Deep Hedging
Machine-driven trading of derivatives under market frictions

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Outline

● Models in an exotic derivatives business
● Teaching a machine to think like a trader
● First steps: toy model trading
● Moving further into the real world
How are models used in an exotic derivatives business?

Pricing new trades

- Classical risk-neutral models are ubiquitous

\[ V_0 = E^Q \left[ \frac{V_T}{B_T} \right] \]

- Disregard any existing portfolio and price the derivative under the assumption that perfect replication is possible

- Apply **local** adjustments: hedging costs (trader’s estimate), model limitation adjustments, …

- For larger trades, consider **global** adjustments depending on existing portfolio: credit charge, concentration charge, etc.
How are models used in an exotic derivatives business?

**Hedging**

- Compute the price with the usual classical model

\[ V_0 = E^Q \left[ \frac{V_T}{B_T} \right] \]

- Then compute “greeks”

\[ \frac{\partial V_0}{\partial X} \]

  - For factors which are stochastic in the model, and parameters which aren’t (e.g. interest rates in a local volatility model)

- Based on the greeks, decide which hedging instruments to buy/sell

  - The right hedge is *not* just the model risk
  - Traders adjust the actual traded risk with “experience/skill”
  - He/she needs to be aware of transaction costs, market dynamics (such as vol-spot correlation), concentration and liquidity risk…
How are models used in an exotic derivatives business?

Apply constraints

- Internal: control the risks we take, ensure efficient use of capital
- External: regulatory, legal

Examples:

- Direct risk and stress limits based on the model:
  \[
  L < \frac{\partial V_0}{\partial X} < U
  \]
  \[
  V_0(X) - V_0(X + \text{stress}) < M
  \]

- Limits on CVaR
- Capital requirements – many determining factors
- Short selling bans

These constraints are not usually part of the valuation model
Beyond the classical approach

- We want to increase automation in the business
- The risk management model needs to do more

- It should include transaction costs, lack of liquidity, and constraints
- This means accepting that perfect replication is impossible…
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Teaching a machine to think like a trader

Trading inputs and outputs

- Risk management

Portfolio of derivatives

Market state: prices of hedging instruments, cost of trading, liquidity,

Alternative Data: News, historic trading pattern

In the real world, we accept risk when we trade

Buy/sell decision

Aiming to optimize future PnL distribution
Teaching a machine to think like a trader

Trading inputs and outputs

- Pricing

Existing portfolio of derivatives

Market state: prices of hedging instruments, cost of trading, liquidity,

Alternative Data: News, historic trading pattern

Aim to charge enough that the trade has a positive impact on the hedged portfolio’s profit/loss distribution

Pricing is subjective and nonlinear

New trade details

New trade price

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How to compare profit/loss distributions?

- We could use classical “Markoviz” portfolio optimization
  
  - Maximize expected return while penalizing variance
    
    \[ E(w_T) = \mathbb{E}[w_T] - \lambda \text{Var}[w_T] \]

- Note that \( E(w_T) \) is a function on the distribution of terminal wealth

- But mean-variance is not a good measure if terminal wealth is not normally distributed
  
  - Exist on-monotone cases where \( X \geq Y \) but \( E(X) < E(Y) \)
How to compare profit/loss distributions?

- What are sensible conditions for our risk-adjusted value function $E(w_T)$?

- Monotonicity
  \[ X \geq Y \Rightarrow E(X) \geq E(Y) \]

- Convexity
  \[ E(\alpha X + (1 - \alpha)Y) \geq \alpha E(X) + (1 - \alpha)E(Y), \alpha \in [0,1] \]

- Cash invariance
  \[ E(X + c) = E(X) + c \]

\(-E(\cdot)\) is a **convex risk measure**

We are risk-averse

More is better

There is no risk adjustment for cash
How to compare profit/loss distributions?

- We will mostly use the *entropic* measure: \( E(X) = -\frac{1}{\lambda} \ln \mathbb{E}[e^{-\lambda X}] \)
- Equivalent to mean-variance for small risk-aversion parameter \( \lambda \):
  \[
  -\frac{1}{\lambda} \ln \mathbb{E}[e^{-\lambda X}] = \mathbb{E}[X] - \frac{1}{2} \lambda \text{Var}[X] + \cdots
  \]
- Example: \( X \sim \mathcal{N}(0,1) \)
- Plot risk-adjusted value

Bounded by the two extreme risk-adjusted values: risk-neutral and worst-case
Hedging

- We can now express preferences on future profit/loss distributions
- *Hedging* is the act of buying and selling “hedging instruments” to optimize that distribution
- A *hedging strategy* is a function:
  \[ a_t^\pi = a_t^\pi(t, s_t) \]
  - \( \pi \) parameterized the strategy; e.g. as the weights of neural networks
  - Liquid instruments with observable prices
  - State \( s_t \) is the history of everything including our current book
- It tells us how much of each hedging instrument to buy or sell at each time \( t \), for every possible state \( s_t \)
- Not all actions are possible – in general \( a_t^\pi \) will be subject to limits which are also state-dependent (e.g. short-sell constraints)
Hedging

- How does the hedging strategy \( \pi \) contribute to the terminal profit/loss?

\[
w_T(\pi; z) = \sum_{j=1}^{T} \delta_j^\pi \cdot h_j + z_j - \alpha_j^\pi \cdot H_j - c_j^\pi
\]

- \( \delta_j^\pi \): accumulated current positions ("deltas"),
  \( \delta_j^\pi = \delta_{j-1}^\pi + \alpha_j^\pi \)

- \( h_j \): cashflows generated by hedging instruments

- \( z_j \): cashflows from our exotic derivatives portfolio

- \( c_j^\pi \): transaction costs incurred,
  \( c_j^\pi = c(a_j^\pi, s_j) \)

- \( H_j \): mid prices of hedging instruments

Note: all cash flows are discounted
Hedging

- The terminal profit/loss is not deterministic – our task is to optimize it
- That means maximizing

\[ E[w_T(\pi; z)] = E \left[ \sum_{j=1}^{T} \delta_j^\pi \cdot h_j + z_j - a_j^\pi \cdot H_j - c_j^\pi \right] \]

- Two key challenges:
  - How to generate the distribution
  - How to optimize the hedging function

We apply the value function to the distribution over future real-world states

We need to find the optimal function \( a_j^\pi \) that meets our constraints.

**Path dependency**
The feasible set of allowed actions \( a_j^\pi \) depends on past decisions \( a_{j-1}^\pi \ldots a_0^\pi \).
Market simulators

- To generate the profit/loss distribution for a given strategy, we need to simulate future states of the world
  - Prices of available hedging instruments
  - Corresponding cash flows from exotic derivatives
- We should be simulating in the real-world measure, not $\mathbb{Q}$
  - The real world has “statistical arbitrage”, i.e. with normal risk aversion some trades statistically make money (e.g. shorting options, sell long-dated bonds).
  - Deep Hedging will attempt to take advantage of these opportunities.
    - Absence of arbitrage $\Rightarrow$ absence of statistical arbitrage (e.g. GBM with drift)
    - Existence of arbitrage $\Rightarrow$ existence of statistical arbitrage (e.g. if risk-aversion is very high)
- For the experiments presented here, we will use classical $\mathbb{Q}$ models
Optimizing the hedging strategy

- We use a deep neural network to represent the strategy $a_j^\pi$

![Diagram of a neural network with inputs and outputs]

- Inputs:
  - Current market state
  - Relevant product state variables

- LSTM cells to capture path dependence
  - Potentially important when we have transaction costs
  - Allows memory of our previous hedging decisions

Prices of all hedging instruments
Harvested automatically
Deep Hedging

Simulate many future states of the world

Compute risk-adjusted value on a batch of paths for neural network strategy

Update network parameters by following gradient

\[ \frac{\partial E(w_T(\pi; z))}{\partial W_i} \]
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Start simple

- Hedge a short at-the-money 30-day European call
  \[ z_T = -(S_T - K)^+ \]
- Generate paths in Black-Scholes
- Check the impact of transaction costs, risk aversion, and risk limits
Toy model trading

Risk Aversion (Entropy)

- Vanilla option delta
- 10bps cost
- No limits
- Entropic value
- Black-Scholes simulator
Toy model trading

Transaction costs (Entropy)

- Vanilla option delta
- No limits
- Entropic value
- Risk aversion 10
- Black-Scholes simulator
Trading limits

- Vanilla option delta
- 0.01% proportional cost
- Entropic value
- Risk aversion 10
- Black-Scholes simulator
Risk measure

- Vanilla option PnL distribution
- 0.01% cost
- No limits
- Black-Scholes simulator
Forward-starting options

- Increase the complexity: simulate with Heston model
- Compute optimal spot-only hedges for forward-starting options
  \[ z_T = \max\left(0, \frac{S_T}{S_t} - K\right) \]
- 15-day forward start
- 45-day maturity
- Daily hedging
- Entropic value with risk aversion 50
- No limits
Forward-starting options

- Impact of transaction costs on incremental and total delta
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“Autocallable” note

- Popular retail payoff:
  - Client is short a down-and-in put paid at maturity
  - Upper knockout barrier
  - Fixed coupons until KO
- 0.1% transaction costs
- No limits
- Risk aversion 20
- Entropic value
- Monthly hedging
Portfolio of autocallables

- Based on a real portfolio
- 0.1% transaction costs
- No limits
- Local volatility simulator
- Monthly hedging

Training convergence
Moving further into the real world

## Market simulator

- Go beyond “classical” models – build a statistical model instead

**Challenge:** avoid arbitrage when simulating options

- Simulate discrete local volatilities to avoid static arbitrage
- Dynamic arbitrage still a challenge
Market simulator

- How do we build a full statistical market simulator that reflects real-world drifts but is arbitrage-free?
  - Does it need to be fully arbitrage-free?
- What about rare events?
  - A statistical simulator is not likely to capture these well
- In particular, we want the model to behave well in a stress scenario, and to price in the risk appropriately
  - Should we insert stress events into the market simulator?
  - With what probability? Historical likelihood?
- For equities we focused on spot and volatility – there’s lots more
  - Rates, spreads, FX, …
Conclusions

- We formalized the task of pricing and managing the risk of an exotic derivatives portfolio
- Obtaining the optimal hedging strategy is a difficult problem
- Representing the strategy as a neural network makes it tractable
  - Optimization typically takes minutes on CPU for the toy examples here
- So far it works for:
  - Vanillas, cliquets, barrier options, large portfolios
  - With transaction costs and risk limits
  - Simulators based on classical pricing models (Black Scholes, local volatility, Heston)
Many more interesting challenges ahead

- Developing statistical ($\mathbb{P}$) market simulators for options
- Do we need to compute hedging strategies all the way to maturity?
  - Can we come up with an efficient way to represent a portfolio of exotics as a state?
- How do we choose our risk measure? Can we derive effective real-world risk-measures from the choices people make?
- Go beyond equities: FX, rates, etc.
- Ultimate goal: automated pricing and hedging of exotic derivatives
References and thanks

● Credits:
  ▪ Hans Buehler, Lukas Gonon, Josef Teichmann, Hans Buehler, Jonathan Kochems, Barani Mohan, Blanka Horvath, Len Bai, Pradeepta Das

● Paper:
  ▪ Note that the architecture has moved on since the paper