Investors’ Attention and Stock Market Volatility

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Michael Hasler

swiss:finance:institute

Princeton Workshop, Lausanne 2011
Prerequisites
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Consider the dividend process $\frac{d\delta_t}{\delta_t} = f_t dt + \sigma_\delta dZ^\delta_t$: 
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Price of the short-term asset

Price of the long-term asset
Prerequisites

Consider the dividend process \( \frac{d\delta_t}{\delta_t} = f_t dt + \sigma_\delta dZ^\delta_t \):
Motivation I

Source: Binsbergen et al. (forthcoming, AER 2011).
Our Aim

To show that the *fluctuating investors’ attention* explain the behavior of the short-term asset returns and of the market returns.
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To show that the *fluctuating investors’ attention* explain the behavior of the short-term asset returns and of the market returns.

Our Model

A pure exchange Lucas economy with an unobservable fundamental and a signal (*flow of news*). The attention is modeled as the correlation between the unobserved fundamental and the signal.
Outline

I Model

II Results

III Conclusions
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II Results

III Conclusions
The risky asset is a claim to the dividend process $\delta$:

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• The dividend expected growth rate, $f$, is unobservable and behaves according to:

$$df_t = \lambda \left( \bar{f} - f_t \right) dt + \sigma_f dZ^f_t$$
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The representative agent observes a signal, $s$, with the following dynamics:

$$ds_t = \Phi_t dZ^f_t + \sqrt{1 - \Phi_t^2} dZ^s_t$$
Investors’ Attention: A Sample Path

\[ \frac{d\delta_t}{\delta_t} = f_t dt + \sigma_\delta dZ^\delta_t \]
Investors’ Attention: A Sample Path

\[ \phi_t = \int_0^t e^{-\omega(t-u)} \frac{d\delta_u}{\delta_u} \]

\[ \frac{d\delta_t}{\delta_t} = f_t dt + \sigma_\delta dZ_\delta^\delta \]
**Investors’ Attention: A Sample Path**

\[ \Phi_t \equiv \frac{\psi}{\psi - (1 - \psi)e^{\Lambda(\phi_t - f/\omega)}} \]

\[ \phi_t = \int_0^t e^{-\omega(t-u)} \frac{d\delta_u}{\delta_u} \]

\[ \frac{d\delta_t}{\delta_t} = f_t dt + \sigma_\delta dZ^\delta_t \]
The dynamics of the post filtering state vector:

\[
\frac{d\delta_t}{\delta_t} = \hat{f}_t dt + (\sigma_\delta 0) dW_t
\]

\[
d\hat{f}_t = \lambda \left( \bar{f} - \hat{f}_t \right) dt + \left( \frac{\gamma_t}{\sigma_\delta} \sigma_f \Phi_t \right) dW_t
\]

\[
d\phi_t = \omega \left( \frac{\hat{f}_t}{\omega} - \phi_t \right) dt + (\sigma_\delta 0) dW_t
\]

\[
d\gamma_t = \left( \sigma_f^2 \left( 1 - \Phi_t^2 \right) - 2\lambda \gamma_t - \frac{\gamma_t^2}{\sigma_\delta^2} \right) dt
\]
### GMM Estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std Error</th>
<th>t-stat</th>
<th>p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\delta$</td>
<td>0.015</td>
<td>0.001347</td>
<td>11.18</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{f}$</td>
<td>0.026</td>
<td>0.002937</td>
<td>8.98</td>
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<tr>
<td>$\lambda$</td>
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<td>0.290070</td>
<td>2.97</td>
<td>0.003</td>
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<tr>
<td>$\sigma_f$</td>
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<tr>
<td>$\psi$</td>
<td>0.524</td>
<td>0.052686</td>
<td>9.95</td>
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</table>
Equilibrium

- We compute the equilibrium using the techniques from Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989).
Equilibrium

- We compute the equilibrium using the techniques from Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989).
- The state vector is not affine ⇒ we perform an accurate quadratic approximation.

\[
S_t = E_t \int_0^\infty \xi_s \delta_s ds
\]

The stock return volatility is computed by applying Itô's lemma:

\[
\sigma_t = \frac{1}{S_t} \frac{\partial S_t}{\partial x_t} \text{diff}(x_T t)
\]
Equilibrium

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- After the approximation, we can use the theory of affine processes to compute the price of the risky asset:

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$$\sigma_t = \frac{1}{S_t} \frac{\partial S_t}{\partial x_t} \text{diff} \left( x_t^T \right)$$
Outline

I Model

II Results

III Conclusions
Fundamental Volatility and Market Volatility

![Scatter Plot and Linear Fit](image)

- **Scatter Plot**
- **Linear Fit**

<table>
<thead>
<tr>
<th>Estimate</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0003</td>
<td>-248.192</td>
<td>0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.0054</td>
<td>633.573</td>
<td>0</td>
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$R^2 = 0.988$
Fundamental Volatility and Market Volatility

![Scatter Plot and Linear Fit](image)

### Results

#### Fundamental Volatility and Market Volatility

**Scatter Plot and Linear Fit**

#### Estimation

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#### R-squared

- $R^2 = 0.988$
Fundamental Volatility and Market Volatility

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & Estimate & Standard Error & t Statistic & P-Value & \(R^2\) \\
\hline
\(\alpha\) & -0.0704 & 0.0003 & -248.192 & 0 & 0.988 \\
\(\beta\) & 3.43696 & 0.0054 & 633.573 & 0 & \\
\hline
\end{tabular}
\end{table}
Investors’ Attention and Market Volatility

![Scatter Plot with Quadratic Fit]

Estimate | Standard Error | t Statistic | P-Value
--- | --- | --- | ---
$\alpha$ | 0.0854 | 595.3390 | 0
$\beta_1$ | -0.0120 | -19.3191 | 0
$\beta_2$ | 0.0520 | 100.759 | 0
Investors’ Attention and Market Volatility

Results

Scatter Plot

Quadratic Fit

Estimate Standard Error t Statistic P-Value

$\alpha = 0.0854 \quad 0.0001 \quad 595.3390 \quad 0 \quad 0.979$

$\beta_1 = -0.0120 \quad 0.0006 \quad -19.3191 \quad 0$

$\beta_2 = 0.0520 \quad 0.0005 \quad 100.759 \quad 0$

Attention and Volatility

Andrei and Hasler

Princeton Workshop 2011
Results

Investors’ Attention and Market Volatility

![Scatter Plot with Quadratic Fit](scatter_plot.png)

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One Simulated Path

![Graph showing the Simulated Path of Short Term Asset's Price and Dividend](image-url)
Short Term Asset Vol / Market Vol

![Scatter Plot](image)

- **Volatility Ratio**
- **Scatter Plot**
- **Linear Fit**

<table>
<thead>
<tr>
<th>Attention</th>
<th>Volatility Ratio</th>
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<tbody>
<tr>
<td>0</td>
<td>0.46</td>
</tr>
<tr>
<td>0.2</td>
<td>0.48</td>
</tr>
<tr>
<td>0.4</td>
<td>0.50</td>
</tr>
<tr>
<td>0.6</td>
<td>0.52</td>
</tr>
<tr>
<td>0.8</td>
<td>0.54</td>
</tr>
<tr>
<td>1</td>
<td>0.56</td>
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</tbody>
</table>

**Estimates**

- **$r^2$**
  - 0.4553
  - 496.583
  - 0
  - 0.901
- **$\alpha$**
  - 0.1008
  - 76.287
  - 0

**Attention and Volatility Andrei and Hasler Princeton Workshop 2011**
Short Term Asset Vol / Market Vol

Attention

Volatility Ratio

Scatter Plot

Linear Fit

Estimate  t Statistic  P-Value

$\alpha$  0.4553  496.583  0  0.901

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Results

Short Term Asset Vol / Market Vol

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## CAPM: Short Term Asset

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**Return predictability: Market Returns**

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<tr>
<td>$\alpha$</td>
<td>0.0165</td>
<td>4.8229</td>
<td>0</td>
<td>0.005</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.0055</td>
<td>-4.732</td>
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### Return predictability: Short Term Asset Returns

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<tr>
<td>$\alpha$</td>
<td>0.0068</td>
<td>9.346</td>
<td>0</td>
<td>0.017</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.0101</td>
<td>-8.982</td>
<td>0</td>
<td></td>
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Outline

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II Results

III Conclusions
Conclusions

- In a pure exchange economy with an unobservable fundamental, fluctuating attention generates GARCH effects both for the market returns and for the short-term asset returns.
- The volatility is low when the attention is low and vice versa.
- The short term asset volatility increases more with the attention than the stock volatility.
- The short-term asset has a $\beta$ lower than 1 and its returns are predictable, as recent empirical results suggest.
- Additional implications that we explore in subsequent work: comovement of asset returns, amplification of the difference of beliefs.