairly stable throughout the crisis; see, e.g., Copeland, Martin, and Walker (2010) and Krishnamurthy, Nagel, and Orlov (2011).

The second proxy is the Fontaine and Garcia (2011) liquidity factor. This factor is estimated from the cross section of on-the-run premiums for Treasuries, which in turn depend on the funding advantage (or specialness) of on-the-run Treasuries in the repo market; see, e.g., Duffie (1996) and Jordan and Jordan (1997). These liquidity proxies, denoted Repospr, and FGt, respectively, are displayed in Panels A and B in Fig. 7.

Government bond market liquidity. We consider two proxies for government bond market liquidity. The first proxy is the Hu, Pan, and Wang (2010) liquidity factor, which is a daily aggregate of Treasury price deviations from fair value. Their argument is that lower liquidity allows more noise in the yield curve, as prices can deviate more from fundamental values before arbitrageurs step in to profit from misvaluations. The second proxy is the spread between government bonds and government-sponsored agency bonds with lower liquidity but the same credit risk. Following Longstaff (2004), we use the spread between yields on Resolution Funding Corporation (Reacorp) bonds and off-the-run Treasuries (specifically, we use the ten-year par yield spread). These liquidity

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32 There does exist a trading platform for EUR interbank deposits, e-Mid. However, the maturities of the traded deposits are almost exclusively overnight (see Angelini, Nobili, and Picillo, 2009), while we are interested in liquidity measures for longer-term deposits.

33 In addition to wider spreads, larger initial margins (or haircuts) would indicate lower repo market liquidity. However, in contrasts to haircuts on structured product collateral, haircuts on Treasuries and agency-issued MBSs were fairly stable throughout the crisis; see, e.g., Copeland, Martin, and Walker (2010) and Krishnamurthy, Nagel, and Orlov (2011).

34 In her analysis of the three-month EURIBOR-OIS spread, Schwartz (2010) uses a similar spread between yields on KfW bonds (guaranteed by the German government) and German government bonds as a proxy for liquidity. We experimented with this spread (again, the ten-year par
Table 5
Default component, model versus regression.

The table shows the default component of the term structure of interbank risk in the US dollar market obtained with two different approaches. “Model” shows the default components obtained with the \( \Lambda(2, 2.1) \) specification. “Regression” shows the default components obtained by regressing each interest rate spread [London Interbank Offered Rate minus overnight indexed swap (LIBOR-OIS) or interest rate swap minus overnight indexed swap (IBS-OIS)] on a constant and the credit default swap (CDS) spread with the same maturity (except in case of the three-month LIBOR-OIS spread, when we regress on the six-month CDS spread due to the absence of a three-month CDS spread). The table displays means and, in parentheses, standard deviations of the default components. \( \text{SPREAD}_{3M} \) and \( \text{SPREAD}_{6M} \) denote the spread term structures indexed to three-month and six-month LIBOR. Units are basis points. \( \hat{\beta} \) shows the regression coefficients on the CDS spreads along with Newey and West (1987) \( t \)-statistics using ten lags, in parentheses. Each time series consists of 895 daily observations from August 9, 2007 to January 12, 2011, except those marked with \( \dagger \), which consist of 643 daily observations from July 28, 2008 to January 12, 2011.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>3M</th>
<th>6M</th>
<th>1Y</th>
<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: ( \text{SPREAD}_{3M} ) Model</td>
<td>28.1</td>
<td>25.2</td>
<td>24.0</td>
<td>23.8</td>
<td>23.9</td>
<td>24.1</td>
<td>24.1</td>
<td>28.6</td>
<td>28.6^{\dagger}</td>
</tr>
<tr>
<td>Regression</td>
<td>24.1</td>
<td>24.8</td>
<td>24.8</td>
<td>24.1</td>
<td>21.7</td>
<td>19.9</td>
<td>19.9</td>
<td>18.9</td>
<td>18.9^{\dagger}</td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>0.36</td>
<td>0.35</td>
<td>0.31</td>
<td>0.28</td>
<td>0.23</td>
<td>0.20</td>
<td>0.20</td>
<td>0.16</td>
<td>0.16^{\dagger}</td>
</tr>
<tr>
<td>Panel B: ( \text{SPREAD}_{6M} ) Model</td>
<td>45.9</td>
<td>43.1</td>
<td>40.9</td>
<td>40.5</td>
<td>40.7</td>
<td>41.0</td>
<td>41.0</td>
<td>48.6</td>
<td>48.6^{\dagger}</td>
</tr>
<tr>
<td>Regression</td>
<td>42.2</td>
<td>40.1</td>
<td>37.5</td>
<td>35.6</td>
<td>32.3</td>
<td>30.0</td>
<td>30.0</td>
<td>24.2</td>
<td>24.2^{\dagger}</td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>0.62</td>
<td>0.57</td>
<td>0.48</td>
<td>0.42</td>
<td>0.35</td>
<td>0.30</td>
<td>0.30</td>
<td>0.20</td>
<td>0.20^{\dagger}</td>
</tr>
</tbody>
</table>

By construction, the regression residuals measure the variation in liquidity that is orthogonal to default risk. We then regress \( \xi_t \) on these unspanned liquidity components.

Because \( F_G \), \( HPW \), \( DFL_{t} \), and \( DFL_{fin} \), are available only until December 31, 2009, we use data up to this date in all regressions. Furthermore, since \( F_G \), \( DFL_{t} \), and \( DFL_{fin} \), are available only at a monthly frequency, we run all regressions on monthly data. We convert from daily to monthly time series by averaging the daily observations over the month, which mirrors the construction of \( DFL_{t} \), and \( DFL_{fin} \). Very similar results are obtained by using end-of-month observations. Finally, to avoid spurious results due to the high persistence of the unspanned liquidity components, we run the second-step regressions in first differences (unit root tests are available upon request).

Results. Table 6 displays pair-wise correlations between monthly changes in the unspanned liquidity components and monthly changes in \( \xi_t \). In general, the unspanned liquidity components are moderately correlated, except for \( HPW \), and \( RC_{sprt} \), which are relatively highly correlated, and \( DFL_{t} \) and \( DFL_{fin} \), which are almost perfectly correlated.

Table 7 displays results from univariate and multivariate regressions of monthly changes in \( \xi_t \) on monthly changes in the unspanned liquidity components. Consider first the USD market (Panel A of Table 7). In the univariate regressions, the coefficients on all the liquidity proxies are positive. \( F_G \) is marginally significant, \( DFL_{t} \) and \( DFL_{fin} \), are significant at conventional levels, and the rest are highly significant. Adjusted \( R^2 \)'s lie between 0.10 and 0.49. In a multivariate regression with all liquidity proxies except \( DFL_{fin} \), the adjusted \( R^2 \) increases to 0.62 but, due to multicollinearity, several of the liquidity proxies become insignificant and have the wrong sign.\(^{36}\) Removing the least significant regressors results in a specification with only \( Reposprt \) and \( RC_{sprt} \), both highly significant, and an adjusted \( R^2 \) of 0.64.

Consider next the EUR market (Panel B of Table 7). The results are generally consistent with those of the USD market. In the univariate regressions the adjusted \( R^2 \)'s vary between 0.03 and 0.44, with all but the two corporate bond liquidity proxies being significant. In the regression specification with only \( Reposprt \) and \( RC_{sprt} \), both are highly significant and the adjusted \( R^2 \) reaches 0.70.

Taken together, the results lend support to the conjecture that the non-default component of interbank risk largely captures liquidity effects not spanned by default risk.

5.6. Pricing of interbank risk

The model allows us to estimate the compensation that market participants require for bearing interbank risk. These results are necessarily tentative, because the relatively short sample period implies that the market price of risk parameters are estimated with some uncertainty.

(proxied by \( HPW \) and \( RC_{sprt} \), respectively, are displayed in Panels C and D in Fig. 7.

Corporate bond market liquidity. As proxies for corporate bond market liquidity, we use the Dick-Nielsen, Feldhutter, and Lando (2012) liquidity factors. These factors are aggregates of several bond-specific liquidity measures and liquidity risk measures. We consider both their liquidity factor for the overall corporate bond market and their liquidity factor for bonds issued by financial institutions. The latter liquidity factor is particularly interesting because bond issuance (covered or uncovered) represents an important source of longer-term funding for banks. These liquidity proxies are displayed in Panels E and F in Fig. 7 and are denoted \( DFL_{t} \) and \( DFL_{fin} \), respectively.

Approach. We focus on the \( \lambda(2, 2.1) \) specification, in which the non-default component is driven by \( \xi_t \). Because \( \xi_t \) captures the part of interbank risk that is unspanned by default risk, the relevant question is the extent to which \( \xi_t \) is related to the factors of funding and market liquidity, which are unspanned by default risk. For this reason, we first regress the liquidity proxies on the first two principal components of the CDS term structure of the panel.\(^{35}\)

\( (footnote continued) \)

yield spread) in our analysis of the EUR market, but found that the Refcorp-Treasury spread had better explanatory power.

\( 35 \) The first two principal components explain more than 99% of the variation in the CDS term structure of the LIBOR and EURIBOR panels.

\( (footnote continued) \)

Regressing on a larger number of principal components does not change the results in any significant way.

\( 36 \) We include only one of the two corporate bond liquidity proxies in the multivariate regression, because of their near-perfect correlation.
When discussing risk premiums, it is important to distinguish between the swap market and the unsecured interbank money market. In both markets, there are premiums associated with interest rate risk, variation in default risk, and unspanned liquidity risk. These premiums are captured by the market prices of risk on the driving Wiener processes. However, in the unsecured interbank money market, there is also a jump risk premium on the default event itself, if the mean loss rate differs under the objective and risk-neutral measures, see Yu (2002) and Jarrow, Lando, and Yu (2005). Because our data set does not allow us to estimate the default event risk premium, we focus on the risk premiums that are available in the swap market.

As we are interested in the compensation for exposure to interbank risk, we consider a swap spread strategy consisting of receiving the fixed rate in an IRS indexed to three-month LIBOR and paying the fixed rate in an OIS of the same maturity. This strategy is expected to be approximately delta-neutral with respect to pure interest rate risk. The time-$t$ value of the strategy is $V_{t}^{\text{SPR}} = V_{t}^{\text{IRS}} - V_{t}^{\text{OIS}}$, where $V_{t}^{\text{IRS}}$ and $V_{t}^{\text{OIS}}$ denote the time-$t$ values of an IRS and OIS, respectively, from the perspective of the party who receives the fixed rate and pays the floating rate. The risk-neutral dynamics of the marked-to-market value of this strategy are given by

$$dV_{t}^{\text{SPR}} = r(t)V_{t}^{\text{SPR}} \, dt + \frac{\partial V_{t}^{\text{SPR}}}{\partial r} \, \sigma_{r} \, dW_{r}(t) + \frac{\partial V_{t}^{\text{SPR}}}{\partial \gamma} \, \sigma_{\gamma} \, dW_{\gamma}(t)$$

$$+ \frac{\partial V_{t}^{\text{IRS}}}{\partial \nu} \sigma_{\nu} \sqrt{\nu(t)} \, dW_{\nu}(t) + \frac{\partial V_{t}^{\text{IRS}}}{\partial \mu} \sigma_{\mu} \sqrt{\mu(t)} \, dW_{\mu}(t)$$

$$+ \frac{\partial V_{t}^{\text{IRS}}}{\partial \xi} \sigma_{\xi} \sqrt{\xi(t)} \, dW_{\xi}(t).$$ (44)

As expected, in our estimated model $\frac{\partial V_{t}^{\text{SPR}}}{\partial r}$ and $\frac{\partial V_{t}^{\text{SPR}}}{\partial \gamma}$ are negligible compared with $\frac{\partial V_{t}^{\text{IRS}}}{\partial \nu}$, $\frac{\partial V_{t}^{\text{IRS}}}{\partial \mu}$, and $\frac{\partial V_{t}^{\text{IRS}}}{\partial \xi}$. Hence, the strategy has an almost pure exposure to interbank risk. With the market price of risk specification given in Eq. (34), the instantaneous Sharpe ratio is

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**Fig. 6.** Default component, model versus regression. The figure displays time series of the model-based and regression-based default components in the US dollar market. Panels A and B display default components of the three-month (3M) and six-month (6M) London Interbank Offered Rate-overnight indexed swap (LIBOR-OIS) spread, respectively. Panels C and D display default components of the five-year (5Y) interest rate swap-overnight indexed swap (IRS-OIS) spread indexed to three-month and six-month LIBOR, respectively. In each panel, the thin black line is the default component obtained with the $A(2,2,1)$ specification, and the thick gray line is the default component obtained by regressing the interest rate spread on a constant and the maturity-matched credit default swap (CDS) spread (except in case of the three-month LIBOR-OIS spread, when we regress on the six-month CDS spread due to the absence of a three-month CDS spread). Units are basis points. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece’s debt to non-investment-grade status by Standard and Poor’s on April 27, 2010. Each time series consists of 895 daily observations from August 9, 2007 to January 12, 2011.

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It follows from Eqs. (11) and (54) that $\frac{\partial V_{t}^{\text{IRS}}}{\partial \nu} = \frac{\partial V_{t}^{\text{IRS}}}{\partial \nu} = \frac{\partial V_{t}^{\text{IRS}}}{\partial \xi} = 0$, so that $\frac{\partial V_{t}^{\text{SPR}}}{\partial r} = \frac{\partial V_{t}^{\text{IRS}}}{\partial r}$, $\frac{\partial V_{t}^{\text{SPR}}}{\partial \gamma} = \frac{\partial V_{t}^{\text{IRS}}}{\partial \gamma}$, and $\frac{\partial V_{t}^{\text{SPR}}}{\partial \xi} = \frac{\partial V_{t}^{\text{IRS}}}{\partial \xi}$. 

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approximately given by

$$SR_{SPR} \approx \frac{\frac{\partial V^{IRS}}{\partial v} \sigma_{\mu} \mu(t) + \frac{\partial V^{IRS}}{\partial \mu} \sigma_{v} v(t) + \frac{\partial V^{IRS}}{\partial \xi(v)} \sigma_{\xi} \xi(t)}{\sqrt{\left(\frac{\partial V^{IRS}}{\partial v} \right)^2 \sigma_{v}^2 \mu(t) + \left(\frac{\partial V^{IRS}}{\partial \mu} \right)^2 \sigma_{\mu}^2 \mu(t) + \left(\frac{\partial V^{IRS}}{\partial \xi(v)} \right)^2 \sigma_{\xi}^2 \xi(t)}}. \tag{45}$$

Fig. 8 displays time series of $SR_{SPR}$ at the one-year, five-year, and ten-year swap maturities in the USD market (results for the EUR market are similar). Due to their fast mean-reversion, $\mu(t)$ and $\xi(t)$ mainly affect expectations about future LIBOR rates in the near term. This implies that, for maturities beyond approximately one year, the spread strategy’s loadings on $W_s(t)$ and $W_r(t)$ have very little dependence on maturity. In contrast, $\mu(t)$ also impacts expectations about future LIBOR rates in the longer term, and the spread strategy’s loading on $W_s(t)$ increases with maturity over the entire maturity range. The sample averages of the market prices of risk on $W_s(t)$ and $W_r(t)$ are $-0.28$ and $-0.20$, respectively, while we are not able to reliably estimate the market price of risk on $W_r(t)$, which is set to zero (see end of Section 4.3). In the first part of the sample period, when unspanned liquidity risk is an important component of interbank risk, the strategy is primarily exposed to $W_s(t)$ for which there is no compensation. This is particularly the case for short swap maturities. Therefore, Sharpe ratios are low and increasing with swap maturity. The sole exceptions is the period around the Bear Stearns near-bankruptcy, when default risk briefly became the main driver of interbank risk. In the second part of the sample period, when default risk is the most important component of interbank risk, the strategy is mainly exposed to $W_s(t)$ and $W_r(t)$, and Sharpe ratios are, therefore, larger during this period. For instance, the instantaneous Sharpe ratio at the five-year swap maturity is estimated to have averaged 0.35 from early 2009 to the end of the sample period. This estimate is at the lower end of the range of Sharpe ratios reported by Duarte, Longstaff, and Yu (2007) for spread arbitrage strategies between IRS and Treasuries. For a pre-crisis sample, they report realized Sharpe ratios between 0.37 and 0.66, depending on maturity.

6. Conclusion

In this paper, we contribute to the rapidly growing literature on the interbank money market by studying the term structure of interbank risk. We follow most existing studies by measuring interbank risk by the spread between a LIBOR rate and the rate on a maturity-matched overnight indexed swap. We show that the spread between the fixed rate on an interest rate swap indexed to, say, three-month LIBOR and an OIS of the same maturity reflects risk-neutral expectations about future three-month LIBOR-OIS spreads and, therefore, future three-month interbank risk. This allows us to infer a term structure of interbank risk from IRS-OIS spreads of different maturities. We develop a dynamic term structure model with default risk in the interbank money market that, in conjunction with information from the credit

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**Fig. 7.** Liquidity proxies. Repospr, denotes the spread between three-month general collateral repo rates for agency-issued MBSs and Treasuries (in basis points). $FG_t$ denotes the Fontaine and Garcia (2011) liquidity factor. $HPW_t$ denotes the Hu, Pan, and Wang (2010) liquidity factor. $RC_{sprt}$ denotes the spread between the fixed rate on an interest rate swap and the rate on a maturity-matched overnight indexed swap. $DFL_t$ and $DFLt_{int}$ denotes the Dick-Nielsen, Feldhutter, and Lando (2012) liquidity factors for the overall corporate bond market and for bonds issued by financial institutions, respectively. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008 and the Lehman Brothers bankruptcy filing on September 15, 2008. Each time series consists of 27 monthly observations from August 2007 to December 2010.
default swap market, allows us to decompose the term structure of interbank risk into default and non-default components. We apply the model to study interbank risk from the onset of the financial crisis in August 2007 until January 2011. We find that, on average, the fraction of total interbank risk due to default risk increases with maturity. At the short end of the term structure, the non-default component is important in the first half of the sample period and is correlated with various measures of funding liquidity and market liquidity. At longer maturities, the default component is the dominant driver of interbank risk throughout the sample period. We also provide tentative results indicating that swap market participants require compensation for exposure to variation in interbank default risk. Our results hold true in both the USD and EUR markets and are robust to different model parameterizations and measures of interbank default risk. In addition to providing insights into the dynamics and determinants of interbank risk, the model provides a framework for pricing, hedging, and risk management of interest rate swaps in the presence of significant basis risk.

### Appendix A. Proof of Eq. (2)

Discounting the integral equation (1) gives

$$\Delta \text{Repos}_t = \text{RC}_{s, t}^{\text{PR}} + \text{HF}_{W, t}^{\text{PR}} + \Delta \text{DFL}_{t}^{\text{PR}} + \Delta \text{DFLfin}_{t}^{\text{PR}}.$$  

### Table 6

**Pair-wise correlations.**

The table displays pair-wise correlations between monthly changes in \( \xi \) and the funding and market liquidity components that are unspanned by default risk. \( \xi \) is estimated using the \( \Delta (2, 1) \) specification. The unspanned liquidity components are the residuals from regressions of liquidity proxies on the first two principal components of the panel credit default swap (CDS) term structure. \( \text{Repos}_t^{\text{PR}} \) denotes the spread between three-month general collateral repo rates for agency-issued MBSs and Treasuries (in basis points). \( \text{FG}_t \) denotes the Fontaine and Garcia (2011) liquidity factor. \( \text{DFL}_t \) denotes the Dick-Nielsen, Feldhutter, and Lando (2012) liquidity factors for the overall corporate bond market and for bonds issued by financial institutions, respectively.

<table>
<thead>
<tr>
<th>Panel</th>
<th>US dollar market</th>
<th>Euro market</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \text{Repos}_t )</td>
<td>( \Delta \text{FG}_t )</td>
<td>( \Delta \text{DFL}_t )</td>
</tr>
<tr>
<td>( \Delta \text{FG}_t )</td>
<td>0.41</td>
<td>0.50</td>
</tr>
<tr>
<td>( \Delta \text{DFL}_t )</td>
<td>0.54</td>
<td>0.45</td>
</tr>
<tr>
<td>( \Delta \text{RCsprt}_t )</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>( \Delta \text{DFLfin}_t )</td>
<td>0.41</td>
<td>0.31</td>
</tr>
<tr>
<td>( \Delta \text{FG}_t )</td>
<td>0.26</td>
<td>0.27</td>
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<td>( \Delta \text{FG}_t )</td>
<td>0.63</td>
<td>0.30</td>
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<td>( \Delta \text{FG}_t )</td>
<td>0.71</td>
<td>0.21</td>
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<td>( \Delta \text{FG}_t )</td>
<td>0.43</td>
<td>0.11</td>
</tr>
<tr>
<td>( \Delta \text{FG}_t )</td>
<td>0.37</td>
<td>0.99</td>
</tr>
</tbody>
</table>

### Table 7

**Non-default component and liquidity.**

The table displays results from regressing monthly changes in \( \xi \) on monthly changes in the funding and market liquidity components that are unspanned by default risk. \( \xi \) is estimated using the \( \Delta (2, 1) \) specification. The unspanned liquidity components are the residuals from regressions of liquidity proxies on the first two principal components of the panel credit default swap (CDS) term structure. \( \text{Repos}_t^{\text{PR}} \) denotes the spread between three-month general collateral repo rates for agency-issued MBSs and Treasuries (in basis points). \( \text{FG}_t \) denotes the Fontaine and Garcia (2011) liquidity factor. \( \text{DFL}_t \) denotes the Dick-Nielsen, Feldhutter, and Lando (2012) liquidity factors for the overall corporate bond market and for bonds issued by financial institutions, respectively.

$$M(t) = e^{-\int_0^t \sigma(V(t)) \, dt} V(t) + \int_0^t e^{-\int_u^t \sigma(V(u)) \, dt} (r(t) - r_c(t)) V(u) \, du$$

is a Q-martingale. We obtain

$$d\left( e^{-\int_0^t \tau(t) \, dt} V(t) \right) = -\tau(t) e^{-\int_0^t \tau(t) \, dt} V(t)$$

Integration by parts then implies

$$d\left( e^{-\int_0^t \tau(t) \, dt} V(t) \right) = d\left( e^{-\int_0^t \tau(t) \, dt} V(t) \right) e^{-\int_0^t \tau(t) \, dt} V(t)$$

$$= e^{-\int_0^t \tau(t) \, dt} V(t) \int_0^t e^{-\int_u^t \tau(t) \, dt} (r(t) - r_c(t)) \, dt$$

$$+ e^{-\int_0^t \tau(t) \, dt} V(t) \int_0^t e^{-\int_u^t \tau(t) \, dt} (r(t) - r_c(t)) \, dt + dm(t).$$

Hence,

$$M(t) = e^{-\int_0^t \tau(t) \, dt} V(t) + \int_0^t e^{-\int_u^t \tau(t) \, dt} (r(t) - r_c(t)) V(u) \, du$$

(47)

is a Q-martingale. We obtain

$$d\left( e^{-\int_0^t \tau(t) \, dt} V(t) \right) = -\tau(t) e^{-\int_0^t \tau(t) \, dt} V(t)$$

(48)

Integration by parts then implies

$$d\left( e^{-\int_0^t \tau(t) \, dt} V(t) \right) = d\left( e^{-\int_0^t \tau(t) \, dt} V(t) \right) e^{-\int_0^t \tau(t) \, dt} V(t)$$

$$= e^{-\int_0^t \tau(t) \, dt} V(t) \int_0^t e^{-\int_u^t \tau(t) \, dt} (r(t) - r_c(t)) \, dt$$

$$+ e^{-\int_0^t \tau(t) \, dt} V(t) \int_0^t e^{-\int_u^t \tau(t) \, dt} (r(t) - r_c(t)) \, dt + dm(t).$$

(49)
Hence, \( e^{-\int_0^T r_s \, ds} V(t) \) is a Q-martingale, and because \( V(T) = X \) we conclude that
\[
e^{-\int_0^T r_s \, ds} V(t) = \mathbb{E}_t^Q \left[ e^{-\int_0^T r_s \, ds} X \right],
\]
which proves Eq. (2).

**Appendix B. Extended doubly stochastic framework**

Here, we briefly recap and extend the standard doubly stochastic framework for modeling default times in our setting.

The main aspect of our extension is that we can incorporate an arbitrary number of default times in one framework. We assume that the filtered probability space \((\Omega, \mathcal{F}, \mathcal{F}_t, Q)\) carries an i.i.d. sequence of standard exponential random variables \(\varepsilon(t_0) \sim \text{Exp}(1)\), for \(t_0 \geq 0\), which are independent of \(\mathcal{F}_\infty\). For every \(t_0 \geq 0\), we let \(\tau(t_0)\) be a non-negative \(\mathcal{F}_t\)-adapted intensity process with the property
\[
\int_{t_0}^t \lambda(t_0, s) \, ds < \infty
\]
for all finite \(t \geq t_0\). We then define the random time
\[
\tau(t_0) = \inf \left\{ t > t_0 \mid \int_{t_0}^t \lambda(t_0, s) \, ds \geq \tau(t_0) \right\} > t_0.
\]
\(\tau(t_0)\) is not an \(\mathcal{F}_t\)-stopping time but becomes a stopping time with respect to the enlarged filtration \(\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t\) where \(\mathcal{H}_t = \bigvee_{s \leq t} \sigma(H(t_0, s) \wedge \tau(t))\) is the filtration generated by all \(\tau(t_0)\)-indicator processes \(H(t_0, t) = 1_{(\tau(t_0) \leq t)}\). The \(\mathcal{G}_t\)-stopping times \(\tau(t_0)\) are then \(\mathcal{F}_t\)-doubly stochastic in the sense that
\[
\mathbb{E}^Q \left[ Y \mathbb{1}_{(\tau(t_0) > T)} \mid \mathcal{G}_t \right] = \mathbb{E}^Q \left[ \frac{Y}{\mathbb{E}^Q \left[ \exp(-\int_0^\tau \lambda(t_0, s) \, ds) \mid \mathcal{G}_t \right]} \right]
\]
for all \(\mathcal{F}_t\)-measurable non-negative random variables \(Y\); see, e.g., Filipović (2009, Lemma 12.2).

**Appendix C. Pricing formulas for the affine model**

We derive the pricing formulas for the affine model used in this paper. It is evident from the system of stochastic differential equations composed of Eqs. (26), (30), and (33) that the partial state vectors \((r(t), \gamma(t))\), \((\mu(t), \lambda(t_0, t))\), \((\mu(t), \lambda(t_0, t))\), and \((\gamma(t), \epsilon(t))\) form independent autonomous affine jump-diffusion processes. Hence, the subsequent exponential-affine expressions (54), (56), and (62) follow directly from the general affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2) and the fact that \(r_1(t) = r(t) + \Lambda\), see Eq. (21). The following formulas are for the full \(\lambda(2, 2, 2)\) model. The nested versions, \(\lambda(2, 1, 1)\), are obtained by setting the respective model parameters, \(\kappa, \theta, \sigma, \kappa_1, \theta_1, \sigma_1\), equal to zero and setting \(\epsilon(t) \equiv \theta_2\) and \(\mu(t) \equiv \theta_3\), respectively.

**Lemma C.1.** The time \(t\)-price of the collateralized zero-coupon bond maturing at \(T\) equals
\[
P_c(t, T) = \mathbb{E}_t^Q \left[ e^{-\int_0^T r_s \, ds} \right] = \exp \left\{ A(T-t) + B_c(T-t) r(t) + B_e(T-t) \gamma(t) \right\},
\]
where the functions \(A\) and \(B = (B_c, B_e)^\top\) solve the system of Riccati equations
\[
\begin{align*}
\partial_t A(t) &= \frac{\sigma^2}{2} \sigma_c B_c(t)^2 + \rho \sigma \sigma_c B_c(t) B_e(t) + \frac{\sigma^2}{2} B_e(t)^2 + \kappa_1 \theta B_e(t) - \Lambda \\
\partial_t B_c(t) &= -\kappa_c B_c(t) - 1 \\
\partial_t B_e(t) &= -\kappa_e B_e(t) + \kappa_c B_c(t) \\
A(0) &= 0, \quad B(0) = 0.
\end{align*}
\]

**Lemma C.2.** The time \(t\)-value of an unsecured loan with notional 1 in Eq. (18) equals
\[
B(t_0, T) = \mathbb{E}_{t_0}^Q \left[ e^{-\int_0^T (r_s + \epsilon(t_0, s)) \, ds} \right] = P_c(t_0, T) \exp \left( C(T-t_0) + D_c(T-t_0) \mu(t_0) + D_e(T-t_0) \lambda(t_0) \right)
\]
where the functions \(C\) and \(D = (D_c, D_e, D_\lambda)^\top\) solve the system of Riccati equations
\[
\begin{align*}
\partial_t C(t) &= \kappa_c \partial_t D_c(t) + \kappa_1 \Lambda D_c(t) + \Lambda \\
\partial_t D_c(t) &= \frac{\sigma^2}{2} D_c(t)^2 - \kappa_c D_c(t) + \frac{D_\lambda(t)}{\zeta_c} \\
\partial_t D_e(t) &= -\kappa_e D_e(t) + \kappa_1 \kappa_c D_c(t) + \kappa_1 D_\lambda(t) - \frac{D_\lambda(t)}{\zeta_c}.
\end{align*}
\]
\begin{align*}
\frac{d}{dt} D_\mu(t) &= \frac{\sigma^2}{2} D_\mu(t) - \kappa_\mu D_\mu(t) + \kappa_\nu D_\nu(t) \\
\frac{d}{dt} D_\nu(t) &= -\kappa_\nu D_\nu(t) + 1 \\
C(0) &= 0, \quad D(0) = 0. \tag{57}
\end{align*}

Proof. We write
\begin{equation}
B(t_0, T) = E_{t_0} \left[ t_0^{r(t_0)} \right] = E_{t_0} \left[ \exp \int_{t_0}^T (r(t) - \lambda_0(t, T)) dt \right]. \tag{58}
\end{equation}

Now the claim follows from the general affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2). Note that \( D_\mu(t) < 0 \) for all \( t > 0 \). Hence, the rational function on the right-hand side of the equation for \( \frac{d}{dt} D_\mu(t) \) is well defined and derived by
\begin{align*}
\int_0^\infty \left( e^{D_\mu(t)} - 1 \right) \zeta e^{\zeta t} \, d\zeta &= \zeta \int_0^\infty \left( e^{\zeta t} - 1 \right) \zeta \, d\zeta - 1 \\
&= \frac{D_\mu(t)}{\zeta_\mu - D_\mu(t)} - 1 \\
&= \frac{D_\mu(t)}{\zeta_\mu - D_\mu(t)}. \quad \square \tag{59}
\end{align*}

We obtain the following exponential affine expression for the \((T-t_0)\)-maturity LIBOR rate \( L(t_0, T) \).

**Corollary C.3.** The \((T-t_0)\)-maturity LIBOR rate given in Eq. (19) equals
\begin{equation}
L(t_0, T) = \frac{1}{T-t_0} \left( P(t_0, T) \right)^{-1} \exp \left[ -\left( C(T-t_0) - D_\mu(T-t_0) k_\nu(t_0) - D_\nu(T-t_0) \lambda - 1 \right) \right] \\
\times \exp \left[ -E(T-t_0) - F(t_0) k_\nu(t_0) - F(t_0) \kappa_\nu \right] \\
\tag{60}
\end{equation}
with \( C(T-t_0) \) and \( D(T-t_0) \) given in Lemma C.2 and where the functions \( E \) and \( F = \left( F_\mu, F_\nu \right)^T \) solve the Riccati equations
\begin{align*}
\frac{d}{dt} E(t) &= \kappa_\mu F_\mu(t) \\
\frac{d}{dt} F_\mu(t) &= \frac{\sigma^2}{2} F_\mu(t)^2 - \kappa_\mu F_\mu(t) - 1 \\
\frac{d}{dt} F_\nu(t) &= \frac{\sigma^2}{2} F_\nu(t)^2 - \kappa_\nu F_\nu(t) + \kappa_\nu F_\mu(t) \\
E(0) &= 0, \quad F(0) = 0. \tag{61}
\end{align*}

Proof. In view of Eq. (31) and the affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2), the multiplicative residual term is given by
\begin{equation}
\frac{1}{\Xi(t_0, T)} = \exp \left[ \frac{\kappa_\nu}{\kappa_\mu} \left( F(t_0) - F(t_0) k_\nu \right) + \frac{\sigma^2}{2} \right], \tag{62}
\end{equation}
where the functions \( E \) and \( F = \left( F_\mu, F_\nu \right)^T \) solve the Riccati equations (61). The corollary now follows from Eq. (19) and Lemma C.2. \( \square \)

In view of Eq. (7) we also need a closed form expression for
\begin{equation}
I = E_T \left[ \exp \left( \int_{t_0}^{r(t_0)} dt \right) L(t_0, T) \right]. \tag{63}
\end{equation}

for time points \( t \leq t_0 < T \). Using the tower property of conditional expectations we calculate
\begin{align*}
I &= E_T \left[ \exp \left( \int_{t_0}^{r(t_0)} dt \right) E_T \left[ \exp \left( \int_{t_0}^{r(t_0)} dt \right) (T-t_0) L(t_0, T) \right] \right] \\
&= E_T \left[ \exp \left( \int_{t_0}^{r(t_0)} dt \right) P(t_0, T) (T-t_0) L(t_0, T) \right] \\
&= \left( E_T \left[ \exp \left( \int_{t_0}^{r(t_0)} dt \right) P(t_0, T) \right] \right) \times \left( E_T \left[ \exp \left( \int_{t_0}^{r(t_0)} dt \right) \exp \left[ C(T-t_0) - D_\mu(T-t_0) k_\nu(t_0) - D_\nu(T-t_0) \lambda \right] \right] \right) \\
&= \left( P(t_0, T) \exp \left[ C(T-t_0) - D_\mu(T-t_0) k_\nu(t_0) - D_\nu(T-t_0) \lambda \right] \right) \times \left( \exp \left[ -E(T-t_0) - F(t_0) k_\nu(t_0) - F(t_0) \kappa_\nu \right] \right) \\
&= \left( \exp \left[ -E(T-t_0) - F(t_0) k_\nu(t_0) - F(t_0) \kappa_\nu \right] \right) \left( P(t_0, T) \exp \left[ C(T-t_0) - D_\mu(T-t_0) k_\nu(t_0) - D_\nu(T-t_0) \lambda \right] \right) \\
&= \left( \exp \left[ -E(T-t_0) - F(t_0) k_\nu(t_0) - F(t_0) \kappa_\nu \right] \right) \times \left( P(t_0, T) \right) \times \left( \exp \left[ C(T-t_0) - D_\mu(T-t_0) k_\nu(t_0) - D_\nu(T-t_0) \lambda \right] \right) \\
&= \left( \exp \left[ -E(T-t_0) - F(t_0) k_\nu(t_0) - F(t_0) \kappa_\nu \right] \right) \times \left( P(t_0, T) \right) \times \left( \exp \left[ C(T-t_0) - D_\mu(T-t_0) k_\nu(t_0) - D_\nu(T-t_0) \lambda \right] \right).
\end{align*}

The conditional expectations on the right-hand side of the last equality can easily be obtained in closed form using the affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2).

It remains to be checked whether the above conditional expectations are well defined. Sufficient admissibility conditions on the model parameters are provided by the following lemma, the proof of which is in the online Appendix.

**Lemma C.4.** (i) Suppose \( k_\mu \geq 0 \), and define
\begin{align*}
\theta_\mu &= \sqrt{\kappa_\mu^2 + \frac{\sigma^2}{\zeta_\mu \kappa_\mu + 1}} \tag{65} \\
\kappa_\mu^2 &= \frac{2}{\zeta_\mu \kappa_\mu + 1} \tag{66} \\
C_\mu &= \frac{2 \kappa_\mu C_\nu}{\theta_\mu \left( \exp \left[ \frac{C_\nu}{\theta_\mu} \right] - 1 \right)} \tag{67} \\
\theta_\mu &= \sqrt{\kappa_\mu^2 + \frac{2 \kappa_\mu^2 C_\nu}{\theta_\mu \left( \exp \left[ \frac{C_\nu}{\theta_\mu} \right] - 1 \right)}} \tag{68} \\
\kappa_\mu &> \frac{1}{2} \sigma^2 C_\nu \tag{69}
\end{align*}
and
\begin{align*}
\kappa_\mu &\geq \frac{\sigma^2 C_\nu e^{-\left( k_\mu / 2 \right)^2}}{2} + \frac{4 \kappa_\mu^2 \sigma^2}{2 \kappa_\mu \sigma^2} \left[ 2 F_1 \left( \frac{1}{\theta_\mu}, \frac{\kappa_\mu}{2 \kappa_\mu}, \frac{2 \kappa_\mu^2 C_\nu - 2 \kappa_\mu}{2 \kappa_\mu^2 C_\nu}, \frac{\sigma^2 C_\nu}{\kappa_\mu \sigma^2} \right) \right. \\
&\left. + e^{-\left( k_\mu / 2 \right)^2} \right] F_1 \left( \frac{1}{\theta_\mu}, \frac{\kappa_\mu}{2 \kappa_\mu}, \frac{2 \kappa_\mu^2 C_\nu - 2 \kappa_\mu}{2 \kappa_\mu^2 C_\nu}, \frac{\sigma^2 C_\nu}{\kappa_\mu \sigma^2} \right) \right) \tag{70}
\end{align*}
where \( F_1 \) denotes the Gauss hypergeometric function and
\begin{align*}
\tau^* &= \frac{1}{k_\nu} \log \max \left\{ \frac{2 \kappa_\mu - \sigma^2 C_\nu}{2 \kappa_\mu}, \frac{2 \kappa_\mu - \sigma^2 C_\nu}{2 \kappa_\mu} \right\} \tag{71}
\end{align*}
then
\[ E^Q\left[ \exp\left[ -D(t-T_0)u(t_0) - D_\nu(T_0)u(t_0) \right] \right] < \infty. \] (72)

(II) Define
\[ \theta_\xi = \sqrt{\kappa_\xi^2 + 2\sigma^2_\xi}, \] (73)
\[ C_\xi = \frac{2(e^{\theta_\xi(T_0)} - 1)}{\theta_\xi(e^{\theta_\xi(T_0)} + 1) + \kappa_\xi(e^{\theta_\xi(T_0)} - 1)}, \] (74)
\[ \theta_\epsilon = \sqrt{\kappa_\epsilon^2 + 2\sigma^2_\epsilon \kappa_\epsilon}, \] (75)
\[ C_\epsilon = \frac{2\kappa_\epsilon C_\epsilon (e^{\theta_\epsilon(T_0)} - 1)}{\theta_\epsilon(e^{\theta_\epsilon(T_0)} + 1) + \kappa_\epsilon(e^{\theta_\epsilon(T_0)} - 1)}. \] (76)

If conditions (69) and (70) hold for \( C_\xi, \kappa_\xi, \sigma_\xi, C_\mu, \kappa_\mu, \sigma_\mu \) replaced by \( C_\xi, \kappa_\epsilon, \sigma_\epsilon, C_\kappa, \kappa_\kappa, \sigma_\kappa \), respectively, then
\[ E^Q\left[ \exp\left[ -F_\xi(T_0)\xi(T_0) - F_\epsilon(T_0)\epsilon(T_0) \right] \right] < \infty. \] (77)

**Remark C.5.** Note that \( \nu^2 = 0 \) if and only if \( k_{\epsilon}^2/(2\kappa_\epsilon \sigma_\epsilon^2) \geq C_\epsilon \). In this case, Eq. (70) reads as \( \kappa_\mu \geq \sigma_\mu^2 C_\mu \), which is automatically satisfied as is shown at the end of the proof of Lemma C.4.

For the CDS coupon rate calculations, we need the respective exponential affine expressions for (23)–(25). For \( I_1(t_0, T) \) we obtain
\[ l_1(t_0, T) = \sum_{i=1}^{N} (t_i - t_{i-1}) e^{-\kappa_{\mu}(t_i-t_0)} B(t_0, t_i). \] (78)

In both formulas for \( l_2(t_0, T) \) and \( V_{prot}(t_0, T) \), the following expression shows up:
\[ f(t_0, u) = E^Q\left[ e^{-\int_{t_0}^{T} (\theta(t,s)+\lambda(t,s)) ds} \lambda(t_0, u) \right]. \] (79)

**Lemma C.6.** We have
\[ J(t_0, u) = (g(u-t_0) + h_v(u-t_0)u(t_0) + h_m(u-t_0)p(t_0)) \] (80)
\[ + h_\nu(u-t_0)\lambda) e^{-\kappa_{\mu}(t_0-t_\nu)} B(t_0, u), \]
where the functions \( g \) and \( h = (h_v, h_m, h_\nu)^T \) solve the linear inhomogeneous system of ordinary differential equations
\[ \partial_t g(t) = \kappa_{\mu} g(t) + \kappa_{\nu} h_\nu(t) \] (81)
\[ \partial_t h_v(t) = \sigma^2_{\nu} D_\nu(t) h_v(t) - \kappa_{\nu} h_v(t) + \left( \frac{c_{\nu}}{\lambda_{\nu} - D_\nu(t)} \right)^2 \] (82)
\[ \partial_t h_m(t) = \sigma^2_{\nu} D_\nu(t) h_m(t) - \kappa_{\mu} h_m(t) + \kappa_{\nu} h_v(t) \] (83)
\[ \partial_t h_\nu(t) = -\kappa_\nu h_\nu(t) \] (84)
\[ g(0) = 0, \quad h_v(0) = (0, 0, 1)^T \]
and where the functions \( D = (D_\nu, D_\mu, D_\lambda)^T \) are given in **Lemma C.2**.

**Proof.** We first decompose \( J(t_0, u) = P_c(t_0, u)l(t_0, u) \) with
\[ l(t_0, u) = E^Q\left[ e^{-\int_{t_0}^{T} \lambda(t,s) ds} \lambda(t_0, u) \right]. \] (85)

which we can compute by differentiating the respective moment generating function \( I(t, u) = \frac{d}{dv} E^Q\left[ e^{-\int_{t_0}^{T} \lambda(t,s) ds} \lambda(t_0, u) \right] \big|_{v=0}. \) (86)

The affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2) gives us
\[ E^Q\left[ e^{-\int_{t_0}^{T} \lambda(t,s) ds} \lambda(t_0, u) \right] = \exp(G(u-t_0, v) + H_\nu(u-t_0, v)\mu(t_0)) \] (87)
\[ + H_m(u-t_0, v)\mu(t_0) + H_m(u-t_0, v)\lambda), \] (88)
where the functions \( G \) and \( H = (H_v, H_m, H_\nu)^T \) solve the system of Riccati equations
\[ \partial_t G(\tau, v) = \kappa_v g(\tau, v) + \kappa_\nu H_v(\tau, v) \] (89)
\[ \partial_t H_v(\tau, v) = \sigma^2_v H_v(\tau, v)^2 - \kappa_v H_v(\tau, v) + \frac{H_m(\tau, v)}{\xi_v - H_v(\tau, v)} \] (90)
\[ \partial_t H_m(\tau, v) = \sigma^2_v H_m(\tau, v)^2 - \kappa_v H_m(\tau, v) + \kappa_v H_v(\tau, v) \] (91)
\[ \partial_t H_\nu(\tau, v) = -\kappa_\nu H_\nu(\tau, v) - 1 \] (92)
\[ G(0, v) = 0. \quad H_v(0, v) = (0, 0, 0)^T. \] (93)

Hence, from Eq. (93) we obtain
\[ I(t_0, u) = \left(g(u-t_0) + h_v(u-t_0)v(t_0) + h_m(u-t_0)p(t_0) + h_\nu(u-t_0)\lambda) \right) \] (94)
\[ \exp(G(u-t_0, 0) + H_v(u-t_0, 0)v(t_0)) \] (95)
\[ + H_m(u-t_0, 0)p(t_0) + H_m(u-t_0, 0)\lambda), \]
where \( g(\tau) = (d/dv) G(\tau, v) \big|_{v=0} \) and \( h = (h_v, h_m, h_\nu)^T \) is given by \( h(\tau) = (d/dv) H(\tau, v) \big|_{v=0} \). Note that \( G(\tau, 0) = C(\tau) - \tau A \) and \( H_v(\tau, 0) = D(\tau) \), see **Lemma C.2**. Differentiating both sides of the system (85) in \( v \) at \( v=0 \) shows that the functions \( g \) and \( h \) solve the linear inhomogeneous system of ordinary differential equations (81). Thus, the lemma is proved.

**Appendix D. Maximum likelihood estimation and the unscented Kalman filter**

If the pricing function in Eq. (39) were linear, \( h(X_t; \theta) = h_0(\theta) + H(\theta)X_t \), the Kalman filter would provide efficient estimates of the conditional mean and variance of the state vector. Let \( \hat{X}_{t-1} = \hat{E}_{t-1}[X_t] \) and \( \hat{Z}_{t-1} = \hat{E}_{t-1}[Z_t] \) denote the expectation of \( X_t \) and \( Z_t \) respectively, using information up to and including time \( t-1 \), and let \( P_{t-1} \) and \( F_{t-1} \) denote the corresponding error covariance matrices. Furthermore, let \( \hat{X}_t = \hat{E}_t[X_t] \) denote the expectation of \( X_t \) including information at time \( t \), and let \( P_t \) denote the corresponding error covariance matrix. The Kalman filter consists of two steps: prediction and update. In the prediction step, \( \hat{X}_{t-1} \) and \( P_{t-1} \) are given by
\[ \hat{X}_{t-1} = P_0 + \Phi_X \hat{X}_{t-1} \] (96)
\[ P_{t-1} = P_0 + \Phi_X P_{t-1} \Phi_X^T + Q_t. \] (97)

The change of order of differentiation and expectation is justified by dominated convergence. Indeed, it follows from Duffie, Filipović, and Schachermayer (2003, Theorem 2.16) that \( E^Q\left[ \exp\left[ -\int_{t_0}^{T} \lambda(t,s) ds \right] \lambda(t_0, u) \right] \) is finite for all \( u \) in some neighborhood of zero.
and \( \hat{Z}_{t-1} \) and \( F_{t-1} \) are in turn given by

\[
\hat{Z}_{t-1} = h(\hat{X}_{t-1}; \theta)
\]

\[
F_{t-1} = HP_{t-1}H^T + \Sigma.
\]

In the update step, the estimate of the state vector is refined based on the difference between predicted and observed quantities, with \( \hat{X}_t = E_t[X_t] \) and \( P_t \) given by

\[
\hat{X}_t = \hat{X}_{t-1} + W_t (Z_t - \hat{Z}_{t-1})
\]

\[
P_t = P_{t-1} - W_t F_{t-1} W_t^T,
\]

where

\[
W_t = P_{t-1} H^T F_{t-1}^{-1}
\]

is the covariance between pricing and filtering errors.

In our setting, the pricing function is nonlinear for all the instruments included in the estimation, and the Kalman filter has to be modified. Nonlinear state space systems have traditionally been handled with the extended Kalman filter, which effectively linearizes the measure equation around the predicted state. However, in recent years the unscented Kalman filter has emerged as a very attractive alternative. Instead of approximating the measurement equation, it uses the true nonlinear measurement equation and instead approximates the distribution of the state vector with a deterministically chosen set of sample points, called sigma points, that completely capture the mean and covariance of the state vector. When propagated through the nonlinear pricing function, the sigma points capture the mean and covariance of the data accurately to the second order (third order for true Gaussian states) for any nonlinearity.\(^{40}\)

More specifically, a set of \( 2L + 1 \) sigma points and associated weights are selected according to the following scheme:

\[
\hat{X}_{t-1}^0 = \hat{X}_{t-1}, \quad w^0 = \frac{\kappa}{L + \kappa}
\]

\[
\hat{X}_{t-1}^i = \hat{X}_{t-1} + \left( \sqrt{(L + \kappa)P_{t-1}} \right)_i, \quad w^i = \frac{1}{2(L + \kappa)}, \quad i = 1, \ldots, L
\]

\[
\hat{X}_{t-1}^i = \hat{X}_{t-1} - \left( \sqrt{(L + \kappa)P_{t-1}} \right)_i, \quad w^i = \frac{1}{2(L + \kappa)}, \quad i = L + 1, \ldots, 2L,
\]

where \( L \) is the dimension of \( \hat{X}_{t-1} \), \( \kappa \) is a scaling parameter, \( w^i \) is the weight associated with the \( i \)th sigma-point, and \( \left( \sqrt{(L + \kappa)P_{t-1}} \right)_i \) is the \( i \)th column of the matrix square root. Then, in the prediction step, Eqs. \((89)\) and \((90)\) are replaced by

\[
\hat{Z}_{t-1} = \sum_{i=0}^{2L} w^i h(\hat{X}_{t-1}^i; \theta)
\]

\[
F_{t-1} = \sum_{i=0}^{2L} w^i \left( h(\hat{X}_{t-1}^i; \theta) - \hat{Z}_{t-1} \right) \left( h(\hat{X}_{t-1}^i; \theta) - \hat{Z}_{t-1} \right)^\top + \Sigma.
\]

The update step is still given by Eqs. \((91)\) and \((92)\), but with \( W_t \) computed as

\[
W_t = \sum_{i=0}^{2L} w^i \left( h(\hat{X}_{t-1}^i; \theta) - \hat{Z}_{t-1} \right) F_{t-1}^{-1}.
\]

When implementing the Kalman filter, we follow usual practice and initialize \( \hat{X}_t \) and \( P_t \) at their unconditional values. Another implementation issue is that a subset of the state variables \( (\nu_t, \mu_t, \xi_t, \text{and } \epsilon_t) \) are restricted to be non-negative, but because this is not taken into account in the update step \((91)\), negative values could occur. In these cases, we replace negative values in \( \hat{X}_t \) by zeros. This solution appears to be fairly standard in the empirical term structure literature and is discussed in detail in Chen and Scott (2003).

In terms of maximizing the log-likelihood function, we initially apply the Nelder-Mead algorithm and later switch to the gradient-based BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm. The optimization is repeated with several different plausible initial parameter guesses, to minimize the risk of not reaching the global optimum.

**Appendix E. Supplementary data**

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jfineco.2013.03.014.

**References**


Federal Reserve Bank of New York and Massachusetts Institute of Technology.


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\(^{40}\) For comparison, the extended Kalman filter estimates the mean and covariance accurately to the first order. The computational costs of the extended Kalman filter and the unscented Kalman filter are of the same order of magnitude.